THESE

PRESENTEE

A L'UNIVERSITE DE PARIS-SUD

CENTRE D'ORSAY

POUR OBTENIR

LE GRADE DE DOCTEUR ES-SCIENCES PHYSIQUES

PAR

Ignatios ANTONIADIS

SUJET : AU-DELA DU MODELE STANDARD $U(1) \times SU(2) \times SU(3)$

soutenue le : 10 Juin 1983 devant la Commission d'examen

MM. MEYER Philippe — Président
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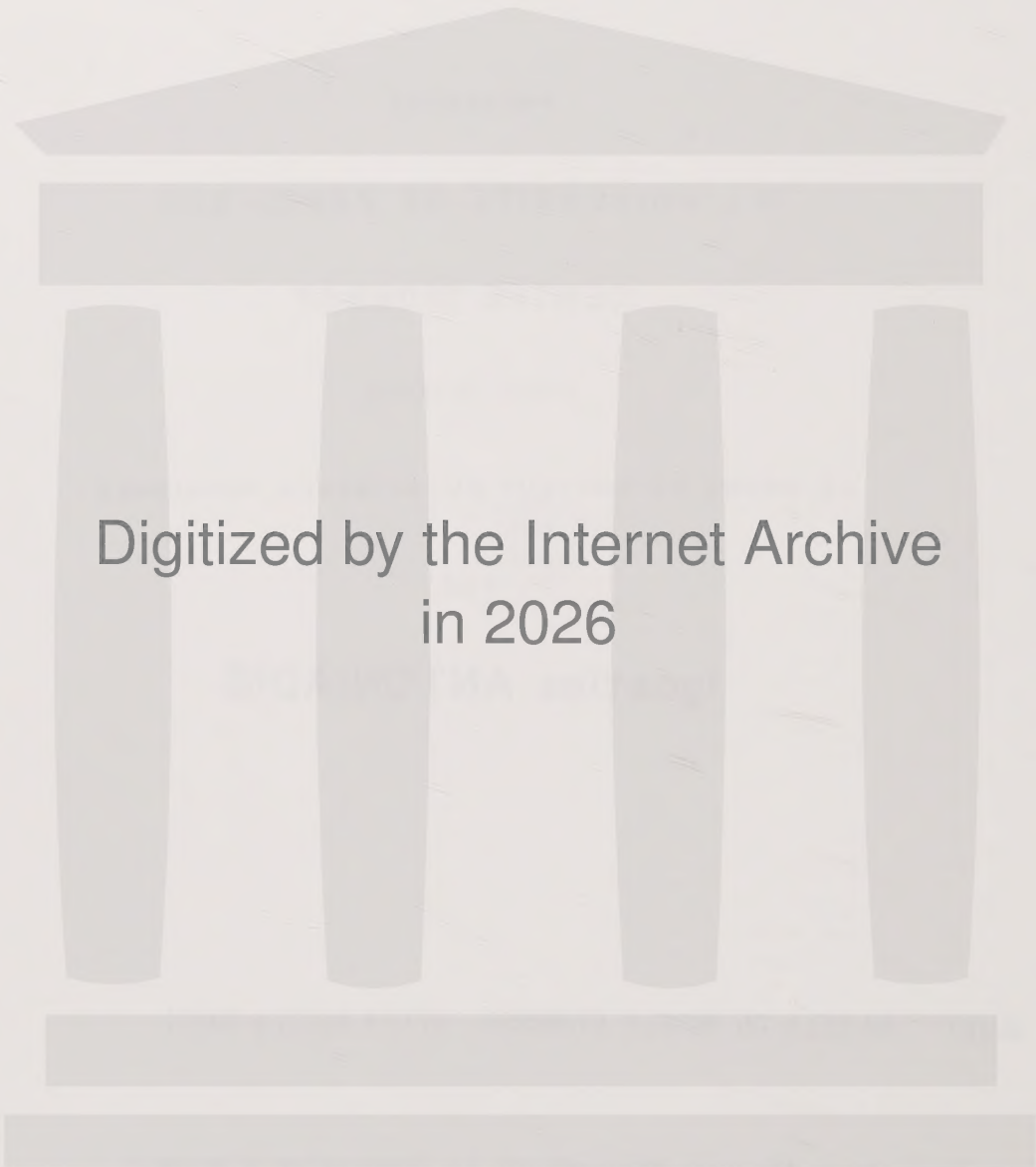
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CHAPITRE I

INTRODUCTION

I - INTRODUCTION

Le plus grand progrès théorique dans la physique des particules élémentaires pendant ces dernières années a été indiscutablement le développement des théories de jauge ; c'est-à-dire des théories qui ont une invariance par rapport à un ensemble de transformations locales (dépendantes de l'espace-temps). Un consensus général s'est établi selon lequel les théories de jauge sont l'outil de base pour l'étude des particules élémentaires.

Aux énergies actuelles, on décrit toutes les interactions entre les particules élémentaires à l'aide d'une théorie quantique des champs basée sur le groupe de jauge $SU(3)_c \times SU(2)_L \times U(1)_y$. Le groupe de couleur $SU(3)_c$ définit la chromodynamique quantique (QCD)[1] qui décrit les interactions fortes, tandis que le groupe $SU(2)_L \times U(1)_y$ décrit ensemble les interactions électromagnétiques et faibles[2]. Un grand nombre des prédictions du modèle standard $SU(3)_c \times SU(2)_L \times U(1)_y$ ont été vérifiées par l'expérience. D'une part, la chromodynamique quantique en théorie des perturbations prédit des résultats, pour une grande variété de processus faisant intervenir des hadrons et des leptons à haute énergie, en fonction d'un seul paramètre ayant les dimensions d'une masse. En outre, dans le secteur non perturbatif, le développement des théories de jauge sur le réseau [3] a fourni une méthode numérique pour calculer le spectre des masses des hadrons et les premiers résultats obtenus sont assez encourageants [4]. D'autre part, au-delà de la théorie de Fermi, le modèle des interactions électro-faibles $SU(2)_L \times U(1)_y$ décrit très bien les expériences où intervient le courant faible neutre, et les propriétés de désintégration des particules charmées. Tout récemment au CERN, dans l'expérience des collisions proton-antiproton avec une énergie dans le centre de masse de 540 GeV, quelques évènements de production du boson W, qui transmet les interactions faibles, ont été mise en évidence [5] !

Bien que tous les problèmes théoriques relatifs au modèle standard n'aient pas été résolus, et qu'on attende des confirmations venant des expériences en cours et à venir, il y a aussi beaucoup de questions qui se posent au-delà du modèle standard : quelle est l'origine du groupe $SU(3) \times SU(2) \times U(1)$? Est-ce que les trois constantes de couplage sont vraiment indépendantes ? Quelle est la situation à plus haute énergie ?

Pourquoi la charge électrique apparaît-elle toujours comme multiple d'une unité ($\pm$ 1/3) ? Existe-t-il des connexions entre les quarks et les leptons ? Combien de générations y-a-t-il ? Les scalaires de Higgs qui sont nécessaires pour la renormalisabilité de la théorie sont-ils fondamentaux ou composés ? Quelle est la relation éventuelle de la gravitation avec les autres interactions ?

Une méthode plus ou moins standard pour répondre à ces questions consiste à chercher une plus grande symétrie qui se manifesterait à haute énergie et qui serait brisée spontanément (ou très doucement) à une certaine échelle de masse telle que la théorie ainsi construite soit compatible avec les phénomènes à basse énergie.

Dans cet esprit, l'exemple le plus caractéristique pour les symétries locales est le processus de "grande unification" [6,7] qui consiste à définir une théorie de jauge ne dépendant que d'une constante de couplage unifiant toutes les interactions à très haute énergie, sauf la gravitation. Le groupe de jauge unifié G est spontanément brisé à une énergie de l'ordre de 10^{15} GeV en SU(3) x SU(2) x U(1). Les théories grandement unifiées expliquent la quantification de la charge électrique et traitent symétriquement les quarks et les leptons qui se trouvent dans la même représentation irréductible du groupe unifié. Elles prédisent [8-10] la valeur de l'angle faible qui décrit les courants neutres ainsi que divers rapports de masses entre les quarks et les leptons. Dans ces théories, le nombre baryonique et le nombre leptonique ne sont pas séparément conservés et par conséquent, le proton est instable avec un temps de vie de l'ordre de 10^{31} ans. Elles ont aussi des conséquences cosmologiques assez intéressantes [11]. Malgré tous ces avantages, les théories grandement unifiées introduisent de nouveaux problèmes : comment choisir le groupe unifié ? Comment lever l'arbitraire qui existe dans le secteur des scalaires de Higgs ? Comment comprendre le "grand désert", sans aucun phénomène nouveau entre 100 et 10^{15} GeV ? Comment faire co-exister naturellement dans une même théorie des particules dont les masses diffèrent par un rapport 10^{13} (problème de la "hiérarchie de jauge") ?

Dans le même but d'unification, mais dans une direction différente, les théoriciens ont introduit la supersymétrie [12] qui autorise des

transformations réciproques des bosons en fermions. Les théories supersymétriques ont été introduites dans l'espoir de relier les bosons de jauge avec les fermions et les scalaires de Higgs. Cependant, pour construire des modèles acceptables phénoménologiquement on doit doubler au moins le spectre des particules actuellement connues [13]. En plus de l'avantage esthétique d'avoir un système où les fermions et les bosons sont reliés, l'ensemble des prédictions de ces théories dépend de très peu de paramètres. On espère aussi que la supersymétrie sera utile pour résoudre le problème de la hiérarchie de jauge dans les théories grandement unifiées [14]. Un grand nombre de problèmes liés à la cohérence interne de ces théories reste cependant à résoudre. Finalement la supersymétrie locale est directement liée à la gravitation ; ce fait renforce l'espoir que la supergravité [15] peut jouer un rôle important pour trouver une théorie unifiant toutes les interactions de la nature.

Dans cette thèse, je présente des travaux relatifs à ces problèmes. En premier lieu, avec mes collègues, j'ai travaillé sur les théories grandement unifiées. Tout d'abord, nous avons trouvé une description précise du phénomène de la brisure spontanée de symétrie basée sur le groupe de renormalisation. Avec ce traitement, on tient exactement compte des effets de différentes masses et en particulier des masses des bosons lourds de jauge et des scalaires de Higgs. Ensuite, nous avons appliqué cette méthode pour calculer les prédictions de la théorie unifiée basée sur le groupe minimal SU(5) en incluant également les effets de la deuxième brisure du groupe SU(2) x U(1). Nous avons prédit la valeur précise de l'angle faible θ_w et de la masse des bosons de jauge lourds qui seraient responsables de la désintégration du proton. Il est assez remarquable que la prédiction pour $\sin^2 \theta_w$ soit en accord avec la valeur expérimentale extraite des expériences du courant neutre.

Nous avons aussi travaillé sur les théories supersymétriques. La chromodynamique quantique évidemment n'est pas supersymétrique. Néanmoins, elle possède une supersymétrie interne si on considère les quarks comme spineurs de Majorana et si on les choisit dans la représentation adjointe. Par conséquent, en faisant un changement numérique des facteurs de groupe, on trouve une symétrie entre les quarks et les gluons. Nous avons montré que

cette symétrie se manifeste en particulier dans une relation simple entre les dimensions anormales des opérateurs de Wilson, qui sont des singlets de saveur, et de "twist" deux. Nous avons vérifié sa validité jusqu'au deuxième ordre de la théorie des perturbations, dans un schéma de renormalisation qui conserve la supersymétrie.

Nous avons ensuite étudié les prédictions des extensions supersymétriques de la chromodynamique quantique. Nous avons étendu les prédictions habituelles de QCD pour les fonctions de structure, jusqu'au deuxième ordre de la théorie des perturbations en présence de gluinos légers (partenaires supersymétriques des gluons). Il se trouve que les modifications induites par les gluinos sur les violations de l'invariance d'échelle prédites par la théorie ne sont pas significativement importantes. En conséquence, les données expérimentales actuelles sont aussi compatibles avec l'existence des gluinos légers. Nous avons aussi extrait la distribution en x de gluinos dans le nucléon. Par la suite, nous avons étudié la production de "jets" à grand moment transverse dans les réactions proton-antiproton de l'expérience SPS au CERN, ainsi que l'asymétrie de polarisation dans le processus de production d'un photon direct. Notre conclusion est que les effets des gluinos sont très petits. En revanche, l'introduction des quarks scalaires (partenaires supersymétriques des quarks) implique des modifications importantes. Pour la production de "jets", on trouve une section efficace deux fois plus grande que celle prédite par QCD. Pour la production du photon direct, on trouve une asymétrie de l'ordre de 10 %, alors que QCD prédit une polarisation nulle.

Nous avons aussi étudié les théories supersymétriques grandement unifiées. Nous avons trouvé une méthode simple pour traiter les effets de masse et nous avons donné la prédiction correspondante pour la masse des bosons de jauge lourds et pour l'angle faible dans le modèle le plus simple. Finalement, nous avons travaillé sur des problèmes théoriques de la renormalisation des théories supersymétriques en terme des composants de superchamps et nous avons trouvé que ces propriétés deviennent extrêmement simples dans les jauges axiales.

Dans la dernière partie de cette thèse, nous présentons un problème dont l'idée maîtresse est l'opposé de celle de la "grande unification".

Supposons qu'à haute énergie il n'y ait aucune symétrie et examinons la possibilité d'obtenir les symétries de la nature comme points fixes attractifs aux basses énergies [16]. Dans certains cas des symétries globales traités auparavant, la propriété d'être un attracteur ou non a permis de privilégier certains groupes de symétrie[17-19]. Afin de pouvoir éventuellement lever l'indétermination théorique du choix du groupe de jauge des interactions entre les particules, nous avons étudié la stabilité infrarouge des théories de jauge. Malheureusement, les résultats obtenus semblent négatifs.

En conclusion, au-delà du modèle standard, beaucoup de difficultés et de problèmes se posent. Les théories grandement unifiées et la supersymétrie constituent une direction probable pour unifier toutes les interactions fondamentales entre les particules élémentaires.

Le plan de cette thèse est le suivant :

Dans le chapitre II, nous rappelons brièvement le modèle standard et ses prédictions principales.

Dans le chapitre III, nous décrivons les motivations et les propriétés générales des théories grandement unifiées et nous présentons une méthode précise permettant de traiter les effets de la brisure spontanée d'un groupe de symétrie G en un produit de facteurs $G_1 \times G_2 \times \ldots$, basée sur le groupe de renormalisation.

Dans le chapitre IV, nous appliquons cette méthode à la théorie unifiée du groupe minimal SU(5). Nous donnons la prédiction de la masse M_x des bosons de jauge lourds qui produisent la désintégration du proton, du temps de vie du proton, et de l'angle faible effectif $\theta_w(\mu)$ dépendant de l'énergie μ.

Au chapitre V, nous étudions un autre problème du modèle standard, celui de la violation forte de la symétrie CP (conjugaison de charge x parité)[20]. Nous discutons les différentes solutions possibles proposées jusqu'à maintenant et en particulier nous donnons des arguments phénoménologiques qui excluent l'existence de l'axion à la Peccei-Quinn[20,21].

Dans le chapitre VI, nous présentons les traits généraux de la supersymétrie, ses propriétés, le formalisme de superchamps et quelques exemples de théories supersymétriques. Nous étudions une symétrie d'origine supersymétrique entre les quarks et les gluons de la chromodynamique quantique ordinaire. Nous examinons finalement le problème de la renormalisation de la supersymétrie N = 1 avec un seul multiplet vecteur de jauge, dans la jauge axiale.

Au chapitre VII, se trouvent les prédictions principales des théories supersymétriques. Nous discutons d'abord la brisure de la supersymétrie et nous décrivons des extensions supersymétriques du modèle standard. Nous examinons les modifications que les extensions supersymétriques de QCD apportent aux violations de l'invariance d'échelle pour les fonctions de structure de la diffusion profondément inélastique, à la production des "jets" à grand moment transverse dans les réactions proton-antiproton avec une énergie dans le référenciel du centre de masse de 540 GeV, et à l'asymétrie de polarisation de la production d'un photon direct dans la même expérience. Nous donnons ensuite les prédictions des théories supersymétriques grandement unifiées en comparaison avec celles des théories grandement unifiées ordinaires.

Dans le dernier chapitre VIII, nous présentons l'idée de l'"anti-grande unification", ses motivations, les résultats précédents et nous étudions la stabilité infrarouge des théories de jauge.

CHAPITRE II

MODELE STANDARD $SU(3)_c \times SU(2)_L \times U(1)_y$ ET SES PREDICTIONS PRINCIPALES

II - MODELE STANDARD $SU(3)_c \times SU(2)_L \times U(1)_y$ ET SES PREDICTIONS PRINCIPALES

II.1 - La chromodynamique quantique

La chromodynamique quantique (QCD) est une théorie de jauge basée sur le groupe de couleur $SU(3)_c$[1]. Elle décrit un octet des gluons et N_f triplets des quarks, N_f étant le nombre de saveurs. Elle contient un seul paramètre Λ dont la dimension est celle d'une masse, et dont la valeur numérique dépend du schéma de renormalisation choisi. Expérimentalement, la valeur de $\Lambda_{\overline{MS}}$ est de l'ordre de 200 MeV[22]. La chromodynamique quantique, comme toutes les théories de jauge non abéliennes qui ne contiennent pas trop de "champs de matière" (fermions et scalaires), a la propriété remarquable d'être "asymptotiquement libre"[23]. Ceci signifie que la constante de couplage effective qui caractérise l'intensité des forces entre constituants devient négligeable à très courtes distances (à haute énergie) et infiniment forte à grandes distances, où les quarks et les gluons sont confinés à l'intérieur des hadrons.

La quasi-totalité des prédictions de la chromodynamique quantique perturbative sont obtenues grâce au théorème de "factorisation"[24] suivant lequel la dynamique de la plupart des réactions d'énergie beaucoup plus grande que Λ , est séparée en deux régimes :

- celui des petites distances qui peut être calculé en théorie des perturbations en utilisant le groupe de renormalisation[25] et la propriété de la liberté asymptotique ;
- celui des grandes distances qui est responsable de la formation des hadrons et qui ne peut qu'être paramétrisé à l'aide de fonctions déterminées expérimentalement.

Dans cet esprit, la chromodynamique quantique apparaît comme la théorie qui généralise [26] l'image simple du modèle des partons[27]. Elle prédit assez bien[28] :

(i) Le rapport R de la section efficace d'annihilation électron-positron en hadrons et celle d'annihilation en muon-antimuon.

(ii) Les violations de l'invariance d'échelle observées dans la diffusion profondément inélastique des leptons sur les nucléons et dans les

processus d'annihilation semi-inclusive électron-positron.

(iii) Les sections efficaces des réactions hadron-hadron à haute énergie et à grand moment transverse des particules sortantes, observées .

(iv) La structure en "jets" aux processus hadron-hadron et électron-positron.

D'autre part, il n'existe aucun moyen analytique de traiter le problème de formation des états liés (qui est un problème non perturbatif) et de vérifier si la chromodynamique quantique conduit effectivement au confinement des quarks, et reproduit le spectre de masses des hadrons. Néanmoins, pendant les dernières années le grand développement des techniques de calcul sur le réseau[3] semble une méthode prometteuse pour approcher ces problèmes. Les premiers résultats obtenus[4], en utilisant un réseau relativement petit, vont dans la bonne direction et renforcent l'espoir que la chromodynamique quantique est la théorie fondamentale des interactions fortes.

II.2 - Le modèle des interactions électro-faibles

Ce modèle[2] qui unifie les interactions électromagnétiques et faibles est une théorie qui réalise la brisure spontanée de la symétrie de jauge $SU(2)_L \times U(1)_y$ en $U(1)_{em}$. Il décrit quatre bosons de jauge, un triplet $(W_\mu^+, W_\mu^-, W_\mu^3)$ et un singlet B_μ de $SU(2)_L$. Les bosons neutres physiques sont le photon A_μ et celui qui transmet les courants neutres Z_μ° . On les trouve en faisant une rotation des bosons non physiques B_μ et W_μ^3 d'un angle θ_w appelé l'angle faible.

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = R_\theta \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \quad \text{avec } R_\theta = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \qquad \text{(II.1)}$$

Les bosons de jauge W_μ^+, W_μ^- et Z_μ° acquièrent des masses lourdes par l'intermédiaire du mécanisme de Higgs[29]. Dans le modèle le plus simple, il y a un seul doublet complexe de scalaires. Après la brisure spontanée de la symétrie, il apparaît trois bosons de Goldstone[30] qui correspondent aux trois générateurs brisés du groupe $SU(2)_L \times U(1)_y$ en $U(1)_{em}$. Mais à cause du mécanisme de Higgs, ces trois bosons de Goldstone disparaissent du spectre de la théorie, tandis que les bosons $W^\pm$ et Z° deviennent massifs. Le mécanisme de Higgs est le seul moyen connu qui

permette d'avoir des bosons de jauge massifs en gardant la renormalisabilité de la théorie (dans le cas non abélien). Le prix à payer est l'existence nécessaire de particules scalaires fondamentales ayant des interactions entre elles, avec des masses et des couplages en général arbitraires. Dans le modèle standard qui est le plus simple, il reste un scalaire de Higgs physique, sa masse et ses couplages de Yukawa avec les fermions étant des paramètres libres.

Examinons maintenant les fermions. Leurs composantes gauches forment des doublets de $SU(2)_L$ tandis que leurs composantes droites sont des singulets. Tous les fermions acquièrent des masses après la brisure spontanée de symétrie. La condition de ne pas avoir d'anomalie[31] - nécessaire pour le renormalisabilité de la théorie - impose que le nombre de quarks (modulo les triplets de couleur) doit être le même que le nombre de leptons[32] :

$$\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ b \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L , \ \ldots \ u_R, d_R, c_R, s_R, b_R, t_R, \ \ldots \qquad \text{(II.2a)}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L , \ \ldots \ e_R, \mu_R, \tau_R, \ \ldots \qquad \text{(II.2b)}$$

Avec trois générations de familles et des neutrinos de masse nulle, ce modèle a treize paramètres libres qui deviennent seize si l'on tient compte des trois angles de mélange[33] entre les six saveurs de quarks. Les 13 paramètres sont les neuf masses des fermions, la charge électrique e, l'angle faible θ_w, la masse M_w du boson $W^{\pm}$ et la masse du scalaire de Higgs M_H. La contrainte obtenue imposant que la théorie reproduise bien à basse énergie les résultats de l'interaction de Fermi des courants chargés ne laisse indéterminés que deux paramètres, l'angle faible θ_w et la masse du boson de Higgs M_H, en plus de la masse du quark top qui n'est pas encore trouvé expérimentalement.

Le modèle standard prédit l'existence de courants neutres ayant une intensité proportionnelle à $\dfrac{e}{\sin^2\theta_w \cos^2\theta_w} [T_3 - \sin^2\theta_w Q]$ où T_3 est l'isospin faible et Q la charge électrique du fermion considéré. Ces courants neutres ont été effectivement observés expérimentalement avec un $\sin^2\theta_w$ environ égal à .22[34].

Après la découverte récente du boson de jauge intermédiaire W dans l'expérience $p - \bar{p}$ au CERN[5], on attend pour les années suivantes une grande variété de résultats expérimentaux permettant de tester toutes les propriétés et les prédictions du modèle standard. En particulier quelques questions très importantes d'un point de vue théorique sont :

(i) Où le quark top se trouve-t-il ?

(ii) Quelle est la structure du secteur des scalaires de Higgs, nécessaires pour la renormalisabilité de la théorie ?

(iii) La théorie des interactions électro-faibles est-elle effectivement décrite par le modèle le plus simple, ou la nature est-elle plus compliquée (possibilité d'existence de plusieurs Z° etc) ?

CHAPITRE III

GRANDE UNIFICATION

III - GRANDE UNIFICATION

III.1 - Motivations et propriétés générales des théories grandement unifiées

Une des idées les plus attrayantes de ces dernières années est celle de l'unification de toutes les interactions entre particules élémentaires. L'hypothèse de la "grande unification"[6,7] introduit un groupe semi-simple G qui est spontanément brisé en une ou plusieurs étapes en $SU(3)_c \times SU(2)_L \times U(1)_y$, lui même brisé par la suite en $SU(3)_c \times U(1)_{em}$

$$G \xrightarrow[M_1 > M_2 > \cdots > M_x]{} SU(3)_c \times SU(2)_L \times U(1)_y \xrightarrow[M_w \sim 100\ GeV]{} SU(3)_c \times U(1)_{em}$$

La masse M_x doit être très grande pour qu'aux basses énergies, les interactions autres que celles du modèle standard soient pratiquement inobservables.

Dans les théories grandement unifiées basées sur un groupe simple G, il y a une seule constante de couplage g; mais les intensités des interactions fortes, faibles et électromagnétiques sont très différentes. Cette dissociation[8] de la constante de couplage unique g en trois constantes différentes vient justement du phénomène de la brisure spontanée de symétrie.

Une autre conséquence de ces théories est la quantification de la charge électrique Q. En effet Q doit correspondre à un générateur non brisé du groupe unifié G[8]. Par conséquent TrQ = 0, ce qui implique que la somme de toutes les charges des particules à l'intérieur d'une représentation s'annule. C'est pourquoi les leptons et les quarks de chaque famille doivent nécessairement appartenir à la même représentation. Pour illustrer un peu les idées, examinons la théorie unifiée la plus simple.

Cherchons un groupe simple minimal G qui contient SU(3) x SU(2) x U(1). Son rang sera alors égal à 4 qui est celui de SU(3) x SU(2) x U(1). (Le rang d'un groupe est le nombre maximal des générateurs commutant entre eux). Le seul groupe qui satisfait ces conditions tout en ayant des représentations complexes est SU(5)[7].

Dans SU(5), les fermions de chaque famille appartiennent aux représentations $\bar{5}$ et 10. Leur décomposition en termes des nombres quantiques de $SU(3)_c \times SU(2)_L$ est :

$$\bar{5} = (\bar{3},1) + (1,2)$$
$$= (d_i^c)_L + (e, \nu_e)_L \quad \text{et} \tag{III.1a}$$

$$10 = (\bar{3},1) + (3,2) + (1,1)$$
$$= (u_i^c)_L + (u_i, d_i)_L + e_L^c \ , \tag{III.1b}$$

où i = 1,2,3 compte la couleur, "L" indique les composantes gauches des spineurs et "c" est l'opération de conjugaison de charge. Les composantes gauches d'un spineur transformé par la conjugaison de charge correspondent aux composantes droites de l'antiparticule.

Les bosons de jauge appartiennent à la représentation adjointe du groupe :

$$24 = (8,1) + (1,3) + (1,1) + (3,2) + (\bar{3},2) \tag{III.2}$$

Ils se décomposent en 8 gluons de $SU(3)_c$, $W_\mu^\pm$, W_μ^3 et B_μ de $SU(2)_L \times U(1)_y$ et ensuite 6 bosons lourds et leurs antiparticules qui sont des triplets de couleur et des doublets de SU(2) :

$$(X_i^+, Y_i^+) \quad \text{et} \quad (X_i^-, Y_i^-) \quad , \ i = 1,2,3 \ .$$

Finalement, il y a le secteur de Higgs. Pour briser SU(5) en $SU(3)_c \times SU(2)_L \times U(1)_y$ on a besoin d'un multiplet Φ de Higgs dans la représentation adjointe. Représentons-le comme une matrice hermitique de trace nulle. Ce multiplet doit avoir une valeur moyenne dans le vide non nulle, proportionnelle à la matrice correspondante au générateur de $U(1)_y$:

$$\langle \Phi \rangle \sim \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ 0 & & & & -3/2 \end{pmatrix} \times U \tag{III.3}$$

où U est de l'ordre de 10^{15}GeV. Ce multiplet ne peut cependant pas donner une masse aux fermions. La raison est que Φ , qui est un multiplet 24, ne peut pas former des invariants de groupe(singulets) avec les représentations de fermions $\bar{5}$ et 10 puisque :

$$\bar{5} \times 10 = 5 + 45 \text{ et}$$
$$10 \times 10 = \bar{5} + \overline{45} + 50 \quad . \tag{III.4}$$

La façon la plus économique consiste à introduire un multiplet de Higgs supplémentaire, H,dans la représentation fondamentale 5 (complexe) :

$$5 = (3,1) + (1,2) \tag{III.5}$$

Les trois premiers acquièrent une masse très lourde tandis que les deux derniers constituent le doublet du modèle standard, grâce auquel $SU(2)_L \times U(1)_y$ se brise en $U(1)_{em}$. C'est pourquoi H doit prendre une valeur moyenne non nulle dans le vide :

$$\langle H \rangle \sim \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \times U_o \quad , \tag{III.6}$$

avec U_o de l'ordre de 10^2GeV. La charge électrique Q correspond au générateur du groupe non brisé $U(1)_{em}$:

$$Q = \begin{pmatrix} -1/3 & & & 0 & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1 & \\ 0 & & & & 0 \end{pmatrix} \tag{III.7}$$

En utilisant la forme de Q (III.7), on trouve facilement les charges des différentes particules

$Q_{e^-} = -1$, $Q_d = \frac{1}{3} Q_{e^-}$, $Q_u = \frac{2}{3} Q_{e^+}$, $Q_\nu = 0$, $Q_{X^+} = \frac{4}{3}$, $Q_{Y^+} = \frac{1}{3}$ etc.

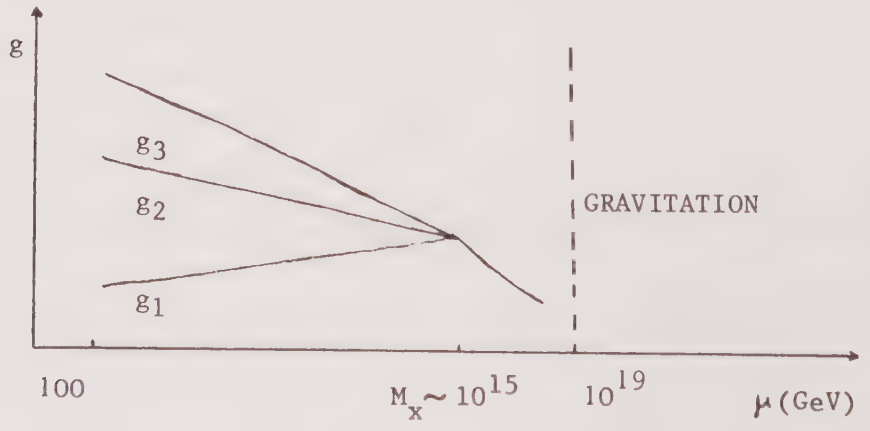
III.2 - Approximation de la fonction $-\theta$

Pour étudier les conséquences et les prédictions des théories grandement unifiées, il nous faut un moyen de traiter les effets des masses dans l'évolution des constantes de couplage. L'image la plus simple est celle de l'approximation des effets de seuil par des fonctions $-\theta$[8].

D'après le théorème de découplage[35], si on considère une théorie des particules lourdes de masse M et des particules légères, et qu'on s'intéresse à des processus avec des énergies caractéristiques beaucoup plus petites que M et impliquant seulement des particules externes légères, toutes les particules lourdes intermédiaires se découplent en laissant des contributions qui s'annulent comme $\frac{1}{M^2}$. En utilisant ce théorème, on peut approximer une théorie unifiée spontanément brisée en utilisant différentes théories effectives selon l'énergie μ :

$$\text{Théorie effective} = \begin{cases} \text{groupe unifié } G & \text{presque exacte quand } \mu \gg M_x \\ & \qquad\qquad\qquad\qquad\qquad\qquad (III.8) \\ SU(3)_c \times SU(2)_L \times U(1)_y & \\ & \text{presque exacte quand } \mu \ll M_x \end{cases}$$

où M_x est la masse des bosons de jauge superlourds X et Y. L'approximation $-\theta$ consiste à extrapoler la validité de (III.8) jusqu'à la région $\mu \approx M_x$ et à poser $M_x = 0$ si $\mu > M_x$ et $M_x = \infty$ si $\mu < M_x$. En conséquence, dans la région $\mu < M_x$ il y a les trois constantes de couplage g_1, g_2 et g_3 de la théorie $U(1)_y \times SU(2)_L \times SU(3)_c$ tandis que dans la région $\mu > M_x$, il y a seulement la constante g de la théorie unifiée G. Cela implique nécessairement que $g_1 = g_2 = g_3 = g$ au point $\mu = M_x$.



Par les deux égalités $g_1 = g_2 = g_3$ à $\mu = M_x$, on peut calculer la valeur de M_x et prédire $\sin^2\theta_w$. Suivant les définitions et les propriétés du modèle standard $\sin^2\theta_w$ est donné par la formule [8]:

$$\sin^2\theta_w = \frac{g_1^2}{g_1^2 + g_2^2} \quad . \tag{III.9}$$

La constante de couplage g_1 correspond au générateur d'hypercharge $U(1)_y$ et g_2 aux générateurs de $SU(2)_L$. Puisque tous sont des générateurs du groupe unifié G, ils doivent avoir la même normalisation, par exemple $\mathrm{Tr}\, Y^2 = \mathrm{Tr}T_3^2$. Mais à l'intérieur du modèle standard, cette normalisation n'existe pas, par ex. $\sum_{\text{fermions}} T_{3f}^2 \neq \sum_{\text{fermions}} Y_f^2$. Donc, pour incorporer $SU(2)_L \times U(1)_y$ dans le groupe G, il faut dilater l'hypercharge par un facteur $C = \left(\sum_{\text{fermions}} T_{3f}^2 \Big/ \sum_{\text{fermions}} Y_f^2 \right)^{1/2}$. On peut vérifier facilement que $C^2 = \frac{5}{3}$. D'autre part, dans la formule (III.9), si on identifie g_2 avec la constante de couplage g de $SU(2)_L$, g_1 doit être dilaté par 1/C pour être identifié avec la constante de couplage de $U(1)_y$ dans G et donc pour $\mu = M_x$ on a :

$$\sin^2\theta_w = \frac{g^2/C^2}{g^2 + g^2/C^2} = \frac{1}{1 + C^2} = \frac{3}{8} \quad . \tag{III.10}$$

En utilisant maintenant les équations du groupe de renormalisation, on peut trouver la valeur de $\sin^2\theta_w$ à l'énergie $\mu \approx 100$ GeV. Le résultat est environ .2 qui est très près de la valeur mesurée expérimentalement. Néanmoins, pour faire une comparaison précise, il faut tenir compte d'une part des effets de masse[9,10] dans la prédiction théorique et d'autre part des corrections radiatives[34] correspondantes à l'expérience considérée.

La valeur de la masse M_x, des bosons lourds est déterminée par l'intersection des courbes $g_1(\mu)$, $g_2(\mu)$ et $g_3(\mu)$, dont les pentes sont calculables grâce aux fonctions-β de SU(3) x SU(2) x U(1) dont on connaît les valeurs à basse énergie, de l'ordre de 100 GeV. A cause de la dépendance

logarithmique en μ de l'évolution des constantes de couplage $g_i(\mu)$ on trouve une valeur de M_x très grande, de l'ordre de 10^{14} GeV. Une fois cette masse déterminée, on peut prédire la durée de vie du proton. Celui-ci peut se désintégrer, puisque les quarks et les leptons appartiennent à la même représentation, et sont couplés par l'intermédiaire des bosons lourds X et Y, de masse d'ordre M_x. Par conséquent, les processus qui donnent la désintégration du proton sont supprimés par un facteur $\sim \frac{1}{M_x^2}$ et donc le temps de vie du proton τ_p sera proportionnel à M_x^4 / m_p^5 , compris entre 10^{30} et 10^{32} années [8-10]. Les expériences récentes semblent pour l'instant donner une borne inférieure très proche des ces prédictions[36]. Bien que le modèle minimal SU(5) viole séparément la conservation des nombres baryonique B et leptonique L, il prédit une conservation du nombre B-L qui correspond à une symétrie globale qui n'est pas spontanément brisée. A cause de cette symétrie, on a des règles de sélection comme par exemple $p \rightarrow e^+ X$ mais $p \not\rightarrow e^- X$ etc.

Finalement, les théories grandement unifiées prédisent des rapports de masse entre les quarks et les leptons. Par exemple dans SU(5), il y a deux couplages de Yukawa différentes (section III.1). L'une donne une masse au quark u et l'autre au quark d et à l'électron. On a donc trois relations, pour les trois générations, qui sont satisfaites à $\mu = M_x$: $m_d = m_e$, $m_s = m_\mu$ et $m_b = m_\tau$. En utilisant l'évolution du groupe de renormalisation on trouve[6,10] que $m_{b/m_\tau} \sim 2.85$ et $\frac{m_s}{m_d} = \frac{m_\mu}{m_e} \sim 200$. La première est en bon accord avec le rapport expérimental tandis que les deux dernières relations sont très mal vérifiées.

Terminons cette section par quelques problèmes spécifiques des théories grandement unifiées. Il y a tout d'abord l'arbitraire du choix du groupe unifié G, si l'on ne pose pas la restriction de rang minimum. On doit alors choisir parmi plusieurs groupes possibles, plusieurs possibilités de brisure, et on a beaucoup de paramètres indéterminés surtout dans le secteur de Higgs, etc. Un autre problème est celui de la hiérarchie de jauge. Dans la théorie, il y a deux échelles de masses M_W et M_x qui ont un rapport très grand $M_{x/M_W} \sim 10^{12}$ sans qu'aucune raison profonde n'explique ce grand nombre. De plus, si l'on impose à l'ordre le plus bas de la théorie des perturbations cette hiérarchie de masses, les corrections radiatives la détruisent en général, à moins d'imposer, ordre par ordre, des conditions de

renormalisation extrêmement peu naturelles. Prenons par exemple, le potentiel de Higgs dans SU(5), décrit au chapitre précédent. Nous avons un multiplet Φ qui acquière une valeur moyenne dans le vide très grande $\langle\Phi\rangle \sim 10^{15}$ GeV et un multiplet H avec $\langle H\rangle \sim 100$ GeV. Dans le potentiel, il y aura nécessairement des termes de mélange $\lambda H^2\Phi^2$ qui donneront au H une masse superlourde $\lambda^{1/2}\langle\Phi\rangle \sim 10^{15}$ GeV. Or, H doit contenir le doublet léger du modèle standard. Pour que la masse du $W^{\pm}$ et du Z soit d'environ 100 GeV, il faudra alors ajuster les paramètres du potentiel 10^{-28} près, et ceci à tous les ordres de la théorie des perturbations (problème appelé "fine tuning").

III.3 - Une formulation précise pour la brisure spontanée de symétrie

Dans cette approche, au lieu d'utiliser des approximations comme celle de la fonction $-\theta$, nous traitons les effets de la brisure spontanée de symétrie d'une façon exacte. Notre formalisme est basé sur les propriétés suivantes :

(i) L'égalité entre les trois constantes de couplage effectives $g_1 = g_2 = g_3$ est valable asymptotiquement dans la limite $\frac{\text{masses}}{\text{énergie}} \to 0$.

(ii) La constante de couplage est unique. C'est pourquoi, même si nous voulons tenir compte de toutes les deux étapes de la brisure du groupe unifiée G (à M_X et à M_W), elle ne doit être définie que par une seule condition de renormalisation.

(iii) La différence observée entre les intensités des constantes de couplage $g_1(\mu)$, $g_2(\mu)$, $g_3(\mu)$ vient de différentes définitions du même objet (la constante de couplage g de la théorie spontanément brisée). Les différentes constantes $g_i(\mu)$ sont alors reliées entre elles par une renormalisation finie.

(iv) La renormalisation finie entre les différentes constantes de couplages ne peut pas être traitée perturbativement à cause de la présence des grands logarithmes($\ln \frac{M_X^2}{\mu^2} \sim 60$ quand $\mu \sim 100$ GeV). C'est pourquoi, on a besoin d'une méthode de resommation en utilisant le groupe de renormalisation.

A PRECISE FORMULATION OF SPONTANEOUS BREAKING OF GRAND UNIFIED THEORIES

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We present a precise formulation using the renormalization group, of the phenomenon of spontaneous breaking of a semi-simple group G to a product of factors $G \to G_1 \times G_2 \times \ldots$. Our treatment takes exactly into account the effect of the different masses, in particular that of the superheavy gauge and Higgs bosons. Therefore it allows for an accurate determination of these parameters.

All interactions among elementary particles seem to be described by gauge theories. At present energies the generally accepted gauge group is $U(1) \times SU(2) \times SU(3)$. The hypothesis of "grand-unification" postulates that this is the remnant of a larger semi-simple group G which is broken spontaneously at some mass scale M [1,2]. Much above M physics will be described by a truely unified theory with a single coupling constant. The large differences among the strengths of strong, electromagnetic and weak interactions, as measured today, are attributed to the fact that each one follows a different renormalization group equation. Since the dependence of the effective coupling constants on the mass-scale is only logarithmic, M turns out to be very large [3]. In most models a value of the order of 10^{14}–10^{15} GeV is found. This result has two important consequences: on the practical side it guarantees the longevity of the proton since τ_p is proportional to M^4. From a theoretical point of view it explains why the three observed groups appear to be decoupled, as if they were three independent interactions, although they come from the same unified theory. On the other hand this gigantic value of M tells us that the grand unified theory will remain inaccessible to direct experimental investigation for any foreseeable future. The only exception to this rather sad perspective seems to be precisely the study of baryon decay which, by a happy coincidence, is predicted to be very close to the present experimental limit. Therefore it is very important to estimate the value of M as accurately as possible and a considerable theoretical effort has been devoted to this end [3,4]. The purpose of the present paper is twofold: first, to present a clear and precise formulation, using the renormalization group, of the phenomenon of spontaneous breaking of a semi-simple group G to a product of factors $G \to G_1 \times G_2 \times \ldots$. In particular, our treatment takes exactly account of the effect of the different mass scales. Our second purpose is to write down the general form of the equations which allow the calculation of M. We shall present our analysis for the case of [2] $SU(5) \to SU(3) \times SU(2) \times U(1)$ and we shall ignore, for pedagogical purposes, the second breaking of $SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1)$. This means that we shall have all fermions massless and we shall only include Higgs scalars belonging to the adjoint representation of SU(5) [2].

We start from the lagrangian:

$$\mathcal{L} = -\tfrac{1}{4}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \bar{\psi} i \not{D} \psi + \tfrac{1}{2}\mathrm{Tr}(D_\mu \Phi D^\mu \Phi) + \tfrac{1}{2}m^2\, \mathrm{Tr}(\Phi^2) - \tfrac{1}{4}h_1 [\mathrm{Tr}(\Phi^2)]^2 - \tfrac{1}{2}h_2\, \mathrm{Tr}(\Phi^4)\,, \quad (1)$$

where by ψ we denote all fermions belonging to the representations $\bar{5}$ and 10. The lagrangian (1) contains four independent parameters, namely the mass m^2 and the three coupling constants, g, h_1 and h_2. After spontane-

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ous symmetry breaking via the translation $\Phi \to \Phi + (v/\sqrt{2})\lambda_{24}$, we obtain the following mass terms:

(i) $M^2 = \frac{5}{3}v^2g^2$ for the vector gauge bosons $G_9 ... G_{20}$,

(ii) $M_3^2 = \frac{1}{3}v^2h_2$ for the scalars $\phi_1 ... \phi_8$,

(iii) $M_2^2 = \frac{4}{3}v^2h_2$ for the scalars $\phi_{21} ... \phi_{23}$,

(iv) $M_1^2 = 2v^2(h_1 + \frac{7}{15}h_2)$ for the scalar ϕ_{24}.

In terms of the parameters of (1) v^2 is given, in the tree approximation, by

$$v^2 = m^2[h_1 + \tfrac{7}{15}h_2]^{-1} . \tag{2}$$

We choose to renormalize the broken theory by imposing the following seven renormalization conditions:

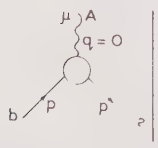
(i) Three conditions for the masses M^2, M_1^2 and M_2^2 which will be chosen physical, as the poles of the corresponding propagators.

(ii) Three wave function renormalization conditions for fermions, vector bosons and scalars, respectively. In fact, for fermions, we need as many conditions as the number of different irreducible representations that appear in (1). For example, the condition on the scalars reads:

$$\frac{\mathrm{d}}{\mathrm{d}p^2}\Gamma_A^{(2)}(p^2)\bigg|_{p^2=-\mu^2} = 1 , \tag{3}$$

and similarly for fermions and gauge bosons. μ^2 is a subtraction point. However, since the SU(5) symmetry is broken, we must specify on which component of Φ we impose the condition. This is done with the index A which can in principle run from 1 to 24. The conservation of SU(3) × SU(2) × U(1) implies that we need only to distinguish four distinct cases (A = 1, ..., 8; 9, ..., 20; 21, ..., 23; 24). Once the condition is imposed on any of the components, all the other Green's functions become finite and calculable. The same remarks apply to the vector bosons.

(iii) Finally, we must impose one coupling constant renormalization condition and we choose it to be the value of the fermion–vector boson three-point function at $p^2 = -\mu^2$



$$\Big|_{p^2=-\mu^2} = \mathrm{i}g_A\,\gamma^\mu T^A_{ab} , \tag{4}$$

where, again, the index A denotes the particular vector boson we have used.

We want to emphasize that, once these three kinds of renormalization conditions have been imposed, the perturbation theory is completely defined and all Green's functions are finite and calculable as formal power series in g_A. Furthermore it is not possible to impose any further conditions. Notice in particular that there exists only one coupling constant, as one should expect from a grand unified theory. Does this mean that one could compute a QCD process like a deep-inelastic structure function as a power series in the weak interaction coupling constant? Formally, the answer is yes, but, in practice, this is not so. Formal perturbation theory guarantees that, if in (4) we choose A to denote one of the SU(2) gauge bosons, all other three-point functions are finite and calculable. In particular, if $\Gamma_3^{(3)}$ is the three-point function with an SU(3) external gauge boson we can write [‡1]:

$$\Gamma_3^{(3)}(p^2 = -\mu^2) = g_2 + A_1g_2^3 + A_2g_2^5 + ... , \tag{5}$$

where the A_n's are finite and calculable functions of μ^2 and the masses of the theory. However, it is easy to check that A_n is of the form:

$$A_n \sim [\ln(M^2/\mu^2)]^n , \tag{6}$$

which, for $M/\mu \approx 10^{14}$ gives $[65]^n$. In other words, although the series (5) is well defined, it is useless for practical computations.

The remedy to this difficulty is well-known. We renormalize the same broken SU(5) theory in three different ways where the index A in the conditions (3) and (4) denote the bosons of U(1), SU(2) or SU(3). This gives us three perturbation expansions in powers of g_1, g_2 or g_3, always of the same theory, but now each one is suited to particular processes. The values of the g_i's will be fixed by experiment, but we must always remember that we are talking about one and the same theory, so we write the analog of eq. (5):

$$g_i = F_{ij}(g_j, M^2/\mu^2, \alpha) , \tag{7}$$

where with α we mean the ratios M_1^2/M^2 and M_2^2/M^2. In the limit of exact SU(5) symmetry, i.e. when $M^2/\mu^2 \to 0$, all coupling constants must be equal. This only happens at infinite energy

$$F_{ij}(g_j, 0, \alpha) = g_j . \tag{8}$$

In order to obtain a renormalization group equation

[‡1] We use the following notation: For A = 1, ..., 8 we call $g_A = g_3$; A = 21, ..., 23 $g_A = g_2$; $g_{A=24} = g_1$.

for the functions F_{ij} we take the total derivative with respect to μ^2 of both sides of eq. (7). In the Landau gauge we obtain:

$$\beta_i(F_{ij}, \lambda, \alpha) = \left[\lambda \frac{\partial}{\partial \lambda} + \beta_j(g_j, \lambda, \alpha) \frac{\partial}{\partial g_j} \right] F_{ij} , \qquad (9)$$

where

$$\beta_k(g_k, \lambda, \alpha) = \mu^2 \, \mathrm{d}g_k/\mathrm{d}\mu^2 , \quad \lambda = M^2/\mu^2 . \qquad (10)$$

The differential equation (9) with the boundary condition (8) is our basic equation. The β-functions are calculable, at any given order in perturbation theory and they are of the form:

$$\beta_k(g_k, \lambda, \alpha) = b_k^0(\lambda, \alpha) g_k^3 + \ldots . \qquad (11)$$

Notice that the b coefficients, contrary to those of F_{ij} itself, do not contain large logarithms. This can be easily understood since the β-functions, as defined by eqs. (10) and (4), possess well-defined limits both for $M^2 \to \infty$, when they become the β-functions of SU(3), SU(2) or U(1) [5] and $M^2 \to 0$, when they become all equal to that of SU(5).

Eq. (9) can be solved by standards methods. Its characteristic system of ordinary differential equations is:

$$-\mathrm{d}\lambda/\lambda = \mathrm{d}g_j/\beta_j(g_j, \lambda, \alpha) = \mathrm{d}F_{ij}/\beta_i(F_{ij}, \lambda, \alpha) . \qquad (12)$$

We find two independent first integrals which we denote by $f_j(g_j, \lambda, \alpha)$ and $f_i(F_{ij}, \lambda, \alpha)$. They are constants on every orbit of the system and can be computed once the β-functions are known. For example, the solution of the first among eqs. (12) gives

$$g_j = g_j(\lambda, \alpha, c) , \qquad (13)$$

where c is an integration constant. We can invert this relation with respect to c and write

$$c = f_j(g_j, \lambda, \alpha) . \qquad (14)$$

We can think of this equation as describing the motion of a point particle on its trajectory. The initial conditions are of the form $g_j = g_{j0}$ for $\lambda = \lambda_0$. This determines c. A first integral is any arbitrary function of c. For future convenience let us choose this function such that $c = g_{j0}$ for $\lambda = \lambda_0$. Therefore we obtain

$$g_{j0} = f_j(g_{j0}, \lambda_0, \alpha) \quad \text{or} \quad g_j = f_j(g_j, \lambda_0, \alpha) . \qquad (15)$$

Similarly we find the other first integral by solving the second equation and we choose

$$F_{ij}(g_j, \lambda_0, \alpha) = f_i(F_{ij}(g_j, \lambda_0, \alpha), \lambda_0, \alpha) . \qquad (16)$$

The general solution of (9) can now be written as:

$$f_i(F_{ij}, \lambda, \alpha) = \Phi(f_j(g_j, \lambda, \alpha)) , \qquad (17)$$

where $\Phi(x)$ is an arbitrary function of x. It is determined by the boundary condition (8). Using first (15) and (16) we obtain:

$$F_{ij}(g_j, \lambda_0, \alpha) = \Phi(g_j) . \qquad (18)$$

In (18) we can take the limit $\lambda_0 \to 0$. This could not be done directly in the solutions of (12) because the integral $\int \mathrm{d}\lambda/\lambda$ is singular at $\lambda = 0$. However in (18) this singularity cancels and, using (8), we find

$$x = \Phi(x) . \qquad (19)$$

The general solution is now given by

$$f_i(g_i, \lambda, \alpha) = f_j(g_j, \lambda, \alpha) , \qquad (20)$$

which expresses any coupling constant ($F_{ij} = g_i$) in terms of any other. This completes the solution of eq. (9).

Before discussing the applications of our eq. (20) let us illustrate the procedure using the one-loop β-functions given by (11). In this case the variables in eqs. (12) are separated and the solutions are

$$c = g_j^{-2} - 2 \int_{\lambda_0}^{\lambda} x^{-1} \, \mathrm{d}x \, b_j^0(x, \alpha) , \qquad (21a)$$

$$c' = F_{ij}^{-2} - 2 \int_{\lambda_0}^{\lambda} x^{-1} \, \mathrm{d}x \, b_i^0(x, \alpha) . \qquad (21b)$$

Notice that in these expressions the limit $\lambda_0 \to 0$ cannot be taken. We need a function of c (and one of c') satisfying the conditions (15) and (16). They are:

$$f_j(g_j, \lambda, \alpha) = \left[g_j^{-2} - 2 \int_{\lambda_0}^{\lambda} x^{-1} \, \mathrm{d}x \, b_j^0(x, \alpha) \right]^{-1/2} , \qquad (22a)$$

$$f_i(F_{ij}, \lambda, \alpha) = \left[F_{ij}^{-2} - 2 \int_{\lambda_0}^{\lambda} x^{-1} \, \mathrm{d}x \, b_i^0(x, \alpha) \right]^{-1/2} , \qquad (22b)$$

and the general solution, eq. (20), becomes:

$$g_i^{-2} = g_j^{-2} + 2 \int_{0}^{\lambda} x^{-1} \, \mathrm{d}x \, [b_i^0(x, \alpha) - b_j^0(x, \alpha)] . \qquad (23)$$

Let us notice again that now the limit $\lambda_0 \to 0$ exists because the difference of the β-functions vanishes for $M^2 \to 0$.

We must now use, as input, the experimentally measured effective strengths of strong, electromagnetic and weak interactions at moderate (say $p^2 \approx 10$–$100\ \mathrm{GeV}^2$) energies. Using the renormalization group for $\mu^2 = -p^2$, we write $g_i = \bar{g}_i(p^2)$ and $\lambda = -M^2/p^2$. The two independent equations given by the relations (20) (i, j: 1,2,3) contain three unknown parameters, namely λ as well as the Higgs masses M_1^2/M^2 and M_2^2/M^2 denoted by α. However, it turns out that, at least at the one-loop level, the dependence on α is very weak. If we ignore it for the moment, then the two equations can be used to determine λ and to predict the value of $\sin\theta_w$. Although a complete investigation requires the introduction of a 5-plet of Higgs scalars and the breaking of $SU(2) \times U(1)$, it already seems that the value of $\sin\theta_w$ is insensitive to any of these details as well as the values of the parameters. The well-known prediction of $\sin^2\theta_w \approx 0.2$ remains essentially unchanged. On the contrary, the value of λ (i.e. M^2) seems to depend strongly on both the low-energy (fermion and W-masses etc.) and the super-Higgs parameters and a precise calculation will be reported elsewhere.

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CHAPITRE IV

PREDICTIONS DES THEORIES GRANDEMENT UNIFIEES

IV - PREDICTIONS DES THEORIES GRANDEMENT UNIFIEES

Nous allons utiliser la méthode expliquée au chapitre (III-3) pour calculer de façon précise la masse M_x des bosons superlourds X, le temps de vie du proton τ_p et la quantité qui paramétrise les expériences du courant neutre, $\sin^2\theta_w$. On considère la forme suivante de la brisure spontanée du groupe unifié G :

$$G \longrightarrow SU(5) \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_y \longrightarrow SU(3)_c \times U(1)_{em} \quad \text{(IV.1)}$$
$$M_1 > M_2 > \cdots \quad M_x \qquad\qquad M_w$$

Nous avons étudié, d'une façon générale, les effets de brisure dans ces théories. Pour cela, nous avons besoin de la dépendance en masses des fonctions-β . Nous les avons calculées exactement, au niveau d'une boucle, et approximativement au niveau de deux boucles. En effet, les prédictions du deuxième ordre peuvent être bornées inférieurement et supérieurement en utilisant deux types d'approximation de la fonction - θ . Nous avons utilisé la jauge de " t'Hooft-Landau dans laquelle le paramètre de jauge ne varie pas en fonction de l'énergie [37] et pour laquelle les équations du groupe de renormalisation sont plus simples. Les résultats que nous avons trouvés en incluant toutes les incertitudes théoriques sont :

$$M_x = 3.1 \times 10^{14 \pm .3 - .2 (N_H - 1) + .1 (f-3)} \left(\frac{\Lambda_{\overline{MS}}}{.2} \right)^{1.03} \text{GeV}, \quad \text{(IV.2)}$$

où N_H est le nombre de doublets de Higgs légers et f le nombre de familles. Nous avons vu au chapitre précédent que le temps de vie du proton est $\tau_p = C_p M_x^4/m_p^5$. La constante C_p/m_p^5 a été calculée par plusieurs groupes [38]. Leur résultat, qui dépend beaucoup des incertitudes provenant des problèmes de la physique hadronique, est environ $10^{-28 \pm .6}$ $\frac{\text{ans}}{(\text{Gev})^4}$.

En utilisant (IV.2), on trouve

$$\tau_p = 10^{30 \pm 1.7 - .8(N_H-1) + .4(f-3)} \left(\frac{\Lambda_{\overline{MS}}}{.2} \right)^{4.1} \text{ans} \ . \quad \text{(IV.3a)}$$

La limite expérimentale actuelle provenant de l'expérience de Michigan

$$\tau_p^{exp} \gtrsim 3 \times 10^{31} \text{ ans} \quad \text{(IV.3b)}$$

si bien que la prédiction principale de SU(5) frôle la contradiction avec l'expérience.

Pour la prédiction de $\sin^2\theta_w$, on a montré que au moins dans SU(5), il est possible de définir d'une façon naturelle un angle faible effectif $\theta_w(\mu)$ dépendant de l'énergie μ, au-dessus et au-dessous de la masse des bosons des interactions faibles, M_w, et on donne la prédiction pour la courbe de $\sin^2\theta_w(\mu)$. Pour $\mu=\sqrt{20}$ GeV on trouve :

$$\sin^2\theta_w\ (\sqrt{20}\ \text{GeV}) = .2163 \pm .0071 + 0.004\ (N_H-1) - .0014(f-3) - .0056\ \ln\frac{\Lambda_{\overline{MS}}}{.2} \qquad \text{(IV.4a)}$$

Pour comparer cette prédiction avec la valeur expérimentale de $\sin^2\theta_w^{exp}$, il faut tenir compte des corrections radiatives pour les processus considérés. Cela a été fait dans la ref. [34], où on trouve :

$$\sin^2\theta_w^{exp}\ (\sqrt{20}\ \text{GeV}) = .216 \pm .010, \qquad \text{(III.4b)}$$

ce qui est en accord remarquable avec la prédiction théorique.

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SYMMETRY-BREAKING EFFECTS IN GRAND UNIFIED THEORIES

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We study the threshold effects in a spontaneously broken gauge theory. In particular we present a computation of the symmetry-breaking effects in the SU(5) grand unified theory including those of the breaking of $SU(2)\times U(1)$. As an application we give a precise value of the superheavy gauge boson mass M_X. We also show that it is possible in SU(5) to define a natural effective weak angle $\theta_w(\mu)$ for any scale μ, below as well as above M_W, and we give the predicted curve for $\sin^2\theta_w(\mu)$.

1. Introduction

The known interactions seem to be described by QCD and the Weinberg–Salam (WS) model which both are gauge theories. It is therefore natural to base grand unification [1, 2] on a simple gauge group $G \supset SU(3)_c \times [SU(2)\times U(1)]_{WS}$ in order that there be a single coupling constant. However the forces associated to the strong, weak and electromagnetic interactions are of vastly different strengths. The central idea [3] behind grand unification is that the splitting of the unique coupling constant of the group G arises from the difference of the masses of the gauge particles resulting from the spontaneous breaking of G.

In grand unified theories, very heavy particles must exist in order to get the observed splitting between the strong and electromagnetic coupling constants. If no new physics interferes drastically with the known interactions, the mass scale of the heavy particles must be as high as 10^{14}–10^{15} GeV. It is perhaps not a coincidence that attempts of quark-lepton unification call for such a mass scale in order to ensure proton stability. The weak angle θ_w, which measures the splitting between the SU(2) coupling constant and the electromagnetic one, is then predicted to be $\sin^2\theta_w \sim 0.20$ [3], which is very close to the experimental values. There are very few practical and significant low-energy tests of grand unified theories so, considerable effort [4–13] has been devoted to estimate, more reliably, $\sin^2\theta_w$ and the leptoquark mass M_X on which the proton lifetime, $\tau_p \sim M_X^4$, is strongly dependent. Our paper is concerned with these estimations for the minimal group G = SU(5) and gives a detailed account of results presented briefly elsewhere [14].

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To get the desired precision, we need to go beyond the leading logarithmic approximation of refs. [3, 4] in which threshold effects are crudely approximated by θ functions. This procedure for treating the decoupling of heavy particles is validated by the decoupling theorem [15, 16]. However, this approximation can only give a rough order of magnitude of M_X. A refined evaluation of threshold effects and second-order corrections is highly dependent on the renormalization prescription. A systematic second-order calculation has been performed with a momentum subtraction (MOM) prescription for an SU(5) grand unified theory by Goldman and Ross [6]. However, their attack of the problem had to rely upon numerical integration of complicated coupled equations which contain the threshold effects explicitly. Other strategies [8, 17] avoid mass-dependent β functions by using a modified minimal subtraction scheme. Weinberg's technique [8] relates the minimal subtraction coupling constant of effective gauge theories constructed from a grand unified theory by integrating out the superheavy fields. Weinberg's formulae have been used by Hall [9] and by Ellis et al. [10] to compute M_X and $\sin^2\theta_w$ in grand unified theories principally based on SU(5). Related prescriptions have been formulated by Binétruy and Schücker [11] and by Chang et al. [12]. All these strategies rely on the decoupling theorem to distinguish the regions above and below a threshold M. The matching between the two regions typically neglects terms of order μ^2/M^2 so it is expected to be valid for energies μ small compared to M, in particular for the crossing of the superheavy threshold in grand unified theories. However, some care may be needed for crossing the (WS) threshold.

The crossing of thresholds can be done exactly using the formulation of ref. [18]. We apply their method to give a unified picture of the evolution of the running coupling constants over the whole range of energy, whatever the number of mass scales, because the matching is done at infinity. The β functions have well-defined limits for $M^2 \to 0$ and $M^2 \to \infty$ and we can see the splitting of the unique coupling constant of the theory and how the decoupling of heavy particles takes place. Related proposals have been made in refs. [13, 19, 20]. In this paper we use the Landau gauge where there is no running gauge parameter and get closed analytical expressions for the evolution of coupling constants with all mass effects taken into account exactly up to one loop level. Second-order effects are evaluated in the step approximation with a careful estimation of errors involved in that procedure. As an application, we calculate M_X in SU(5) and our value agrees with previous estimations [9–13]. Although we study the specific example of SU(5) our results are presented in such a way that they can be extended easily to any grand unified gauge group.

We also devote special attention to the problem of defining $\sin^2\theta_w$ in SU(5). The symmetric value $\sin^2\theta_w = \frac{3}{8}$ is modified by renormalization effects to $\sin^2\theta_w = 0.2$ at ordinary energies [3]. In general the effective value of $\sin^2\theta_w(\mu)$ is calculated at the scale $\mu = M_W$ assuming that the $SU(2) \times U(1)$ gauge symmetry is unbroken. In our scheme we can relax this assumption and define $\sin^2\theta_w(\mu)$ below the W

threshold. However, for $\mu \leq M_W$ the prediction will depend on the definition, and the question of relating the theoretical value to the experimental one becomes important. A possible definition would be $\sin^2 \theta_w(\mu) = \alpha_{em}(\mu)/\alpha_w(\mu)$, where α_w measures the strength of the weak charged current. But there is a more natural definition with respect to neutral current experiments. Indeed we find that it is possible to define a neutral current effective charge whose parametrization is exactly that of the tree level:

$$Q_Z = \frac{1}{\cos \theta_w(\mu)} [T_3 - \sin^2 \theta_w(\mu) Q_{em}] .$$

This angle $\sin^2 \theta_w(\mu)$ is related to the non-trivial γ-Z mixing which arises in the SU(5) symmetric γ-Z basis at the scale μ. It turns out that both definitions give very similar results below the W threshold, in particular our prediction at $\mu = M_W$, $\sin^2 \theta_w(M_W)$, is in agreement with previous results [7, 9–13]. But the value $\sin^2 \theta_w(\mu)$ does increase when $\mu < M_W$ and this increase which is due to the WS breaking, is quite large. This correction is very important because now the corrected value of $\sin^2 \theta_w$, with the uncertainties due to the strong coupling constant determination, overlaps the experimental values. Similar corrections have also been found in refs. [19, 21, 22, 24]. However, we must stress here that our paper is only concerned with the radiative corrections controlled by the renormalization group. Radiative corrections contain another part consisting in the contribution of box diagrams and bremsstrahlung terms, which is μ independent but experiment dependent and which must be evaluated before being compared to our prediction. This part has been recently calculated by several authors [23, 25] in the Weinberg–Salam model.

The paper is organized as follows: In sect. 2, we recall some well-known facts about the breaking of SU(5) down to $SU(3)_c \times U(1)$ so as to make this paper self-contained. We present, in sect. 3, our renormalization scheme with special emphasis on the photon-Z^0 mixing. In sect. 4 we describe how to relate the observed strengths of the three interactions at low energy. In sect. 5 we study the threshold functions at the one-loop level and consider how the decoupling operates in the Landau gauge. We discuss the second-order effects in sect. 6. In sect. 7 we apply our formulae for the evolution of the running coupling constants to calculate M_X in SU(5). We also show how to translate the experimental data on α_{em} and α_s into our renormalization scheme. In sect. 8 we calculate $\sin^2 \theta_w(\mu)$ over the whole range of energy scale μ and compare the predictions of several possible theoretical definitions below the W threshold. Finally in sect. 9 we summarize our results and compare with other authors. Appendices A and B contain some technical details and in appendix B we list all the analytic expressions for the threshold functions.

2. Mass spectrum in a spontaneously broken theory [SU(5)]

Given a gauge group G with the vector gauge bosons assigned to the adjoint representation, we first need to specify the irreducible representations which

accommodate the fermions. Then we must break G into $SU(3)_c \times U(1)_{em}$ with some set of Higgs representations which depends on the desired symmetry breaking pattern. Since the Weinberg–Salam model describes the weak and electromagnetic interactions at low energies, it is useful to write down the $SU(3)_c \times SU(2)_L$ decomposition of all these irreducible representations.

For instance, in the case of $G = SU(5)$, it is well known [2] that we need to introduce a $\bar{5}$-plet and 10-plet for each generation of left-handed fermions. It is also well known [4] that the minimal Higgs system which gives the breakings $SU(5) \to SU(3)_c \times SU(2)_L \times U(1) \to SU(3)_c \times U(1)_{em}$ consists of a 24-plet and a complex 5-plet.

The $SU(3)_c \times SU(2)_L$ decomposition of the 24-plet gauge bosons is

$$\begin{aligned} 24 &= (8,1) \oplus (3,2) \oplus (\bar{3},2) \oplus (1,3) \oplus (1,1)\,, \\ W &= (G_{ij}) \oplus (X_i, Y_i) \oplus (Y_i^+, X_i^+) \oplus (W^+, W^3, W^-) \oplus (B^0)\,. \end{aligned} \tag{2.1}$$

The octets (G_{ij}) are identified with the gluons which belong to the adjoint representation of $SU(3)_c$ and are singlet of $SU(2)_L$. The labels i and j stand for the three SU(3) colors. The SU(2) doublets (X_i, Y_i) and (Y_i^+, X_i^+) represent the superheavy vector bosons X and Y; X_i and Y_i are also triplets in $SU(3)_c$. The SU(2) triplet (W^+, W^3, W^-), which is $SU(3)_c$ singlet, represents the WS SU(2) vector bosons. Finally (B^0), which is $SU(3)_c$ and $SU(2)_L$ singlet, represents the WS U(1) vector boson.

In the same way the 24-plet of Higgs is decomposed as

$$\phi = (\phi_{G_{ij}}) \oplus (\phi_{X_i}, \phi_{Y_i}) \oplus (\phi_{Y_i}^+, \phi_{X_i}^+) \oplus (\phi_W^+, \phi_{23}, \phi_W^-) \oplus (\phi_{24})\,. \tag{2.2}$$

The 5-plet Higgs decomposition is,

$$\begin{aligned} 5 &= (3,1) \oplus (1,2)\,, \\ H &= (H_{Y_i}) \oplus (H_W^+, H^0)\,. \end{aligned} \tag{2.3}$$

(H_{Y_i}) is $SU(3)_c$ triplet and $SU(2)_L$ singlet. The $SU(2)_L$ doublet (H_W^+, H^0) is also singlet under $SU(3)_c$. The H^0 Higgs is usually parametrized as $H^0 = \sqrt{\frac{1}{2}}[R_5 - iI_5]$, where R_5 and I_5 are real fields.

The $\bar{5}$-plet of left-handed fermions is decomposed as

$$\begin{aligned} \bar{5} &= (\bar{3},1) \oplus (1,2)\,, \\ \chi &= (d_i^c)_L \oplus (\nu, e)_L\,. \end{aligned} \tag{2.4}$$

d_L^c represents the left-handed charge conjugate d-quark which is triplet under $SU(3)_c$ and singlet under $SU(2)_L$. The $SU(2)_L$ doublet and $SU(3)_c$ singlet, represent the neutrino and the left-handed electron.

Finally the decomposition of the 10-plet fermions is

$$\begin{aligned} 10 &= (\bar{3}, 1)\oplus(3, 2)\oplus(1, 1)\,, \\ \psi &= (u_i^c)_L\oplus(u_i, d_i)_L\oplus(e^c)_L\,, \end{aligned} \tag{2.5}$$

which contains the $SU(3)_c$ triplet u charge conjugate left-handed quark, the $(u_i, d_i)_L$ $SU(2)_L$ doublet (triplet under $SU(3)_c$) and the singlet charge conjugate left-handed electron.

The SU(5) symmetric lagrangian is:

$$\begin{aligned} \mathscr{L} = &-\tfrac{1}{2}\,\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu} - \frac{1}{\xi}\,\mathrm{Tr}\,[\partial W + ig\xi[\phi, V] + ig\xi(HV_0^+ - V_0H^+)]^2 \\ &+\tfrac{1}{2}\,\mathrm{Tr}\,\bar{\eta}\partial\cdot D\eta + (D_\mu H)^+(D^\mu H) + \mathrm{Tr}\,(D_\mu\phi)^2 + \bar{\chi}^+ i\not{D}\chi + \mathrm{Tr}\,\bar{\psi} i\not{D}\psi \\ &+\nu^2(H^+H) - \tfrac{1}{2}\lambda(H^+H)^2 + \mu^2\,\mathrm{Tr}\,\phi^2 - \tfrac{1}{2}\lambda_1(\mathrm{Tr}\,\phi^2)^2 - \lambda_2\,\mathrm{Tr}\,\phi^4 \\ &-2\beta H^+\phi^2 H - 2\alpha\,\mathrm{Tr}\,\phi^2(H^+H) - (\rho\varepsilon_{ijklm}\bar{\psi}^c_{ij}H_k\psi_{lm} + \delta\bar{\chi}_i H_j\psi^c_{ij} + \mathrm{h.c.})\,. \end{aligned} \tag{2.6}$$

The first line represents the pure gauge theory with the gauge-fixing term and the ghosts, then we have the kinetic energy terms of the scalar Higgs H and ϕ and of the fermions χ and ψ as well as their gauge interaction through their covariant derivatives

$$\begin{aligned} D_\mu H &= \partial_\mu - igW_\mu H\,, \\ D_\mu\psi &= \partial_\mu - ig(W\psi + \psi W^\perp)\,, \\ D_\mu\chi &= \partial_\mu + igW^\perp\chi\,, \\ D_\mu\phi &= \partial_\mu - ig[W_\mu, \phi]\,, \end{aligned} \tag{2.7a}$$

and the strength $F^{\mu\nu}$ is

$$F^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu - ig[W^\mu, W^\nu]\,. \tag{2.7b}$$

The remaining terms represent the Higgs potential and the two possible Yukawa couplings of H with the fermions.

After spontaneous symmetry breaking which occurs via the Higgs field translations $\phi\to\phi - V$ and $H\to H - V_0$, some of the particles get masses and some of them remain massless. The vacuum expectation values $V = \langle\phi\rangle$ and $V_0 = \langle H\rangle$ are chosen to be [4]

$$V = v\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & -\frac{3}{2}-\frac{1}{2}\varepsilon & & \\ & & & & -\frac{3}{2}+\frac{1}{2}\varepsilon & \end{bmatrix}, \qquad v\sim O(10^{15}\,\mathrm{GeV})\,, \tag{2.8}$$

$$V_0 = v_0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad v_0 \sim \mathrm{O}(10^2\ \mathrm{GeV}), \qquad \varepsilon \sim \mathrm{O}\left(\frac{v_0^2}{v^2}\right). \tag{2.9}$$

Let us now examine the mass spectrum [4, 25] of the charged particles in the 't Hooft–Landau gauge ($\xi \to 0$). The choice of this particular gauge will be transparent in the following sections.

2.1. GAUGE VECTOR BOSONS

(1) Eight massless and neutral gluons:

$$M_{\mathrm{G}} = 0, \qquad Q_{\mathrm{G}} = 0. \tag{2.10}$$

(2) Twelve charged superheavy vector bosons $\mathrm{X}_i^{\pm}$, $\mathrm{Y}_i^{\pm}$ with

$$M_{\mathrm{X}}^2 = \tfrac{1}{4}(5+\varepsilon)^2 g^2 v^2, \qquad M_{\mathrm{Y}}^2 = \tfrac{1}{4}(5-\varepsilon)^2 g^2 v^2 + \tfrac{1}{2} g^2 v_0^2, \tag{2.11a}$$

$$Q_{\mathrm{X}^{\pm}} = \pm\tfrac{4}{3}, \qquad Q_{\mathrm{Y}^{\pm}} = \pm\tfrac{1}{3}. \tag{2.11b}$$

(3) Two charged massive $\mathrm{W}^{\pm}$ vector bosons:

$$M_{\mathrm{W}}^2 = \tfrac{1}{2} g^2 v_0^2 + \varepsilon^2 g^2 v^2, \qquad Q_{\mathrm{W}^{\pm}} = \pm 1. \tag{2.12}$$

(4) Two neutral vector bosons, one massive (Z) and one massless, the photon (A). These two neutral particles are related with the non-physical W_3 and B^0 as

$$\begin{pmatrix} \mathrm{Z} \\ \mathrm{A} \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} \mathrm{W}^3 \\ \mathrm{B}^0 \end{pmatrix}, \qquad \text{with } \sin\theta_0 = \sqrt{\tfrac{3}{8}}, \tag{2.13a}$$

$$M_{\mathrm{A}}^2 = 0, \qquad M_{\mathrm{Z}}^2 = \tfrac{8}{5}\tfrac{1}{2} g^2 v_0^2, \qquad Q_{\mathrm{A}} = Q_{\mathrm{Z}} = 0. \tag{2.13b}$$

Between the M_{X}, M_{Y}, M_{W} and M_{Z} there exists the relation

$$M_{\mathrm{X}} = \sqrt{M_{\mathrm{Y}}^2 - \tfrac{5}{8} M_{\mathrm{Z}}^2} + \sqrt{M_{\mathrm{W}}^2 - \tfrac{5}{8} M_{\mathrm{Z}}^2}. \tag{2.14}$$

2.2. SCALAR HIGGS BOSONS

(1) Eight neutral superheavy scalar Higgs corresponding to the massless gluons, ϕ_{G}:

$$M_8 = \tfrac{1}{2}\lambda_2 v^2 (5+\varepsilon)(1-\varepsilon), \qquad Q_8 = 0. \tag{2.15}$$

(2) Twelve massless charged Goldstone bosons associated to $\mathrm{X}_i^{\pm}$ and $\mathrm{Y}_i^{\pm}$:

$$\begin{aligned} &\phi_{\mathrm{X}_i}^{\pm} \quad \text{and } \tilde{\phi}_{\mathrm{Y}_i}^{\pm} = \phi_{\mathrm{Y}}^{\pm} \cos\varphi_{\mathrm{Y}} - H^{\pm} \sin\varphi_{\mathrm{Y}}, \\ &M_{\phi_{\mathrm{X}}} = M_{\tilde{\phi}_{\mathrm{Y}}} = 0, \end{aligned} \tag{2.16a}$$

where φ_Y is a mixing angle between the two Higgs

$$\sin\varphi_Y = \sqrt{\frac{5}{8}}\frac{M_Z}{M_Y}. \tag{2.16b}$$

(3) Six charged superheavy Higgs $\tilde{H}^{\pm}_{Y_i}$ which form the remaining orthogonal combination of ϕ_{Y_i} and H_{Y_i}:

$$\tilde{H}^{\pm}_{Y_i} = H^{\pm}_{Y_i}\cos\varphi_Y + \phi^{\pm}_{Y_i}\sin\varphi_Y,$$

with

$$M_3^2 = -\tfrac{5}{2}\beta v^2(1-\varepsilon)\left[1-\frac{\varepsilon}{5}+\frac{2}{5}\frac{v_0^2}{v^2}\frac{1}{5-\varepsilon}\right]. \tag{2.17}$$

(4) Two charged massless Goldstone bosons associated to $W^{\pm}$, $\tilde{H}^{\pm}_W$:

$$\tilde{H}^{\pm}_W = H^{\pm}_W\cos\varphi_W - \phi^{\pm}_W\sin\varphi_W,$$

with

$$\cos\varphi_W = \sqrt{\frac{5}{8}}\frac{M_Z}{M_W}, \qquad M_{\tilde{H}_W} = 0. \tag{2.18}$$

(5) The remaining orthogonal combination of $\phi^{\pm}_W$ and $H^{\pm}_W$, $\tilde{\phi}^{\pm}_W$ gets a superheavy mass M_4

$$\tilde{\phi}^{\pm}_W = \phi^{\pm}_W\cos\varphi_W + H^{\pm}_W\sin\varphi_W,$$

with

$$M_4^2 = 3\beta\left[\frac{v_0^2}{\varepsilon}+2\varepsilon v^2\right]. \tag{2.19}$$

(6) One neutral and massless Goldstone boson I_5 corresponding to the Z gauge boson

$$M_{I_5} = 0. \tag{2.20}$$

(7) Three massive neutral bosons; two of them are superheavy $\tilde{\phi}_{24}$, $\tilde{\phi}_{23}$ and the other one $\tilde{R}_5$ has a light mass M_5:

$$\tilde{R}_5 \to M_5^2 = v_0^2\left[2\lambda - \frac{9}{5}\frac{\beta^2}{\lambda_2} - \frac{6}{5}\frac{(10\alpha+3\beta)^2}{15\lambda_1+7\lambda_2}\right] + O\left(\frac{v_0^4}{v^2}\right), \tag{2.21a}$$

$$\tilde{\phi}_{24} \to M_{24}^2 = (15\lambda_1+7\lambda_2)v^2 + O(v_0^2), \tag{2.21b}$$

$$\tilde{\phi}_{23} \to M_{23}^2 = 10\lambda_2 v^2 + O(v_0^2). \tag{2.21c}$$

These three physical scalar bosons are related via three mixing angles with the R_5,

ϕ_{23} and ϕ_{24} fields:

$$\begin{pmatrix} \tilde{\phi}_{24} \\ \tilde{\phi}_{23} \\ \tilde{R}_5 \end{pmatrix} = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \sin\phi\sin\theta & -\cos\phi\sin\psi - \cos\theta\cos\psi\sin\phi \\ \sin\theta\sin\psi & \cos\theta & -\sin\theta\cos\psi \\ \sin\phi\cos\psi + \cos\theta\sin\psi\cos\phi & -\cos\phi\sin\theta & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi \end{pmatrix} \times \begin{pmatrix} \phi_{24} \\ \phi_{23} \\ R_5 \end{pmatrix}, \tag{2.22a}$$

with

$$\sin\theta = \frac{3}{5\sqrt{2}}\frac{\beta}{\lambda_2}\frac{v_0}{v} + O\left(\frac{v_0^3}{v^3}\right),$$

$$\sin\psi = \frac{1}{3\sqrt{30}}\frac{v_0}{v}\frac{6(10\alpha+3\beta)-(17\lambda_2+15\lambda_1)}{(\lambda_2-\lambda_1)} + O\left(\frac{v_0^3}{v^3}\right), \tag{2.22b}$$

$$\sin\phi = \frac{1}{3\sqrt{30}}\frac{v_0}{v}\frac{(17\lambda_2+15\lambda_1)(7\lambda_2+15\lambda_1)-60\lambda_2(10\alpha+3\beta)}{(7\lambda_2+15\lambda_1)(\lambda_2-5\lambda_1)} + O\left(\frac{v_0^3}{v^3}\right).$$

2.3. FERMIONS

(1) The d-quark and the electron get the same mass:

$$m_d = m_e = -\frac{\sqrt{10}}{4}\frac{\delta_1}{g}M_Z, \qquad Q_d = -\tfrac{1}{3}, \qquad Q_e = -1. \tag{2.23}$$

(2) The u-quark gets a different mass given as

$$m_u = 2\sqrt{5}\frac{\rho_1}{g}M_Z, \qquad Q_u = \tfrac{2}{3}. \tag{2.24}$$

Analogous relations are valid for every family:

$$m_s = m_\mu = -\frac{\sqrt{10}}{4}\frac{\delta_2}{g}M_Z,$$

$$m_c = 2\sqrt{5}\frac{\rho_2}{g}M_Z,$$

$$m_b = m_\tau = -\frac{\sqrt{10}}{4}\frac{\delta_3}{g}M_Z, \tag{2.25}$$

$$m_t = 2\sqrt{5}\frac{\rho_3}{g}M_Z.$$

All the above mass parameters are not independent. Choosing the 14 independent parameters of the theory (with 3-fermion families and neglecting the Cabibbo angles) to be: the gauge coupling constant "g", four superheavy masses "M_X, M_8, M_3, M_{24}" two light masses "M_W, M_5", six fermion masses "$m_e, m_\mu, m_\tau, m_u, m_c, m_t$" and one mixing angle "$\phi$" (between the neutral Higgs), all the other masses, mixing angles and potential coupling constants can be expressed as functions of the 14 independent parameters indicated before.

In the applications we always neglect terms of order $v_0^2/v^2 \sim O(10^{-26})$ compared to unity. In this approximation one has the tree-level relations

$$\begin{aligned} M_Z^2 &= \tfrac{8}{5} M_W^2 , \\ M_Y^2 &= M_X^2 , \\ M_{23}^2 &= M_4^2 = 4M_8^2 . \end{aligned} \tag{2.26}$$

We close this section noting that this spectrum is obtained using the 't Hooft–Landau gauge. If one uses a general covariant gauge the Goldstone bosons become massive.

3. Renormalization, A–Z mixing

The most important consequence of grand unification is that the strength of the three different low-energy interactions (strong, electromagnetic, weak) are described with one gauge coupling constant. The way the three low-energy effective interactions are related will be presented in the following section. Here we examine some consequences of the renormalizability of the spontaneously broken gauge theories which will be used in the next sections.

In the symmetric phase of the theory, which is defined by the lagrangian (2.7) with $\mu^2, \nu^2 < 0$, the renormalization procedure redefines the bare fields, bare coupling constants and bare masses in terms of the renormalized ones as

$$\phi_R^i = Z_{\phi^i}^{1/2} \phi_0^i , \qquad (\phi^i \to W, \phi, H, \chi, \psi) , \tag{3.1a}$$

$$g_R = Z_g g_0 , \tag{3.1b}$$

$$\lambda_R^i = Z_{\lambda^i} \lambda_0^i , \qquad (\lambda^i \to \lambda, \lambda_1, \lambda_2, \alpha, \beta, \rho_i, \delta_i) , \tag{3.1c}$$

$$m_R^{i2} = m_0^{i2} - \delta m^{i2} , \qquad (m_i^2 \to -\mu^2, -\nu^2) . \tag{3.1d}$$

The label 0 denotes bare quantities and the R renormalized ones. Once the Z_{ϕ^i}, Z_g, Z_{λ^i} and δm^{i2} are found, the Green functions of the theory are finite. It is well known that the counterterms of the symmetric phase make the theory in the broken phase also finite. So, using the same wave-function renormalizations Z_{ϕ^i} and the same gauge coupling constant renormalization Z_g and instead of the λ^i and m^i renormalization ($Z_{\lambda^i}, \delta m^i$) using the 13 independent parameters chosen in the previous section, i.e. the mass renormalization δM_X, δM_3, δM_8, δM_W, δM_5, δm_f^i and

the mixing angle renormalization $\delta\phi$, we will find the spontaneously broken SU(5) theory to be finite.

What are the consequences of this observation? First, the wave-function renormalization constants of the left-handed leptons are the same as those of the neutrino and the right-handed d-quark because they belong to the same $\bar{5}$ multiplet. Also, the wave-function renormalization of the right-handed leptons is the same as that of the left-handed d-quark and both of the left and right-handed u-quark as they belong to the same 10 multiplet. Similarly, the photon wave-function renormalization will be the same as that of the Z-boson, $W^{\pm}$ bosons, gluons and superheavy gauge bosons $X_i^{\pm}$, $Y_i^{\pm}$. The same is true for all multiplets W_a, χ, ψ, ϕ, H which appear in the SU(5) model. Second, the gauge coupling constant counterterm Z_g will be the same for the strong, electromagnetic and weak interactions. This seems to be strange because only one β-function can be defined through a unique Z_g,

$$\beta_{SU(5)} = g_R \mu^2 \frac{d}{d\mu^2} Z_g , \tag{3.2}$$

which does not seem to explain the presence of the three distinct β-functions $\beta_{U(1)}$, $\beta_{SU(2)}$, $\beta_{SU(3)}$ in the low-energy region. In the next sections we answer this question and we explain in detail the way to define the three β-functions. We will see for example that $\beta_{U(1)}$ is an SU(5) β-function with a particular definition of the renormalized coupling constant. The same is true for $\beta_{SU(2)}$ and $\beta_{SU(3)}$:

$$\begin{aligned} \beta^{(1)}_{SU(5)} &= \beta_{U(1)} , \\ \beta^{(2)}_{SU(5)} &= \beta_{SU(2)} , \qquad \text{in the low-energy region .} \\ \beta^{(3)}_{SU(5)} &= \beta_{SU(3)} , \end{aligned} \tag{3.3}$$

Working in the spontaneously broken phase of the theory, care must be taken with the redefinition of the physical particles, order by order in perturbation theory. As we saw in sect. 2 the definitions of the physical particles (as the photon A and Z; $\tilde{\phi}_Y$, $\tilde{H}_Y$; $\tilde{\phi}_W$, $\tilde{H}_W$; $\tilde{\phi}_{24}$, $\tilde{\phi}_{23}$, $\tilde{H}_5$) are given as rotations of the nonphysical ones W_3, B; ϕ_Y, H_Y; ϕ_W, H_W; ϕ_{24}, ϕ_{23}, R_5). The case of the A and Z is just an internal rearrangement inside the same multiplet contrary to the other cases (ex. $\tilde{\phi}_Y$, $\tilde{H}_Y$) which mix two different multiplets. In the tree-level approximation the A and Z are given in terms of W_3 and B as

$$\begin{pmatrix} Z \\ A \end{pmatrix}_0 = R_{\theta_0} \begin{pmatrix} W_3 \\ B \end{pmatrix}_0 , \qquad \text{with } R_{\theta_0} = \begin{pmatrix} c_0 & s_0 \\ -s_0 & c_0 \end{pmatrix} , \tag{3.4}$$

where $s_0 = \sin\theta_0 = \sqrt{\frac{3}{8}}$ and $c_0 = \cos\theta_0 = \sqrt{\frac{5}{8}}$.

When the different interactions come in, the above relation is modified, but a similar equation is valid:

$$\begin{pmatrix} Z \\ A \end{pmatrix}_R = R_\theta \begin{pmatrix} W_3 \\ B \end{pmatrix}_R , \qquad \text{with } R_\theta = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} , \tag{3.5}$$

and $s = s_0 + g^2\Delta$.

Since W_3 and B are two components of the same multiplet they need the same wave-function counterterm Z_W:

$$W_R^a = Z_W^{1/2} W_0^a . \tag{3.6}$$

Combining the last equation with eqs. (3.4), (3.5) we get

$$\begin{pmatrix} Z \\ A \end{pmatrix}_R = Z_W^{1/2} R_{(\theta-\theta_0)} \begin{pmatrix} Z \\ A \end{pmatrix}_0 , \qquad R_{(\theta-\theta_0)} = \begin{pmatrix} c_\Delta & s_\Delta \\ -s_\Delta & s_\Delta \end{pmatrix} , \tag{3.7}$$

with $s_\Delta = \sin(\theta - \theta_0)$ and $c_\Delta = \cos(\theta - \theta_0)$.

One needs the rotation $(\theta - \theta_0)$ to redefine the renormalized photon A and Z boson, as explained later on; comparing the renormalization relation (3.7) valid for the SU(5) model with the corresponding relation in the standard WS model $SU(2) \times U(1)$ one finds an essential difference: In the WS case, the renormalization of the (A, Z) is more complicated because the W_3 and B fields appear in two different multiplets, therefore two independent wave-function renormalization constants are needed:

$$\begin{pmatrix} Z \\ A \end{pmatrix}_R = R_\theta \begin{pmatrix} W_3 \\ B \end{pmatrix}_R = R_\theta \begin{pmatrix} Z_2 & 0 \\ 0 & Z_1 \end{pmatrix}^{1/2} \begin{pmatrix} W_3 \\ B \end{pmatrix}_0 , \tag{3.8a}$$

or

$$\begin{pmatrix} Z \\ A \end{pmatrix}_R = R_\theta \begin{pmatrix} Z_2 & 0 \\ 0 & Z_1 \end{pmatrix}^{1/2} R_{-\theta_0} \begin{pmatrix} Z \\ A \end{pmatrix}_0 \qquad \text{in the } SU(2) \times U(1) \text{ model} . \tag{3.8b}$$

Also, $\sin\theta_0$ is not fixed but is an independent parameter of the theory. A similar situation appears in the SU(5) model for example in the case of $\tilde{\phi}_Y$ and $\tilde{H}_Y$ renormalization because these fields are a combination of fields belonging to different multiplets ϕ_Y in 24, H_Y in 5. Let us now examine the renormalization procedure more precisely. But before doing so let us clarify our notation.

(i) We denote by $\Gamma(W_a^\mu, W_b^\nu)$ (fig. 1a) the 1PI two-point function of the gauge bosons and by $\Gamma_T(W_a, W_b)$ and $\Gamma_L(W_a, W_b)$ the coefficients of the decomposition

$$\Gamma(W_a^\mu, W_b^\nu) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Gamma_T(W_a, W_b) + g^{\mu\nu} \Gamma_L(W_a, W_b) , \tag{3.9a}$$

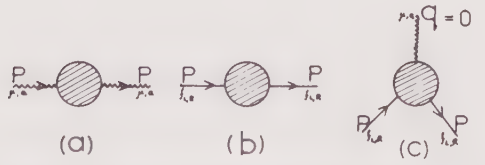

Fig. 1. (a) The 1PI two-point function of the gauge bosons $\Gamma(W_a^\mu, W_b^\nu)$. (b) The 1PI two-fermion function $\Gamma(f_{L,R} f_{L,R})$. (c) The 1PI three-point function of one gauge boson and two fermions $\Gamma(f_{L,R} W_a^\mu f_{L,R})$ at zero transfer momentum.

where p^μ is the external vector momentum. $\Gamma_T(W_a, W_b)$ is related to the wave-function renormalization and $\Gamma_L(W_a, W_b)$ to the vector-boson mass renormalization. It is convenient to define the function $\Pi_{ab}(p^2)$ as the derivative with respect to p^2 of $\Gamma_T(W_a, W_b)$ which has the property to be finite without a mass counterterm subtraction:

$$\Pi_{ab}(p^2) = \frac{\partial}{\partial p^2}\Gamma_T(W_a, W_b)\,. \tag{3.9b}$$

(ii) We denote by $\Gamma(f_{L,R}, f_{L,R})$ the 1PI two-fermion function (fig. 1b) and by $S_{f_{L,R}f_{L,R}}(p^2)$ the $\gamma^\mu(1\pm\gamma_5)/2$ part of the p^μ derivative of $\Gamma(f_{L,R}, f_{L,R})$:

$$\frac{\partial}{\partial p_\mu}\Gamma(f_{L,R}, f_{L,R}) = \gamma^\mu \frac{1\pm\gamma_5}{2} S_{f_{L,R}f_{L,R}}(p^2) + p^\mu \text{ terms}\,. \tag{3.10}$$

The function $S_{f_{L,R}f_{L,R}}(p^2)$ has the same property as $\Pi_{ab}(p^2)$ to be finite without a mass counterterm subtraction.

(iii) Finally we denote by $\Gamma(f_{L,R}W_a^\mu f_{L,R})$ the 1PI three-point function of one gauge boson and two left- or right-handed fermions, and by $V_{f_{L,R}W^a f_{L,R}}(p^2)$ the $\gamma^\mu(1\pm\gamma^5)/2$ coefficient of the $\Gamma(f_{L,R}W_a^\mu f_{L,R})$ evaluated at zero momentum transfer (see fig. 1c):

$$\Gamma(f_{L,R}W_a^\mu f_{L,R})|_{q^\mu=0} = \gamma^\mu \frac{(1\pm\gamma^5)}{2} g V_{f_{L,R}W_a f_{L,R}}(p^2) + p^\mu \text{ terms}\,. \tag{3.11}$$

According to eq. (3.7), one can see that the 1PI two-point function combination of A and Z satisfies the following renormalization relation:

$$\begin{pmatrix}\Gamma(Z^\mu Z^\nu) & \Gamma(Z^\mu A^\nu)\\ \Gamma(A^\mu Z^\nu) & \Gamma(A^\mu A^\nu)\end{pmatrix}_R = Z_W R_{(\theta-\theta_0)}\begin{pmatrix}\Gamma(Z^\mu Z^\nu) & \Gamma(Z^\mu A^\nu)\\ \Gamma(A^\mu Z^\nu) & \Gamma(A^\mu A^\nu)\end{pmatrix}_0 R_{-(\theta-\theta_0)}\,, \tag{3.12a}$$

or

$$\begin{pmatrix}\Gamma_R(Z^\mu Z^\nu) & \Gamma_R(Z^\mu A^\nu)\\ \Gamma_R(A^\mu Z^\nu) & \Gamma_R(A^\mu A^\nu)\end{pmatrix}$$

$$= Z_W\begin{bmatrix} c_\Delta^2\Gamma_0(Z^\mu Z^\nu) + s_\Delta^2\Gamma_0(A^\mu A^\nu) + 2s_\Delta c_\Delta\Gamma_0(A^\mu Z^\nu) \\ s_\Delta c_\Delta[\Gamma_0(A^\mu A^\nu) - \Gamma_0(Z^\mu Z^\nu)] + (c_\Delta^2 - s_\Delta^2)\Gamma_0(A^\mu Z^\nu) \end{bmatrix}$$

$$\begin{bmatrix} s_\Delta c_\Delta(\Gamma_0(A^\mu A^\nu) - \Gamma_0(Z^\mu Z^\nu)) + (c_\Delta^2 - s_\Delta^2)\Gamma_0(A^\mu Z^\nu) \\ s_\Delta^2\Gamma_0(Z^\mu Z^\nu) + c_\Delta^2\Gamma_0(A^\mu A^\nu) + 2c_\Delta s_\Delta\Gamma_0(A^\mu Z^\nu) \end{bmatrix}. \tag{3.12b}$$

At the one-loop level, relations (3.12a, b) become very simple

$$\Gamma_R(Z^\mu Z^\nu) = (1 + Z_W^{(1)})\Gamma_0(Z^\mu Z^\nu)\,, \tag{3.13a}$$

$$\Gamma_R(A^\mu A^\nu) = (1 + Z_W^{(1)})\Gamma_0(A^\mu A^\nu)\,, \tag{3.13b}$$

$$\Gamma_R(A^\mu Z^\nu) = \Gamma_0(A^\mu Z^\nu) + M_Z^2\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\sin(\theta - \theta_0)\,, \tag{3.13c}$$

where $Z_W = 1 + Z_W^{(1)} + O(g^4)$ [in eqs. (3.1a)]. Note that $\Gamma_0(A^\mu Z^\nu)$ is finite at the one-loop level, and $\Gamma_0(Z^\mu Z^\nu)$, $\Gamma_0(A^\mu A^\nu)$ are multiplicatively renormalizable. This is true only at this approximation, at higher orders (see eq. 3.12b) the relation between Γ_0 and Γ_R being more complicated.

In the standard WS model the corresponding relation to eq. (3.12a) becomes much more complicated:

$$\begin{bmatrix}\Gamma(Z^\mu Z^\nu) & \Gamma(Z^\mu A^\nu)\\ \Gamma(A^\mu Z^\nu) & \Gamma(A^\mu A^\nu)\end{bmatrix}_R = R_\theta\begin{pmatrix}Z_2 & 0\\ 0 & Z_1\end{pmatrix}^{1/2} R_{-\theta_0}\begin{bmatrix}\Gamma(Z^\mu Z^\nu) & \Gamma(Z^\mu A^\nu)\\ \Gamma(A^\mu Z^\nu) & \Gamma(A^\mu A^\nu)\end{bmatrix}_0 R_{\theta_0}\begin{pmatrix}Z_2 & 0\\ 0 & Z_1\end{pmatrix}^{1/2} R_{-\theta}\,. \tag{3.14}$$

At the one-loop level it is also true that $\Gamma_0(Z^\mu Z^\nu)$ and $\Gamma_0(A^\mu A^\nu)$ are multiplicatively renormalizable. The difference from the SU(5) model is that the A and Z wave-function renormalizations are combinations of $Z_1^{(1)}$, $Z_2^{(1)}$ and $\sin^2\theta_0$. On the other hand, $\Gamma_0(A^\mu Z^\nu)$ is infinite and it is related to the $\Gamma_R(A^\mu, Z^\nu)$ through $Z_1^{(1)}$, $Z_2^{(2)}$ and $\sin^2\theta_0$ which are also infinite.

Eq. (3.12a) is also valid for the scalar functions $\Pi_{ab}(p^2)$ $(i, j = A, Z)$ defined by eqs. (3.9a, b):

$$\begin{bmatrix}\Pi_{ZZ} & \Pi_{ZA}\\ \Pi_{AZ} & \Pi_{AA}\end{bmatrix} = Z_W R_{(\theta-\theta_0)}\begin{bmatrix}\Pi_{ZZ} & \Pi_{ZA}\\ \Pi_{AZ} & \Pi_{AA}\end{bmatrix} R_{-(\theta-\theta_0)}\,. \tag{3.15a}$$

Let us now consider the way the 1PI three-point functions $\Gamma(fZ^\mu f)$ and $\Gamma(fA^\mu f)$ are renormalized. Using eq. (3.7) one has

$$\begin{pmatrix}\Gamma(fZ^\mu f)\\ \Gamma(fA^\mu f)\end{pmatrix}_R = Z_W^{1/2} Z_f R_{(\theta-\theta_0)}\begin{pmatrix}\Gamma(fZ^\mu f)\\ \Gamma(fA^\mu f)\end{pmatrix}_0\,, \tag{3.15b}$$

where Z_f is the fermion wave function counterterm defined by the equation

$$\Gamma_R(ff) = Z_f\Gamma_0(ff)\,, \tag{3.16a}$$

or

$$S_{ff}(p^2)_R = Z_f S_{ff}(p^2)_0\,. \tag{3.16b}$$

Eq. (3.15) implies that the vertex functions V_{fAf} and V_{fZf} (see eq. 3.10) are renormalized as

$$\begin{pmatrix}V_{fZf}\\ V_{fAf}\end{pmatrix}_R = Z_1 R_{(\theta-\theta_0)}\begin{pmatrix}V_{fZf}\\ V_{fAf}\end{pmatrix}_0\,, \tag{3.17a}$$

where Z_1 is the 1PI vertex counterterm,

$$Z_1 = Z_W^{1/2} Z_f Z_g^{-1}\,, \qquad \text{with } g_R = Z_g g_0\,. \tag{3.17b}$$

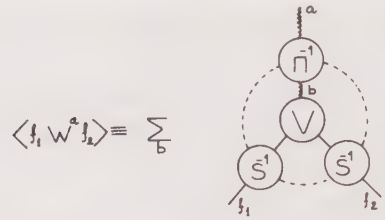

Fig. 2. Diagrammatic representation of the effective charges $\langle f_1 W^a f_2 \rangle$.

Our aim is to determine Z_g. This determination is not unique, and depends on the renormalization condition chosen for the 1PI two- and three-point functions. To relate as much as possible this choice to the measured coupling constants we define a quantity $\langle fW^a f \rangle$, $a = 1, 2, \ldots, 24$, that we call "effective charge of the W^a" as

$$\langle f_1 W^a f_2 \rangle = \sum_b S^{-1/2}_{f_1 f_1} S^{-1/2}_{f_2 f_2} \Pi^{-1/2}_{ab} V_{f_1 w^b f_2} . \tag{3.18}$$

In fig. 2 we give a diagrammatic representation of the effective charges.

They indicate the strength of the interaction between the vector boson W^a and the fermions f_1 and f_2. The renormalization of these objects is very simple and it can be expressed only with Z_g.

(i) A and Z effective charges:

$$\begin{pmatrix} \langle fZf \rangle \\ \langle fAf \rangle \end{pmatrix}_R = Z_g^{-1} R_{(\theta - \theta_0)} \begin{pmatrix} \langle fZf \rangle \\ \langle fAf \rangle \end{pmatrix}_0 . \tag{3.19a}$$

(ii) $W^{\pm}$ effective charge:

$$\langle f_1 W^{\pm} f_2 \rangle_R = Z_g^{-1} \langle f_1 W^{\pm} f_2 \rangle_0 . \tag{3.19b}$$

(iii) Gluon effective charge:

$$\langle fG^a f \rangle_R = Z_g^{-1} \langle fG^a f \rangle_0 . \tag{3.19c}$$

Eqs. (3.19a, b, c) are obtained simply from eqs. (3.15), (3.16b), (3.17b), (3.18). Imposing a renormalization condition to one renormalized effective charge, say $\langle fAf \rangle_R$, Z_g is determined, once the rotation $(\theta - \theta_0)$ is known. The other effective charges $\langle fW^a f \rangle$ are renormalized with the same Z_g and there is no other freedom left for the renormalized quantities. The problem is how can one simply determine θ. This can be done easily. If one calculates the bare neutrino matrix element $\langle \nu A \nu \rangle_0$, one will find that it is different from zero. On the other hand, one knows that the renormalized $\langle \nu A \nu \rangle_R$ must give zero. Using (3.18) and (3.19a) one has

$$\operatorname{tg}(\theta - \theta_0) = \frac{\langle \nu A \nu \rangle_0}{\langle \nu Z \nu \rangle_0} \Leftrightarrow \langle \nu A \nu \rangle_R = 0 . \tag{3.20}$$

One can also use the fact that the left-handed effective charge must be equal to

the right-handed one

$$\langle f_L A f_L\rangle_R = \langle f_R A f_R\rangle_R \,. \tag{3.21}$$

The bare quantities do not, in general, satisfy the above relation. Using eqs. (3.18) and (3.20) one finds

$$\mathrm{tg}\,(\theta-\theta_0) = \frac{\langle f_L A f_L\rangle_0 - \langle f_R A f_R\rangle_0}{\langle f_L Z f_L\rangle_0 - \langle f_R Z f_R\rangle_0} \,. \tag{3.22}$$

Eq. (3.22) generalizes (3.20) and is valid for all fermions. The question that arises here is that in general this angle [eq. (3.22)] may be a fermion-dependent quantity. In appendix A we give the proof that the value of $(\theta-\theta_0)$ is fermion independent at the one-loop level and in the approximation where the first fermion family (where the effective charges are defined) is massless. In the same appendix we demonstrate that the $\langle fZf\rangle_R$, $\langle fAf\rangle_R$, $\langle f_1 W^{\pm} f_2\rangle_R$ and $\langle fG^a f\rangle_R$ have the following forms:

$$\langle fAf\rangle_R = iZ_g^{-1} R_A(p)\sqrt{\tfrac{3}{8}}Q_f \,, \tag{3.23a}$$

$$\langle fZf\rangle_R = iZ_g^{-1} R_Z(p)\sqrt{\tfrac{8}{5}}[T_{3f} - Q_f \sin^2\theta_{\gamma Z}] \,, \tag{3.23b}$$

$$\langle f_1 W^{\pm} f_2\rangle_R = iZ_g^{-1} R_W(p)(T_{\pm})_{12} \,, \tag{3.23c}$$

$$\langle f_1 G^a f_2\rangle_R = iZ_g^{-1} R_G(p)(\lambda^a)_{12} \,, \tag{3.23d}$$

where R_i, $i = A, Z, W, G$ are fermion-independent functions of the momentum p, Q_f is the electric charge of the fermion f, $T \equiv (T_{3f}, T_{+f}, T_{-f})$ is the weak SU(2) generators action on the fermion f and $2\lambda^a$ are the 8 Gell-Mann SU(3) matrices.

We close this section by giving the renormalization conditions we choose

(i) Vector-boson two-point functions:

$$\text{(1)}\ \Gamma_R(W_a^{\mu}, W_a^{\nu})|_{p^2=M_{W_a}^2} = 0 \,, \qquad \text{for the independent mass parameters } M_W, M_X \,. \tag{3.24a}$$

$$\text{(2)}\ \Pi_{aa}(-p^2)_R|_{p^2=-\mu^2} = i \qquad \Rightarrow Z_a^{-1} = \Pi_{aa}^0(\mu^2) \,, \tag{3.24b}$$

for one specific component a.

(ii) Fermion two-point functions:

$$[\Gamma_R(f_R f_R) + \Gamma_R(f_L f_L)]|_{p=m_f} = 0 \,, \qquad \text{for every independent fermion mass parameter} \,, \tag{3.25a}$$

$$S_{ff}(-p^2)|_{p^2=-\mu^2} = i \qquad \Rightarrow Z_f^{-1} = S_{ff}^0(\mu^2) \,, \tag{3.25b}$$

for one component f in each irreducible representation of fermions.

(iii) Effective charge renormalization

$$R_a(-p^2)|_{p^2=-\mu^2} = 1 \qquad \Rightarrow Z_{g_a} = R_a(\mu^2) \,, \tag{3.26}$$

where the index a is one of the components in the adjoint representation.

4. β-function definitions

We are going to describe now the formalism we use to determine the relations between the observed strength of the three interactions at low energies.

As we noted in the previous section, in the SU(5) model, the unique coupling constant g is defined using only one renormalization condition. For example one can choose this condition to be,

$$g_G\langle f_i G^a f_j\rangle_R|_{p^2=-\mu^2} = i\lambda^a_{ij} g_G\,, \tag{4.1}$$

then one obtains a counterterm Z_{g_G} and a renormalized coupling constant g_G such that [see eqs. (3.19c), (3.26)]

$$Z_{g_G}(g_G, \varepsilon, \mu^2, m_i^2) = R_G(\mu^2)\,, \tag{4.2a}$$

$$g_G = Z_{g_G} g_0\,, \tag{4.2b}$$

where we used the dimensional regularization in $n = 4-2\varepsilon$ dimensions.

Using now this counterm we will find the photon (for example) three-point function $\langle fAf\rangle_R$ to be also finite. The problem is that the perturbative expansion of $\langle fAf\rangle_R$ in terms of the coupling constant g_G contains large logarithms and it becomes useless for practical computations. On the other hand, if one defines the unique coupling constant with the photon three-point function $\langle fAf\rangle_R$ as

$$g_A\langle fAf\rangle|_{p^2=-\mu^2} = i g_A\sqrt{\tfrac{3}{8}} Q_f\,, \tag{4.3}$$

which implies that

$$Z_{g_A}(g_A, \varepsilon, \mu^2, m_i^2) = R_A(\mu^2)\,, \tag{4.4a}$$

$$g_A = Z_{g_A} g_0\,, \tag{4.4b}$$

then, in the perturbative expansion of $\langle fAf\rangle$ in powers of $g_A\sqrt{\frac{3}{8}} \equiv e$ no such large logarithms appear; they will appear in the perturbative expansion of $\langle fGf\rangle_R$ in term of e. So one observes that some definitions of the renormalized coupling constants are useless for some matrix elements and quite good for some others.

In ref. [18] a method has been developed to relate g_A and g_G using renormalization group arguments. Let us write the non-perturbative relation between g_A and g_G as

$$g_A = g_A(g_G)\,; \tag{4.5}$$

then $g_A(g_G)$ satisfies the following differential equation:

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta_G(g_G)\frac{\partial}{\partial g_G}\right] g_A(g_G) = \beta_A(g_A(g_G))\,, \tag{4.6a}$$

with

$$\beta_i(g_i) = g_i \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \log Z_{g_i}(g_i, \varepsilon, \mu^2, m_i^2)\,, \tag{4.6b}$$

$$\beta_i(g_i) = \tfrac{1}{2} g_i \left[b_0^i(\mu^2, m_i^2) \left(\frac{g_i}{4\pi} \right)^2 + b_1^i(\mu^2, m_i^2) \left(\frac{g_i}{4\pi} \right)^4 + O(g^6) \right]. \tag{4.6c}$$

Eq. (4.6a) is only true in the 't Hooft–Landau gauge with on-shell mass renormalization. In a general ξ gauge, terms like $\delta(g)\partial/\partial\xi$ appear and make the solution of this equation more complicated. Working always in the 't Hooft–Landau gauge, eq. (4.6a) is equivalent to the following system of differential equations:

$$\frac{dg_A}{\beta_A(g_A)} = \frac{dg_G}{\beta_G(g_G)} = \frac{d\mu^2}{\mu^2}. \tag{4.7}$$

Imposing the asymptotic condition

$$g_A(\mu) \underset{\mu \to \infty}{=} g_G(\mu), \tag{4.8}$$

which represents the uniqueness of the renormalized coupling constant in the limit where $m_i^2/\mu^2 \to 0$ (massless limit or high-energy behaviour), one has the following integral solution of eq. (4.6a):

$$\frac{1}{g_i^2(\lambda)} - \frac{1}{g_i^2(\lambda)} = \int_0^\lambda \frac{dx}{x} \left[\frac{2\beta_i(g_i(x), x, \tau)}{g_i^3(x)} - \frac{2\beta_j(g_j(x), x, \tau)}{g_j^3(x)} \right], \tag{4.9}$$

where $\lambda = M_X^2/\mu^2$ and τ stands for all possible mass ratios. The relation (4.9) for the two-loop β_i functions becomes

$$\frac{1}{\alpha_i(\lambda)} - \frac{1}{\alpha_j(\lambda)} = \frac{1}{4\pi} \int_0^\lambda \frac{dx}{x} \left\{ (b_i^0(x, \tau) - b_j^0(x, \tau) + \left(b_i^1(x, \tau) \frac{\alpha_i(x)}{4\pi} - b_j^1(x, \tau) \frac{\alpha_j(x)}{4\pi} \right) \right\}, \tag{4.10}$$

where $\alpha_i = g_i^2/4\pi$ and b_i^0, b_i^1 are defined in eq. (4.6c). In the following, eq. (4.10) will be our standard equation. It relates two different definitions of the same object (the renormalized gauge coupling constant of the SU(5) theory) up to two-loop level. As we have explained before, several definitions are needed in order to absorb the large logarithms which appear in the perturbative expansion in terms of the "non-natural" renormalized coupling constant. We list here the definitions we used through the effective charges defined by eqs. (3.18), (3.19):

$$\langle fZf \rangle_R|_{p^2 = -\mu^2} = i\sqrt{\tfrac{8}{5}} [T_{3f} - Q_f \sin^2\theta] \to \alpha_Z, \tag{4.11a}$$

$$\langle fAf \rangle_R|_{p^2 = -\mu^2} = i\sqrt{\tfrac{3}{8}} Q_f \to \alpha_A, \tag{4.11b}$$

$$\langle f_i G^a f_j \rangle_R|_{p^2 = -\mu^2} = i\lambda_{ij}^a \to \alpha_G, \tag{4.11c}$$

$$\langle f_1 W^\pm f_2 \rangle_R|_{p^2 = -\mu^2} = iT_{12}^\pm \to \alpha_W, \tag{4.11d}$$

$$\langle f_\alpha W^a f_\beta \rangle_R|_{\substack{p^2 = -\mu^2 \\ m_i^2 = 0}} = i\lambda_{\alpha\beta}^a, \; a = 1, \ldots, 24 \to \alpha_{sym}. \tag{4.11e}$$

The last definition of the renormalized coupling constant (4.11e) gives the symmetric counterterm $Z_{g_{sym}}$ of the unbroken SU(5) theory. One has,

$$Z_{g_{sym}} = R_i(\mu^2, m_i^2, g_{sym}, \varepsilon)|_{m_i^2=0} \equiv R_{sym}(\mu^2, \varepsilon, g_{sym}) , \tag{4.12}$$

where $R_i|_{m_i^2=0}$, $i = A, Z, G, W, \ldots$, is independent of the choice of i. Writing $R_i(\mu^2, m_i^2, g_i, \varepsilon)$ in perturbation as

$$R_i(\mu^2, m_i^2, g_i, \varepsilon) = 1 + R_i^0(\mu^2, m_i^2, \varepsilon)\frac{1}{2}\frac{g_i^2}{(4\pi)^2} + R_i^1(\mu^2, m_i^2, \varepsilon)\frac{1}{2}\frac{g_i^4}{(4\pi)^4} + O(g_i^6) , \tag{4.13}$$

it is straightforward to see that,

$$b_i^0(\mu^2, m_i^2) = \mu^2 \frac{\partial}{\partial \mu^2} R_i^0(\mu^2, m_i^2, \varepsilon) . \tag{4.14}$$

Now using the relation (4.14), eq. (4.10) becomes

$$\left(\frac{4\pi}{\alpha_i(\lambda)} - \frac{4\pi}{\alpha_j(\lambda)}\right) = R_j^0(\lambda, \tau) - R_i^0(\lambda, \tau) + \int_0^\lambda \frac{dx}{x}\left(b_i^1(x, \tau)\left(\frac{g_i}{4\pi}\right)^2 - b_j^1(x, \tau)\left(\frac{g_j}{4\pi}\right)^2\right) , \tag{4.15}$$

where $i, j = A, Z, G, W, SYM$; $\lambda = M_X^2/\mu^2$, $\tau = M_i/M_X$ (all possible mass ratios).

To derive eq. (4.15) we use the property (4.12)

$$R_j^0(\lambda, \tau) - R_i^0(\lambda, \tau)|_{\lambda=0} = 0 . \tag{4.16}$$

5. Threshold functions and decoupling theorem

The decoupling theorem [15, 16] applied to the SU(5) doubly broken gauge theory, tells us that working in a specific energy region one can use an effective theory to describe physical processes. Namely (i) the SU(5) symmetric theory for large energies, $\mu \gg M_X$; (ii) $SU(3)_c \times SU(2)_L \times U(1)$ for intermediate energies $M_X \gg \mu \gg M_W$, and (iii) $SU(3)_c \times U(1)_{em}$ for small energies $\mu \ll M_W$. The existence of these effective theories implies some interesting behaviour for the SU(5) β_i functions defined through the effective charges.

(i) In the large energy region one has the relation

$$\beta_i(g_i, \lambda_X, \lambda_W, \tau) \xrightarrow[\substack{\lambda_X \to 0 \\ \lambda_W \to 0}]{} \beta^{SU(5)}(g) , \qquad \forall i = A, Z, W, G \ldots , \tag{5.1}$$

where $\lambda_X = M_X^2/\mu^2$, $\lambda_W = M_W^2/\mu^2$, $\beta^{SU(5)}$ is the symmetric SU(5) β-function.

(ii) In the intermediate energy region, one has the relations

$$\beta_G(g_G, \lambda_X, \lambda_W, \tau) \xrightarrow[\substack{\lambda_X \to \infty \\ \lambda_W \to 0}]{} \beta^{SU(3)}(g_3; g_1, g_2) , \tag{5.2a}$$

$$\beta_W(g_W, \lambda_X, \lambda_W, \tau) \xrightarrow[\lambda_W \to 0]{\lambda_X \to \infty} \beta^{SU(2)}(g_2; g_1, g_3), \tag{5.2b}$$

$$\frac{1}{g_A^3}\beta_A(g_A, \lambda_X, \lambda_W, \tau) \xrightarrow[\lambda_W \to 0]{\lambda_X \to \infty} \frac{c_0^2}{g_1^3}\beta^{U(1)}(g_1; g_2; g_3) + \frac{s_0^2}{g_2^3}\beta^{SU(2)}(g_2; g_1, g_3), \tag{5.2c}$$

$$\frac{1}{g_Z^3}\beta_Z(g_Z, \lambda_X, \lambda_W, \tau) \xrightarrow[\lambda_W \to 0]{\lambda_X \to \infty} \frac{s_0^2}{g_1^3}\beta^{U(1)}(g_1; g_2, g_3) + \frac{c_0^2}{g_2^3}\beta^{SU(2)}(g_2; g_1, g_3). \tag{5.2d}$$

In eqs. (5.2a–d) the three independent β-functions $\beta^{SU(3)}$, $\beta^{SU(2)}$ and $\beta^{U(1)}$ are given as

$$\frac{2}{g_i}\beta^{SYM(i)} = \left(\frac{g_i}{4\pi}\right)^2 \sum_{j=1}^{3}\left[b_i^0\delta_{ij} + b_{ij}^1\left(\frac{g_j}{4\pi}\right)^2\right] + O(g_k^6), \tag{5.3}$$

where $\beta^{SYM(i)}$ stands for $\beta^{U(1)}$, $\beta^{SU(2)}$, $\beta^{SU(3)}$ for $i = 1, 2, 3$ and the coefficients b_i^0, b_{ij}^1 are given in the end of sect. 6.

Eqs. (5.2c, d) are simply related to the charge relation in SU(5):

$$s_0 Q = -s_0 T_3 + c_0 Y; \qquad c_0 = \sqrt{\tfrac{5}{8}}, \qquad s_0 = \sqrt{\tfrac{3}{8}}, \tag{5.5a}$$

with

$$Q = \begin{bmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & +1 & \\ & & & & 0 \end{bmatrix}, \quad T_3 = \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & -\frac{1}{2} & \\ & & & & +\frac{1}{2} \end{bmatrix},$$

$$Y = \sqrt{\tfrac{3}{5}}\begin{bmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & +\frac{1}{2} & \\ & & & & +\frac{1}{2} \end{bmatrix}. \tag{5.5b}$$

The charge relation (5.5a) is still valid for the effective $SU(3)_c \times SU(2)_L \times U(1)$ which implies the well-known WS relations ($e \equiv g_A s_0$, $g' \equiv g_1 s_0/c_0$)

$$e \equiv s_0 g_A = s g_2, \tag{5.6a}$$

$$g' \equiv g_1 \frac{s_0}{c_0} = \frac{s}{c} g_2, \tag{5.6b}$$

where $s = \sin\theta_W$ and $c = \cos\theta_W$. From eqs. (5.6a, b) one simply has,

$$\frac{1}{g_A^2} = \frac{s_0^2}{g_2^2} + \frac{c_0^2}{g_1^2}, \qquad \frac{1}{g_Z^2} = \frac{c_0^2}{g_2^2} + \frac{s_0^2}{g_1^2}, \tag{5.7}$$

which implies eqs. (5.2c, d).

(iii) Finally in the small energy region where the effective theory is $SU(3)_c \times U(1)^{em}$, one has

$$\beta_A(g_A, \lambda_X, \lambda_W, \tau) \xrightarrow[\lambda_W \to \infty]{\lambda_X \to \infty} \tilde{\beta}^{U(1)^{em}}(g_e; g_3), \tag{5.8a}$$

$$\beta_G(g_G, \lambda_X, \lambda_W, \tau) \xrightarrow[\lambda_W \to \infty]{\lambda_X \to \infty} \tilde{\beta}^{SU(3)}(g_3; g_e). \tag{5.8b}$$

In eqs. (5.8a, b), the two independent β-functions $\tilde{\beta}^{SU(3)}$ and $\tilde{\beta}^{U(1)^{em}}$ are given as

$$\frac{2}{g_i}\tilde{\beta}^{SYM(i)} = \left(\frac{g_i}{4\pi}\right)^2 \sum_{j=e,3} \left[\tilde{b}_i^0 \delta_{ij} + \tilde{b}_{ij}^1 \left(\frac{g_j}{4\pi}\right)^2\right] + O(g_k^6), \tag{5.8c}$$

where $\beta^{SYM(i)}$ stands for $\tilde{\beta}^{U(1)^{em}}$, $\tilde{\beta}^{SU(3)}$ for $i = e, 3$ and the coefficients $\tilde{b}_i^0$, $\tilde{b}_{ij}^1$ are given at the end of sect. 6.

To see how the decoupling theorem works let us examine the properties of the one-loop diagrams which contribute to the one-loop β_i functions. In the limit where we neglect the first family fermion masses the general form of the contribution to $R_i^0(-p^2)$ of a particular diagram has the form,

$$F(-p^2, \varepsilon, \lambda_1, \lambda_2) = \frac{1}{2}\frac{g^2}{(4\pi^2)}\left[\frac{C_F(\varepsilon)}{\varepsilon} - C_F(0)\log(-p^2) + \varphi_F(\lambda_1, \lambda_2) + O(\varepsilon)\right], \tag{5.9a}$$

with

$$\varphi_F(0, 0) = 0. \tag{5.9b}$$

ε is the dimensional regularized parameter $\varepsilon = \frac{1}{2}(4-n)$ and $\lambda_i = m_i^2/p^2$, m_1 and m_2 being the masses of the particles inside the loop. In general three masses appear (vertex-loops), but in the limit where the first fermion family is taken massless only two masses remain; this approximation corresponds to neglecting terms proportional to $(m_t/M_W)^2$ in the β-functions.

The first two terms give the contribution of the symmetric massless theory while the function $\varphi_F(\lambda_1, \lambda_2)$ includes all mass threshold effects. The contribution of F to the corresponding β-function is

$$\Delta b_0 = \mu^2 \frac{\partial}{\partial \mu^2}\left[\frac{C_F(\varepsilon)}{\varepsilon} - C_F(0)\log\mu^2 + \varphi_F(\lambda_1, \lambda_2)\right], \tag{5.10a}$$

$$\Delta b_0 = -C_F(0) + \mu^2 \frac{\partial}{\partial \mu^2}\varphi_F(\lambda_1, \lambda_2), \qquad \lambda_i = \frac{m_i^2}{\mu^2}. \tag{5.10b}$$

The threshold function $\varphi_F(\lambda_1, \lambda_2)$ has the property to behave logarithmically when λ_1 and λ_2 are large and goes to zero when λ_1 and λ_2 are small (eq. 5.9b):

$$\varphi_F \begin{cases} \xrightarrow[\lambda_{1,2}\to 0]{} 0\,, \\ \xrightarrow[\lambda_{1,2}\to\infty]{} C_\varphi \log\lambda_1 + d\varphi\,, \end{cases} \tag{5.11}$$

where $d\varphi$ is a constant number, which depends on the ratio m_1/m_2. So, one has [eqs. (5.10b) and (5.11)]:

$$\Delta b_0 \begin{cases} \xrightarrow[\lambda_{1,2}\to 0]{} -C_F(0)\,, \\ \xrightarrow[\lambda_{1,2}\to\infty]{} -C_F(0) - C_\varphi\,. \end{cases} \tag{5.12}$$

We say that a particular diagram is decoupled in the low-energy region if the pole part is opposite to the large log coefficient of the corresponding threshold function:

$$C_F(0) = -C_\varphi\,. \tag{5.13}$$

In that case the bare quantity F becomes constant when λ_1 and $\lambda_2 \to \infty$

$$F(-p^2, \varepsilon, \lambda_1, \lambda_2) \xrightarrow[\lambda_{1,2}\to\infty]{} \frac{1}{2}\frac{g^2}{(4\pi)^2}\left[\frac{C_F(\varepsilon)}{\varepsilon} - C_F(0)\log M_1^2 + d\varphi\right]. \tag{5.14}$$

A constant contribution of a diagram can always be absorbed in the bare lagrangian by a finite renormalization.

The mass-independent part of $F(\mu^2, \varepsilon, \lambda_1, \lambda_2)$,

$$F_s(\mu^2, \varepsilon) = \frac{1}{2}\frac{g^2}{(4\pi)^2}\left[\frac{C_F(\varepsilon)}{\varepsilon} - C_F(0)\log\mu^2\right], \tag{5.15}$$

represents the symmetric theory contribution and it gives a Δb_s^0

$$\Delta b_s^0 = -C_F(0)\,. \tag{5.16}$$

Eq. (5.1) is a simple consequence of the relation

$$F(\mu^2, \varepsilon, \lambda_1, \lambda_2) \xrightarrow[\lambda_{1,2}\to 0]{} F_s(\mu^2, \varepsilon)\,. \tag{5.17}$$

The decoupling condition (5.13) is not valid, diagram by diagram, in the Landau gauge because the non-physical particles are massless. Indeed the graphs which violate this condition are the following:

(i) the FP ghosts loop (see fig. 3a);

(ii) the Goldstone bosons loop (see fig. 3b);

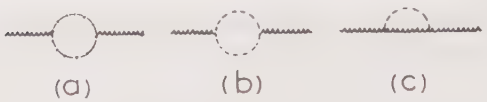

Fig. 3. (a) The FP ghost loop. (b) The Goldstone boson loop. (c) The massive vector boson and Goldstone boson loop $A(\lambda, 0)$.

(iii) the massive vector boson and Goldstone boson loop $A(\lambda, 0)$ (see fig. 3c).

The reason for diagrams (i) and (ii) is obvious. The non-physical particles, which correspond to some heavy bosons, are massless in the Landau gauge ($C_\varphi = 0$) while the pole part is different from zero ($C_F(0) \neq 0$). Diagram (iii) gives a finite contribution to the wave-function renormalization ($C_F(0) = 0$) while in the limit $\lambda \to \infty$ it behaves like a logarithm ($C_\varphi \neq 0$). Note that for a similar graph, where one has a physical massive Higgs boson instead of a Goldstone boson, the decoupling is valid. (In the limit where both masses go to infinity, it behaves like a constant and so $C_\varphi = 0$.)

But we know from general arguments that every diagram $D(\lambda_1, \lambda_2)$ with an internal loop of two gauge bosons, with masses M_1 and M_2, is associated to two tadpoles (which give zero contribution to the effective charges), one loop of the corresponding FP ghosts, one loop of the Goldstone bosons, which are responsible for the masses M_1, M_2 and finally two diagrams of a vector boson and its corresponding Goldstone boson $A(\lambda_1, 0)$, $A(\lambda_2, 0)$.

This combination, in which we have included all the non-physical degrees of freedom, verifies the decoupling condition (5.13). More generally this combination appears in any model of spontaneously broken theory and also in any general gauge ξ. So, we worked out the mechanisms of the decoupling at the one loop level of heavy particles in the Landau gauge for the vector boson propagator with light external vector bosons and heavy internal particles.

For the light fermion propagator and fermion-boson-fermion vertex function, the decoupling is valid graph by graph.

In appendix B we give the analytic expression of all the particular φ_F threshold functions A, B, C, D, E, F, G which are respectively the contribution of the Higgs-boson loop (fig. 4a), the Higgs' loop (fig. 4b), the fermionic loop (fig. 4c), the gauge boson loop (fig. 4d), the fermion-vector boson loop (fig. 4e), the abelian

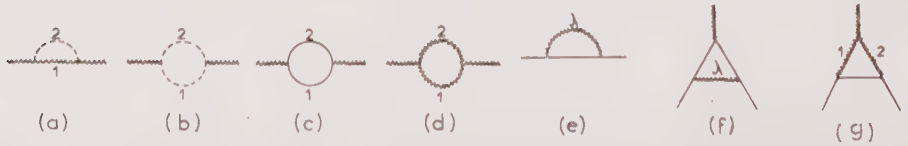

Fig. 4. (a) The gauge boson-Higgs loop contribution $A(\lambda_1, \lambda_2)$. (b) The Higgs loop contribution $B(\lambda_1, \lambda_2)$. (c) Fermionic loop contribution $C(\lambda_1, \lambda_2)$. (d) Gauge boson loop contribution $D(\lambda_1, \lambda_2)$. (e) Gauge boson-fermion loop contribution $E(\lambda)$. (f) Fermion-fermion gauge boson loop vertex contributions $F(\lambda)$. (g) The two gauge boson fermion loop vertex contribution $G(\lambda_1, \lambda_2)$.

type three-point function (fig. 4f), the non-abelian type three-point function (fig. 4g). The $A(\lambda_1, \lambda_2)$ function is the only one to be unsymmetrical in the change $\lambda_1 \leftrightarrow \lambda_2$. This is a characteristic function of the broken theory; it is zero in the zero mass limit.

Because of the abelian structure of E and F, one has (see appendix B)

$$\frac{\partial}{\partial p_\mu} \Gamma_F(p) = g\gamma^\mu V_F(p) + p^\mu \text{ terms}, \tag{5.18}$$

which implies

$$E(\lambda) = F(\lambda). \tag{5.19}$$

Because of this property one can demonstrate (see appendix A) that D, G and E appear in the effective charge definitions always in the following combination:

$$\Phi(\lambda_1, \lambda_2) = D(\lambda_1, \lambda_2) - G(\lambda_1, \lambda_2) - \tfrac{1}{2}E(\lambda_1) - \tfrac{1}{2}E(\lambda_2). \tag{5.20}$$

Φ represents the non-abelian contribution to the effective charges which appears always in the combination (5.20) for any spontaneously broken gauge group. We conclude that one can express all the one-loop β_i functions in any spontaneously broken theory as a linear combination of the logarithmic derivative ($\mu^2 \partial/\partial\mu^2$) of the four independent threshold functions Φ, B, C and A.

For every function one has a family of curves with very similar properties: when $p^2 \to \infty$ they go to zero, while when $p^2 \to 0$ they behave like a logarithm plus a constant. The transition in the threshold region is very rapid (fig. 5). If $\varphi(\lambda_1, \lambda_2)$ denotes one of the functions Φ, B, C, one always has

$$|\varphi(\lambda, 0)| \leq |\varphi(\lambda, r\lambda)| \leq |\varphi(\lambda, \lambda)|, \qquad 0 < r < 1. \tag{5.21}$$

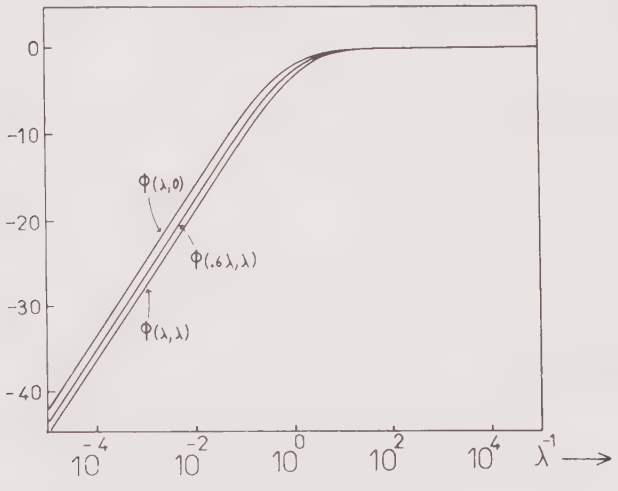

Fig. 5. Family of curves of $\Phi(\lambda, r\lambda)$; $r = 1, 0.6, 0$.

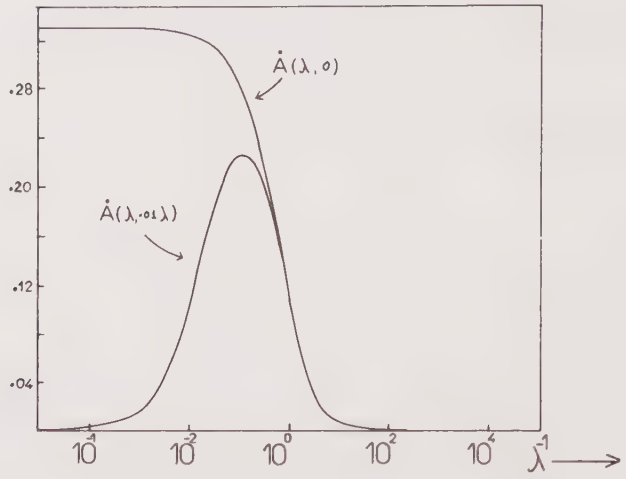

Fig. 6. β-function contribution of $A(\lambda, r\lambda)$ family; $r = 0, 0.01$.

$A(\lambda, 0)$ has similar properties as the other functions, while $A(\lambda, r\lambda)$ for $r \neq 0$ is different. In fact, when $p^2 \to 0$ it behaves like a constant and one has

$$0 \leq |A(\lambda, r\lambda)| \leq |A(\lambda, 0)|, \qquad 0 \leq r < \infty. \tag{5.22}$$

Normalizing the logarithmic behaviour of the different functions to one, the functions Φ, C, B and $A(\lambda, 0)$ are very similar although their analytic expressions are completely different (see appendix B). Their contributions to the various β-functions are proportional to the logarithmic derivatives with respect to p^2. To see what they look like we plot the logarithmic derivative of $A(\lambda, 0)$, $\dot{A}(\lambda, 0)$ (fig. 6). The typical contribution, of all these functions to the various β-functions is a smooth θ-function with a width $K \sim 10^2$. In fig. 6 we also plot $A(\lambda, r\lambda)$, $r \neq 0$ which gives a smooth δ-function contribution. A quite good approximation is to parametrize all the logarithmic derivatives of Φ, B, C, by some simple fit-functions as

$$b_\varphi \equiv p^2 \frac{\partial}{\partial p^2} \varphi(\lambda, r\lambda) = -c_\varphi \theta_{\mathrm{SYM}}(r, \lambda), \tag{5.23a}$$

where

$$\theta_{\mathrm{SYM}}(r, \lambda) = \frac{1}{1 + (1+r)^2 \lambda}, \qquad 0 \leq r \leq 1. \tag{5.23b}$$

Before we close this chapter we list the different behaviours of C, B, A, Φ when $\lambda \to \infty$:

$$C(\lambda, r\lambda) \to \tfrac{4}{3} \log \lambda - \frac{2}{9} + \frac{8}{3} \frac{r}{(1-r)^2} - \frac{4}{3} \frac{r^2(r-3)}{(1-r)^3} \log r, \tag{5.24a}$$

$$C(\lambda, 0) \to \tfrac{4}{3}\log\lambda - \frac{2}{9}, \tag{5.24b}$$

$$C(\lambda, \lambda) \to \tfrac{4}{3}\log\lambda + \tfrac{8}{9}, \tag{5.24c}$$

$$B(\lambda, 0) \to \tfrac{1}{6}\log\lambda + \tfrac{1}{18}, \tag{5.25a}$$

$$B(\lambda, \lambda) \to \tfrac{1}{6}\log\lambda + \tfrac{5}{18}, \tag{5.25b}$$

$$A(\lambda, r\lambda) \to \frac{r-3}{6(1-r)^2} - \frac{1}{3(1-r)^3}\log r, \tag{5.26a}$$

$$A(\lambda, 0) \to \tfrac{1}{3}\log\lambda - \tfrac{2}{9}, \tag{5.26b}$$

$$\Phi(\lambda, r\lambda) \to \frac{89}{36} - \frac{2r}{(1-r)^2} - \tfrac{23}{6}\log\lambda + \frac{17-52r+23r^2}{6(1-r)^3} r\log r, \tag{5.27a}$$

$$\Phi(\lambda, 0) \to -\tfrac{23}{6}\log\lambda + \frac{31}{18}, \tag{5.27b}$$

$$\Phi(\lambda, \lambda) \to -\tfrac{23}{6}\log\lambda - \tfrac{43}{36}. \tag{5.27c}$$

6. g_i relations up to two-loop level

In eq. (4.15) we gave, in integral form, the relation between the different definitions of the renormalized coupling constant up to two-loop level. To solve this equation exactly one needs, in principle, the second-order mass dependence of the β_i functions $b_i^1(M^2/\mu^2)$. These functions are unknown but one can bound them supposing the decoupling theorem (see sect. 5). Let us see that for the case $\mu \gg M_W$. Using the decoupling relations (5.1), (5.2a–d), in the case where all the super-heavy masses are equal to M_X, one has

$$b_i^1\left(\frac{M^2}{\mu^2}\right) g_i^2(\mu) \xrightarrow[\mu \ll M_X]{} \sum_{m=1}^{3} b_{im}^1 g_m^2(\mu), \qquad [\mathrm{SU(3)_c \times SU(2)_L \times U(1)}], \tag{6.1a}$$

$$b_i^1\left(\frac{M^2}{\mu^2}\right) g_i^2(\mu) \xrightarrow[\mu \gg M_X]{} b_{\mathrm{SYM}}^1 g^2(\mu), \qquad [\mathrm{SU(5)}]. \tag{6.1b}$$

Supposing that the width of this transition is K, then $b_i^1(M^2/\mu^2)g_i^2(\mu)$ is bounded by the two functions $B_1^i(\mu^2, g_i)$, $B_2^i(\mu^2, g_i)$:

$$B_1^i(\mu^2, g_i) = \sum_{m=1}^{3} \left[b_{im}^1 \theta\left(\frac{4}{K} M_X^2 - \mu^2\right) + \delta_{im} b_{\mathrm{SYM}}^1 \theta\left(\mu^2 - \frac{4}{K} M_X^2\right) \right] g_m^2(\mu), \tag{6.2a}$$

$$B_2^i(\mu^2, g_i) = \sum_{m=1}^{3} [b_{im}^1 \theta(4KM_X^2 - \mu^2) + \delta_{im} b_{\mathrm{SYM}}^1 \theta(\mu^2 - 4KM_X^2)] g_m^2(\mu). \tag{6.2b}$$

Using the functions B_1^i and B_2^i one can integrate the second-order term which

appears in eq. (4.15). In the case where one of the renormalized coupling constants is the symmetric one, eq. (4.15) is written as ($M_W < \mu < M_X$):

$$\frac{4\pi}{\alpha_i(\mu)} - \frac{4\pi}{\alpha_{SYM}(\mu)} = -R_i(\mu) + R_{SYM}(\mu) - \sum_{m=1}^{3} \frac{b^1_{im}}{b^0_m} \log \frac{\alpha_i(\mu)}{\alpha_i(2M_X)} + \frac{b^1_{SYM}}{b^0_{SYM}} \log \frac{\alpha_s(\mu)}{\alpha_s(2M_X)} \pm \Delta_i(K), \tag{6.3}$$

where b^0_{im}, b^1_{im}, $m = 1, 2, 3$, are the symmetric β-function coefficients of U(1), $SU(2)_L$ and $SU(3)_c$; b^0_{SYM} and b^1_{SYM} correspond to the SU(5) symmetric β-function coefficients, and $\Delta_i(K)$ represents the uncertainties coming from the unknown exact expression of the second-order β_i function.

$$\Delta_i(K) = \frac{\alpha_i(2M_X)}{4\pi} \sum_{m=1}^{3} b^1_{im} \log K. \tag{6.4}$$

Using the equation between $\alpha_{SYM}(\mu)$ and $\alpha_{SYM}(2M_X)$,

$$\frac{4\pi}{\alpha_{SYM}(\mu)} - \frac{4\pi}{\alpha_{SYM}(2M_X)} = -R_{SYM}(\mu) + R_{SYM}(2M_X) - \frac{b^1_{SYM}}{b^0_{SYM}} \log \frac{\alpha_{SYM}(\mu)}{\alpha_{SYM}(2M_X)}, \tag{6.5}$$

one can write eq. (6.3) as ($2M_W < \mu < 2M_X$)

$$\frac{4\pi}{\alpha_i(\mu)} = \frac{4\pi}{\alpha_{SYM}(2M_X)} + R_{SYM}(2M_X) - R_i(\mu) - \sum_{m=1}^{3} \frac{b^1_{im}}{b^0_m} \log \frac{\alpha_i(\mu)}{\alpha_i(2M_X)} \pm \Delta_i(K). \tag{6.6a}$$

Iterating (6.6a) we obtain, for $2M_W < \mu < 2M_X$,

$$\frac{4\pi}{\alpha_i(\mu)} = \frac{4\pi}{\alpha_{SYM}(2M_X)} + R_{SYM}(2M_X) - R_i(\mu) + \sum_{m=1}^{3} \frac{b^1_{im}}{b^0_m} \log \left[1 + \frac{\alpha_{SYM}(2M_X)}{4\pi} (R_m(2M_X) - R_m(\mu)) \right]. \tag{6.6b}$$

We note that for $2M_X < \mu$ one has

$$\frac{4\pi}{\alpha_i(\mu)} = \frac{4\pi}{\alpha_{SYM}(2M_X)} + R_{SYM}(2M_X) - R_i(\mu) + \frac{b^1_{SYM}}{b^0_{SYM}} \log \left[1 - \frac{\alpha_{SYM}(2M_X)}{4\pi} (R_{SYM}(2M_X) - R_{SYM}(\mu)) \right]. \tag{6.6c}$$

From eqs. (6.6a–c) we can obtain the g_i relation up to two-loop level for $2M_X > \mu >$

$2M_W$, in a simple way:

$$\frac{4\pi}{\alpha_i(\mu)}-\frac{4\pi}{\alpha_j(\mu)}=-R_i(\mu)+R_j(\mu)+\sum_{m=1}^{3}\frac{b_{im}^1-b_{jm}^1}{b_m^0}\times\log\left[1+\frac{\alpha_{SYM}(2M_X)}{4\pi}(R_m(2M_X)-R_m(\mu))\right]\pm(\Delta_i(K)-\Delta_j(K))\,. \tag{6.7}$$

Doing a similar approximation for the M_W threshold putting $M_5=M_W$ and $m_t\ll M_W$ one obtains, for every value of μ, the g_i relations

$$\frac{4\pi}{\alpha_i(\mu)}-\frac{4\pi}{\alpha_j(\mu)}=\begin{cases}-R_i(\mu)+R_j(\mu), & \mu>M_X\,,\\ -R_i(\mu)+R_j(\mu)+F_i(\mu)-F_j(\mu), & M_W>\mu>M_X\,,\\ -R_i(\mu)+R_j(\mu)+F_i(2M_W)-F_j(2M_W) & \\ +\tilde{F}_i(\mu)-\tilde{F}_j(\mu)\,, & \mu<M_W\,,\end{cases} \tag{6.8}$$

where

$$F_i(\mu)=\sum_{m=1}^{3}\frac{b_{im}^1}{b_m^0}\log\left[1+\frac{\alpha_{SYM}(2M_X)}{4\pi}(R_m(2M_X)-R_m(\mu))\right]\pm(\Delta_i(K)-\Delta_j(K))\,, \tag{6.9a}$$

$$\tilde{F}_i(\mu)=\sum_{m=1}^{2}\frac{\tilde{b}_{im}^1}{\tilde{b}_m^0}\log\frac{1+(\alpha_{SYM}(2M_X)/4\pi)(R_m(2M_X)-R_m(\mu))}{1+(\alpha_{SYM}(2M_X)/4\pi)(R_m(2M_X)-R_m(2M_W))}\pm(\Delta_i(K)-\Delta_j(K))\,. \tag{6.9b}$$

$\tilde{b}_m^0$, $\tilde{b}_{im}^1$, $m=1,2$, are the symmetric β-function coefficients of $U(1)_{em}$ and $SU(3)_c$, respectively.

Supposing that the width K of the second-order threshold functions is of the same order of magnitude as in the first-order one $K\approx 10^2$, we then see that $\Delta_i(K)$ is about 10 times smaller than the second-order correction ($F_i(\mu)$) for small values of μ. Knowing now that the first-order term (R_i-R_j) is about 40 times bigger than the second-order one, we see that $\Delta_i(K)$ represents a correction of 2.5‰ to the formula given by eq. (6.8).

We note that the value of a coupling constant is given as [see also eq. (6.6b)]

$$\frac{4\pi}{\alpha_i(\mu)}=\frac{4\pi}{\alpha_{SYM}(2M_X)}+R_{SYM}^0(2M_X)-R_i^0(\mu)+\begin{cases}0, & \mu>2M_X\,,\\ F_i(\mu)\,, & 2M_W<\mu<2M_X\,,\\ F_i(2M_X)-\tilde{F}_i(\mu)\,, & \mu<2M_W\,.\end{cases} \tag{6.10}$$

We list here the R_i quantities as well as the symmetric β-function coefficients b_m^0,

b^1_{im}, $\tilde{b}^0_m$, $\tilde{b}^1_{im}$, b^0_{SYM} and b^1_{SYM}:

$$R^0_{\mathrm{SYM}}(\mu) - R^0_{\mathrm{SYM}}(M_{\mathrm{X}}) = b^0_{\mathrm{SYM}} \log \lambda_{\mathrm{X}}\,, \tag{6.11a}$$

$$R^0_{\mathrm{G}}(\mu) - R^0_{\mathrm{SYM}}(\mu) = 2[\Phi(\lambda_{\mathrm{X}}, \lambda_{\mathrm{X}}) + \mathrm{A}(\lambda_{\mathrm{X}}, 0)] + 3B(\lambda_8, \lambda_8) + B(\lambda_3, \lambda_3) + \tfrac{1}{2}C(\lambda_{\mathrm{t}}) + \tfrac{1}{2}C(\lambda_{\mathrm{b}})\,, \tag{6.11b}$$

$$R^0_{\mathrm{A}}(\mu) - R^0_{\mathrm{SYM}}(\mu) = \tfrac{17}{4}[\Phi(\lambda_{\mathrm{X}}, \lambda_{\mathrm{X}}) + A(\lambda_{\mathrm{X}}, 0)] + \tfrac{3}{4}[\Phi(\lambda_{\mathrm{W}}, \lambda_{\mathrm{W}}) + A(\lambda_{\mathrm{W}}, 0)] + \tfrac{3}{4}B(\lambda_4, \lambda_4) + \tfrac{1}{4}B(\lambda_3, \lambda_3) + \tfrac{1}{2}C(\lambda_{\mathrm{t}}) + \tfrac{1}{2}C(\lambda_{\mathrm{b}})\,, \tag{6.11c}$$

$$R^0_{\mathrm{W}}(\mu) - R^0_{\mathrm{SYM}}(\mu) = 3[\Phi(\lambda_{\mathrm{X}}, \lambda_{\mathrm{X}}) + A(\lambda_{\mathrm{X}}, 0)] + \tfrac{5}{4}\Phi(\lambda_{\mathrm{W}}, \lambda_{\mathrm{Z}}) + \tfrac{3}{4}\Phi(\lambda_{\mathrm{W}}, 0) + \tfrac{9}{64}A(\lambda_{\mathrm{Z}}, 0) + A(\lambda_{\mathrm{W}}, \lambda_5) + 2B(\lambda_4, \lambda_4) + \tfrac{1}{2}B(\lambda_5, 0) + \tfrac{3}{4}C(\lambda_{\mathrm{t}}, \lambda_{\mathrm{b}}) + \tfrac{1}{4}C(\lambda_b, 0)\,, \tag{6.11d}$$

$$R^0_{\mathrm{Z}}(\mu) - R^0_{\mathrm{SYM}}(\mu) = \tfrac{15}{4}[\Phi(\lambda_{\mathrm{X}}, \lambda_{\mathrm{X}}) + A(\lambda_{\mathrm{X}}, 0)] + \tfrac{5}{4}\Phi(\lambda_{\mathrm{W}}, \lambda_{\mathrm{W}}) + \tfrac{9}{20}A(\lambda_{\mathrm{W}}, 0) + \tfrac{8}{5}A(\lambda_{\mathrm{Z}}, \lambda_5) + \tfrac{3}{20}B(\lambda_3, \lambda_3) + \tfrac{5}{4}B(\lambda_4, \lambda_4) + \tfrac{4}{5}B(\lambda_5, 0) + \tfrac{3}{10}C(\lambda_{\mathrm{t}}) + \tfrac{1}{2}C(\lambda_{\mathrm{b}})\,, \tag{6.11e}$$

where the threshold functions Φ, B, C, A are defined in sect. 5 and are given in appendix B. The first and second fermion generations have been considered massless.

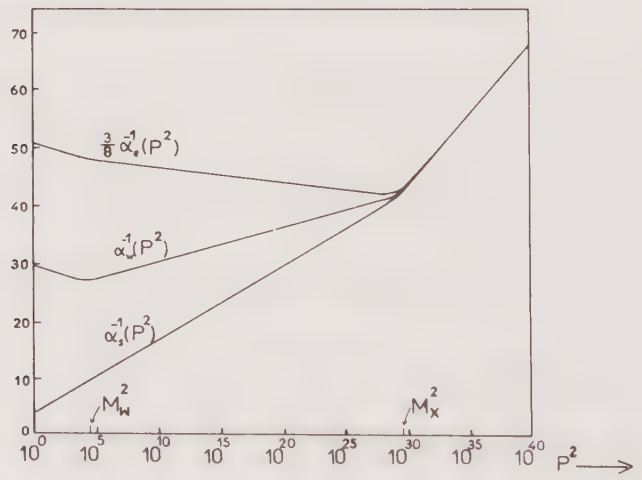

Fig. 7. Plot of the effective coupling constants α_{A}^{-1}, α_{w}^{-1}, α_{s}^{-1} in the region $p^2 = 10^0 - 10^{40}\,(\mathrm{GeV})^2$.

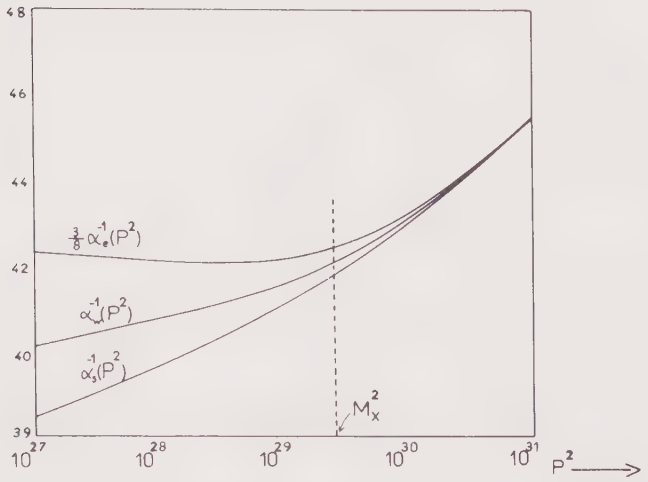

Fig. 8. Plot of the effective coupling constants α_A^{-1}, α_w^{-1}, α_s^{-1} in the M_X energy region $p^2 = 10^{27} - 10^{31}\,(\mathrm{GeV})^2$.

The behaviour of the coupling constants α_s^{-1}, α_w^{-1}, $\alpha_A^{-1} \equiv (\frac{8}{3}\alpha_e)^{-1}$ is represented in fig. 7 over the whole range of energy scale. In fig. 8 we plot them in the M_X energy region to show how different the $\alpha_i(M_X)$ are.

Here we list the symmetric β-functions of $SU(5)^{SYM}$ ($\mu > M_X$), $SU(3)_c \times SU(2)_L \times U(1)$ ($M_W < \mu < M_X$) and $SU(3)_c \times U(1)^{em}$ ($\mu < M_W$).

(i) $SU(5)^{SYM}$, $\mu > M_X$:

$$b^0_{SYM} = -\tfrac{55}{3} + \tfrac{4}{3}N_F + \tfrac{5}{6}N_\phi + \tfrac{1}{6}N_H\,, \tag{6.12a}$$

$$b^1_{SYM} = -\tfrac{850}{3} + \tfrac{698}{3}N_F + \tfrac{175}{3}N_\phi + \tfrac{169}{15}N_H\,, \tag{6.12b}$$

N_F: number of fermion generations, N_ϕ: number of adjoint Higgs representations, N_H: number of fundamental Higgs representations.

(ii) $SU(3)_c \times SU(2)_L \times U(1)$, $M_W < \mu < M_X$:

$$b^0_1 = \tfrac{4}{3}N_F + \tfrac{1}{10}N_H\,, \tag{6.13a}$$

$$b^0_2 = \tfrac{4}{3}N_F + \tfrac{1}{6}N_H - \tfrac{22}{3}\,, \tag{6.13b}$$

$$b^0_3 = \tfrac{4}{3}N_F - 11\,; \tag{6.13c}$$

$$b^0_A = \tfrac{5}{8}b^0_1 + \tfrac{3}{8}b^0_2 = \tfrac{4}{3}N_F + \tfrac{1}{8}N_H - \tfrac{11}{4}\,, \tag{6.14a}$$

$$b^0_Z = \tfrac{3}{8}b^0_1 + \tfrac{5}{8}b^0_2 = \tfrac{4}{3}N_F + \tfrac{17}{120}N_H - \tfrac{55}{12}\,, \tag{6.14b}$$

$$b^0_W = b^0_2 = \tfrac{4}{3}N_F + \tfrac{1}{6}N_H - \tfrac{22}{3}\,, \tag{6.14c}$$

$$b^0_G = b^0_3 = \tfrac{4}{3}N_F - 11\,; \tag{6.14d}$$

$$\begin{bmatrix} b^1_{11} & b^1_{12} & b^1_{13} \\ b^1_{21} & b^2_{22} & b^1_{23} \\ b^1_{31} & b^1_{32} & b^1_{33} \end{bmatrix} = N_F \begin{bmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{bmatrix} + N_H \begin{bmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{bmatrix}; \tag{6.15}$$

$$(b^1_{A1}, b^1_{A2}, b^1_{A3}) = N_F(\tfrac{13}{15}, \tfrac{13}{2}, \tfrac{10}{3}) + N_H(\tfrac{9}{40}, \tfrac{11}{8}, 0) + (0, -17, 0), \tag{6.16a}$$

$$(b^1_{A1} - b^1_{G1}, b^1_{A2} - b^1_{GS}, b^1_{A3} - b^1_{G3}) = N_F(\tfrac{1}{2}, 5, -22) + N_H(\tfrac{9}{40}, \tfrac{11}{8}, 0) + (0, -17, 102), \tag{6.16b}$$

$$(b^1_{W1} - b^1_{A1}, b^1_{W2} - b^1_{A2}, b^1_{W3} - b^1_{A3}) = N_F(-\tfrac{2}{3}, \tfrac{59}{6}, \tfrac{2}{3}) + N_H(\tfrac{3}{40}, \tfrac{19}{24}, 0) + (0, -\tfrac{85}{3}, 0). \tag{6.16c}$$

(iii) $SU(3)_c \times U(1)_e$, $\mu < M_W$:

$$b^0_A = \tfrac{1}{6}(4n_u + n_d + 3n_e), \tag{6.17a}$$

$$b^0_G = \tfrac{2}{3}(n_u + n_d) - 11; \tag{6.17b}$$

$$\begin{bmatrix} b^1_{AA} & b^1_{GA} \\ b^1_{AG} & b^1_{GG} \end{bmatrix} = \begin{bmatrix} \frac{1}{48}(16n_u + n_d + 27n_e) & \frac{1}{12}(4n_u + n_d) \\ \frac{2}{3}(4n_u + n_d) & \frac{38}{3}(n_u + n_d) - 102 \end{bmatrix}. \tag{6.18}$$

n_u: up-type quarks, n_d: down-type quarks, n_e: electron-type leptons.

7. M_X determination

In a previous paper [14] we gave the value of the superheavy vector boson mass M_X. The starting point of the M_X determination is the relation between the different definitions of the renormalized gauge coupling constant. Let us, for instance, examine the relation between the strong and the electromagnetic coupling constant [$\alpha_s \equiv \alpha_G$, $\alpha_e \equiv \frac{3}{8}\alpha_A$, eq. (6.8)]

$$\frac{4\pi}{\alpha_s(\mu)} - \frac{3}{8}\frac{4\pi}{\alpha_e(\mu)} = -R_G(\mu) + R_A(\mu) + F_G(\mu) - F_A(\mu). \tag{7.1}$$

R_G, R_A represent the exact one-loop contribution and F_G, F_A the two-loop correction as explained in sect. 6.

R_G and R_A are given in terms of the threshold functions Φ, B, C and A [eq. (6.11b, c)] and one observes that their difference is fermion independent:

$$-R_G(\mu) + R_A(\mu) = \tfrac{9}{4}[\Phi(\lambda_X, \lambda_X) + A(\lambda_X, 0)] + \tfrac{3}{4}[\Phi(\lambda_W, \lambda_W) + A(\lambda_W, 0)] - 3B(\lambda_8, \lambda_8) + \tfrac{3}{4}B(\lambda_4, \lambda_4) - \tfrac{3}{4}B(\lambda_3, \lambda_3). \tag{7.2a}$$

In the case where $\mu \ll M_X$, $(\lambda_X, \lambda_8, \lambda_4, \lambda_3 \gg 1)$, the functions containing the large

scales behave logarithmically plus a constant term [see eqs. (5.24)–(5.27)] and one has

$$-R_G(\mu)+R_A(\mu)=-\tfrac{67}{8}\log\frac{M_X^2}{\mu^2}-\tfrac{193}{48}-\tfrac{1}{4}\log\frac{a_3a_8^4}{a_4}+\tfrac{3}{4}[\Phi(\lambda_W,\lambda_W)$$

$$+A(\lambda_W,0)]\,,\qquad \mu\ll M_X\,, \tag{7.2b}$$

where $a_i=M_i/M_X$ and $a_4=2a_8$ [eq. (2.26)].

In the more particular case where $\mu\ll M_W$ all functions can be taken as asymptotic and eq. (7.2a) becomes

$$-R_G(\mu)+R_A(\mu)=-\tfrac{67}{8}\log\frac{M_X^2}{\mu^2}-\tfrac{87}{16}-\tfrac{1}{4}\log\frac{a_3a_8^4}{a_4}-\tfrac{21}{8}\log\frac{M_W^2}{\mu^2}\,. \tag{7.2c}$$

As we explained in the previous section, it is a good approximation to derive $F_G(\mu)$ and $F_A(\mu)$ using the decoupling theorem (see sect. 6). Integrating by steps the second-order term of eq. (6.9) one can write $F_G(\mu)-F_A(\mu)$ in the case where we consider three fermion families, as

$$F_G(\mu)-F_A(\mu)=\log\Pi(\mu,2m_b)\Pi(2m_b,2m_t)\Pi(2m_t,2M_W)\Pi(2M_W,2M_X)\,, \tag{7.3a}$$

if

$$\mu\leqslant 2m_b<2m_t<2M_W=2M_5<2M_X=2M_3=2M_8=M_4\,, \tag{7.3b}$$

and

$$F_G(\mu)-F_A(\mu)=\log\Pi(\mu,2M_X)\,, \tag{7.3c}$$

if

$$2m_b<2m_t<2M_W=2M_5\leqslant\mu\leqslant 2M_X=2M_3=2M_8=M_4\,. \tag{7.3d}$$

$\Pi(m_1,m_2)$ is given as

$$\Pi(m_1,m_2)=\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right)^{\gamma_s(m_1,m_2)}\left(\frac{\alpha_e(m_2)}{\alpha_e(m_1)}\right)^{\gamma_A(m_1,m_2)}\,,\qquad m_1,m_2<M_W\,, \tag{7.4a}$$

$$\Pi(m_1,m_2)=\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right)^{\gamma_3(m_1,m_2)}\left(\frac{\alpha_2(m_2)}{\alpha_2(m_1)}\right)^{\gamma_2(m_1,m_2)}\left(\frac{\alpha_1(m_2)}{\alpha_1(m_1)}\right)^{\gamma_1(m_1,m_2)}\,,\quad m_1,m_2>M_W\,, \tag{7.4b}$$

with

$$\gamma_s(m_1,m_2)=\frac{b_{GG}^{1\hat{}}-b_{AG}^1}{b_G^0}\,, \tag{7.5a}$$

$$\gamma_A(m_1,m_2)=\frac{b_{GA}^1-b_{AA}^1}{b_A^0}\,, \tag{7.5b}$$

$$\gamma_i(m_1, m_2) = \frac{b^1_{Gi} - b^1_{Ai}}{b^0_i}, \qquad i = 1, 2, 3 . \tag{7.5c}$$

b^0_G, b^0_A, b^0_i, b^1_{GG}, b^1_{AG}, b^1_{GA}, b^1_{AA}, b^1_{Gi}, b^1_{Ai} are given in sect. 6 by eqs. (6.13)–(6.18) and are evaluated at an energy scale between m_1 and m_2.

Contrary to the one-loop contribution $-R_G + R_A$, the two-loop one $F_G - F_A$ depends on the third fermion family masses m_b and m_t. From eqs. (7.1), (7.2) and (7.4) one obtains the following equation for M_X:

$$M_X \underset{\mu \ll M_X}{=} \mu f(\mu) \exp \frac{4}{67}\left[\frac{3}{8}\frac{4\pi}{\alpha_e(\mu)} - \frac{4\pi}{\alpha_s(\mu)} + \tfrac{3}{4}(\Phi(\lambda_W) + A(\lambda_W, 0)) - \tfrac{193}{48} - \tfrac{1}{4}\log\frac{a_3 a_8^4}{a_4}\right], \tag{7.6a}$$

$$M_X \underset{\mu \ll M_W}{=} \mu f(\mu) \exp \frac{4}{67}\left[\frac{3}{8}\frac{4\pi}{\alpha_e(\mu)} - \frac{4\pi}{\alpha_s(\mu)} - \tfrac{21}{4}\log\frac{M_W}{\mu} - \tfrac{87}{16} - \tfrac{1}{4}\log\frac{a_3 a_8^4}{a_4}\right], \tag{7.6b}$$

where $f(\mu)$ is given as

$$f(\mu) = \exp \tfrac{4}{67}[F_G(\mu) - F_A(\mu)] . \tag{7.7}$$

It is easy to verify that M_X given by eqs. (7.6a, b) is μ independent:

$$\mu^2 \frac{\partial}{\partial \mu^2} \log M_X^2 = 0 . \tag{7.8}$$

To determine M_X one needs $\alpha_e(\mu)$ and $\alpha_s(\mu)$ as well as all the mass parameters appearing in formulae (7.3b) or (7.3d). $\alpha_e(\mu)$ can be determined from the experimental data on $\sigma(e^+e^- \to \text{anything})$ using the dispersion relation. The value of $\alpha_s(\mu)$ is given by the second-order QCD formula as a function of $\Lambda_{\overline{MS}}$ which parametrizes the deep inelastic scaling violation of structure functions, in the energy range $2m_c < \mu < 2m_b$. Both $\alpha_e(\mu)$ and $\alpha_s(\mu)$ determined from the data [27, 28] must be expressed in our renormalization prescription. Let us denote by $\alpha_e(\mu_0)$ and $\alpha_s(\mu_0)$ the values of the electromagnetic and strong coupling constant at a point μ_0 such that the second order $SU(3)_c \times U(1)_{em}$ theory (two-loop) parametrize the experimental data. (μ_0 is sufficiently large that the strong corrections to electromagnetism are calculable perturbatively). Then $\alpha_e(\mu_0)$ and $\alpha_s(\mu_0)$ differ by a finite renormalization from $\alpha_e^D(\mu_0)$ calculated in the dispersion relation scheme [28] and $\alpha_s^{\overline{MS}}(\mu_0)$ calculated in the $\overline{MS}$ scheme [27],

$$\frac{3}{8}\frac{4\pi}{\alpha_e(\mu_0)} - \frac{3}{8}\frac{4\pi}{\alpha_e^D(\mu_0)} = \tfrac{1}{2}\sum_{i=1}^{N_f+N_e} Q_i^2\left(-1 + 6\lambda_i - 12\frac{\lambda_i^2}{\sqrt{1+4\lambda_i}}\log\frac{\sqrt{1+4\lambda_i}+1}{\sqrt{1+4\lambda_i}-1}\right)$$
$$+ \frac{b^1_{AA}}{b^0_A}\log\frac{\alpha_e^D(\mu_0)}{\alpha_e(\mu_0)} + \frac{b^1_{AG}}{b^0_G}\log\frac{\alpha_s^{\overline{MS}}(\mu_0)}{\alpha_s(\mu_0)}, \tag{7.9a}$$

$$\frac{4\pi}{\alpha_s(\mu_0)} - \frac{4\pi}{\alpha_s^{\overline{\rm MS}}(\mu_0)} = -\tfrac{1}{36}(219 - 16N_f) - \sum_{i=1}^{N_f} \tfrac{2}{3} C(\lambda_i)$$

$$+ \frac{b^1_{\rm GA}}{b^0_{\rm A}} \log \frac{\alpha_e^{\rm D}(\mu_0)}{\alpha_e(\mu_0)} + \frac{b^1_{\rm GG}}{b^0_{\rm G}} \log \frac{\alpha_s^{\overline{\rm MS}}(\mu_0)}{\alpha_s(\mu_0)} . \tag{7.9b}$$

Iterating these equations one can see that

$$\begin{bmatrix} \frac{3}{8} \dfrac{4\pi}{\alpha_e(\mu_0)} \\ \dfrac{4\pi}{\alpha_s(\mu_0)} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} \dfrac{4\pi}{\alpha_e^{\rm D}(\mu_0)} \\ \dfrac{4\pi}{\alpha_s^{\overline{\rm MS}}(\mu_0)} \end{bmatrix} + \begin{bmatrix} M_{\rm AA} & M_{\rm AG} \\ M_{\rm GA} & M_{\rm GG} \end{bmatrix} \begin{bmatrix} A_1/b^0_{\rm A} \\ A_2/b^0_{\rm G} \end{bmatrix}, \tag{7.10}$$

where

$$M_{\rm AA} = b^0_{\rm A} + b^1_{\rm AA} \frac{\alpha_{\rm A}^{\rm D}(\mu_0)}{4\pi}, \qquad M_{\rm AG} = b^1_{\rm AG} \frac{\alpha_{\rm G}^{\overline{\rm MS}}(\mu_0)}{4\pi}, \tag{7.11a}$$

$$M_{\rm GA} = b^1_{\rm GA} \frac{\alpha_{\rm A}^{\rm D}(\mu_0)}{4\pi}, \qquad M_{\rm GG} = b^0_{\rm G} + b^1_{\rm GG} \frac{\alpha_{\rm G}^{\overline{\rm MS}}(\mu_0)}{4\pi}, \tag{7.11b}$$

where

$$A_1(\mu_0) = \tfrac{1}{2} \sum_{i=1}^{3N_c+N_f} Q_i^2 \left(-1 + 6\lambda_i - 12 \frac{\lambda_i^2}{\sqrt{1+4\lambda_i}} \log \frac{\sqrt{1+4\lambda_i}+1}{\sqrt{1+4\lambda_i}-1} \right), \tag{7.12a}$$

$$A_2(\mu_0) = -\tfrac{1}{36}(219 - 16N_f) - \sum_{i=1}^{N_f} \tfrac{2}{3} C(\lambda_i) . \tag{7.12b}$$

Choosing μ_0 in the interval of $2m_c < \mu_0 < 2m_b$ one can write $\alpha_s^{\overline{\rm MS}}(\mu_0)$ as function of $\Lambda_{\overline{\rm MS}}$:

$$\frac{4\pi}{\alpha_s^{\overline{\rm MS}}(\mu_0)} = -b^0_{\rm G} \log \frac{\mu_0^2}{\Lambda^2_{\overline{\rm MS}}} + \frac{b^1_{\rm GG}}{b^0_{\rm G}} \log \log \frac{\mu_0^2}{\Lambda^2_{\overline{\rm MS}}} . \tag{7.13}$$

We can then express $M_{\rm X}$ as function of $\alpha_e^{\rm D}(\mu_0)$ and $\alpha_{\overline{\rm MS}}(\mu_0)$ which are experimentally given using eqs. (7.6b) and (7.10):

$$M_{\rm X} = \mu_0 f'(\mu_0) \exp \frac{4}{67} \left[\frac{3}{8} \frac{4\pi}{\alpha_e^{\rm D}(\mu_0)} - \frac{4\pi}{\alpha_{\rm MS}(\mu_0)} + A_1(\mu_0) - A_2(\mu_0) \right.$$

$$\left. - \tfrac{21}{4} \log \frac{M_{\rm W}}{\mu_0} - \tfrac{87}{16} - \tfrac{1}{4} \log \frac{a_3 a_8^4}{a_4} \right] . \tag{7.14}$$

A_1, A_2 are given by eqs. (7.12a, b) and $f'(\mu_0)$ is

$$f'(\mu_0) = \left(\frac{\alpha_s(2m_b)}{\alpha_{\overline{\rm MS}}(\mu_0)} \right)^{\delta_s(\mu_0/2, m_b)} \left(\frac{\alpha_s(2m_t)}{\alpha_s(2m_b)} \right)^{\delta_s(m_b, m_t)} \left(\frac{\alpha_s(2M_{\rm X})}{\alpha_s(2m_t)} \right)^{\delta_s(m_t, M_{\rm X})}$$

$$\times \left(\frac{\alpha_2(2M_{\rm X})}{\alpha_2(2M_{\rm W})} \right)^{\delta_2(M_{\rm W}, M_{\rm X})} \left(\frac{\alpha_1(2M_{\rm X})}{\alpha_1(2M_{\rm W})} \right)^{\delta_1(M_{\rm W}, M_{\rm X})} , \tag{7.15a}$$

with

$$\delta_i = \tfrac{4}{67}\gamma_i = \frac{4}{67}\,\frac{b^1_{Gi} - b^1_{Ai}}{b^0_i}\,. \tag{7.15b}$$

From eq. (7.14) one can determine M_X given $\alpha_e^D(\mu_0)$ and $\alpha_{\overline{MS}}(\mu_0)$. We choose $\Lambda_{\overline{MS}} = 0.2$ which is the present favoured value [27] and $\alpha_e^D(3\text{ GeV}) = 1/133.3$; then we obtain the central value for M_X^c

$$M_X^c = 3.1\times 10^{14}\text{ GeV}\,. \tag{7.16}$$

Taking the derivative with respect to $\Lambda_{\overline{MS}}$ of $\log M_X$,

$$\Lambda_{\overline{MS}}\frac{\partial}{\partial\Lambda_{\overline{MS}}}\log M_X = \tfrac{200}{201}[1 + 0.08\alpha_s(2m_b) + 0.06\alpha_s(2m_t) + 0.34\alpha_s(2M_X)]$$

$$= 1.03\,. \tag{7.17}$$

This implies that

$$M_X = 3.1\left(\frac{\Lambda_{\overline{MS}}}{0.2}\right)^{1.03}\times 10^{14}\text{ GeV}\,. \tag{7.18}$$

Let us come now to the uncertainties that appear to the M_X determination.

(i) $\alpha_{\overline{MS}}$ uncertainties, $\Lambda_{\overline{MS}}$: In eq. (7.18) one can see that M_X is very sensitive to the $\Lambda_{\overline{MS}}$ value.

(ii) α_e^D uncertainties $\delta(\alpha_e^D)^{-1} = \pm 0.5$.

$$M_X = M_X^c\,\exp\pm 0.28\delta(\alpha_e^D)^{-1} = M_X^c(1\pm 0.15)\,. \tag{7.19}$$

(iii) M_W uncertainties:

$$M_X = M_X^c\left(\frac{M_X}{85}\right)^{-0.31} \sim M_X^c(1\pm 0.04) \qquad \text{when } 75 < M_W < 95\,. \tag{7.20}$$

(iv) Second-order step approximation (width K):

$$M_X = M_X^c(K)^{\pm 0.01} = M_X^c(1\pm 0.05)\,, \qquad \text{for } K = 10^{\pm 2}\,. \tag{7.21}$$

(v) Superheavy Higgs masses M_3, M_8:

$$\begin{aligned} &M_X = M_X^c(a_3a_8^3)^{-1/67}, \qquad a_i = M_i/M_X\,,\\ &M_X = M_X^c(1\pm 0.5)\,, \qquad 10^{-3} < a_i < 10^{+3}\,. \end{aligned} \tag{7.22}$$

(vi) Third fermion family masses m_t:

$$M_X = M_X^c\left(\frac{\alpha_s(2m_t)}{\alpha_s(2m_t^0)}\right)^{0.05} \simeq M_X^c\left(1 - 0.004\log\frac{m_t}{20}\right)\,. \tag{7.23}$$

(vii) Fourth fermion family $M_T \simeq M_B \simeq M_L$:

$$M_X = M_X^c\left(\frac{\alpha_s(M_X)}{\alpha_s(M_T)}\right)^{-0.16}\,. \tag{7.24}$$

If $M_T \approx 85$ GeV then the correction is 1.3. One observes that in the case where we have another fermion family the M_X value changes by a maximum factor 1.3.

(viii) $\bar{5}$ Higgs mass M_5

$$M_X = M_X^c \left(\frac{\alpha_2(2M_5)}{\alpha_2(2M_W)}\right)^{-0.024} \left(\frac{\alpha_1(2M_5)}{\alpha_1(2M_W)}\right)^{0.003} \simeq M_X^c \left(1 + 0.0005 \log \frac{M_5}{85}\right). \quad (7.25)$$

All the uncertainties can be expressed in a single form

$$M_X = 3.1 \left(\frac{\Lambda_{\overline{MS}}}{0.2}\right)^{1.03} \exp 0.28\,((\alpha_e^D)^{-1} - 133.3) \left[1 - 0.004 \log \frac{m_t}{20}\right.$$

$$\left. + 0.0005 \log \frac{M_5}{85} - 0.015 \log a_3 a_8^3 - 0.31 \log \frac{M_W}{85} + 0.01 \log K\right]. \quad (7.26)$$

8. $\sin^2 \theta_W$ determination

$\sin^2 \theta_W$ is introduced to define the photon (massless gauge boson) and the Z-vector boson (the massive one) instead of the initial non-physical gauge bosons W_3 and B associated to the $SU(2)_L$ and U(1) subgroups. In sect. 2, working at the tree approximation level we saw that $\sin^2 \theta_0 = \frac{3}{8}$ [eq. (3.14)]. The bare value θ_0 changes order by order in perturbation as we explained in sect. 3, in order to redefine the renormalized photon as the gauge boson which corresponds to $U(1)_{em}$ and couples to the conserved electromagnetic current. The extra rotation $\theta_{\gamma Z} - \theta_0$ is finite to all orders in perturbation and is given by eq. (3.22). It is clear that $\sin \theta_{\gamma Z}(\mu)$ contains large logarithms in the perturbative expansion in powers of g_A. So one needs a renormalization group argument in order to derive $\sin^2 \theta_{\gamma Z}(\mu)$. This can be done easily using the formalism developed in the previous sections. Defining the SU(5) renormalized gauge coupling constant $g_{\gamma Z}$ with the finite renormalization.

$$g_{\gamma Z} = g_A Z_{\gamma Z}(g_A), \quad (8.1)$$

and choosing $Z_{\gamma Z}^{-1}(g_A)$ to be the perturbative expansion of $\sin^2 \theta_{\gamma Z}/\sin^2 \theta_0$ which has a finite coefficient (see sect. 3),

$$Z_{\gamma Z}^{1}(g_A) = \frac{\sin^2 \theta_{\gamma Z}}{\sin^2 \theta_0} = 1 + \frac{g_A^2}{(4\pi)^2} R_{\gamma Z} + \left(\frac{g_A^2}{(4\pi)^2}\right)^2 R_{\gamma Z}^1 + \cdots, \quad (8.2)$$

one then can relate the two definitions $g_{\gamma Z}$ and g_A using the solution of the differential equation. Using now the fact that the running $\sin^2 \theta_{\gamma Z}(\mu)$ is given by

$$\sin^2 \theta_{\gamma Z}(\mu) \equiv s_{\gamma Z}^2(\mu) = s_0^2 \frac{g_A^2(\mu)}{g_{\gamma Z}^2(\mu)}, \quad (8.3)$$

one obtains

$$s_{\gamma Z}^2(\mu) = \frac{3}{8} + \frac{\alpha_e(\mu)}{4\pi} [R_{\gamma Z} + F_{\gamma Z}], \quad (8.4a)$$

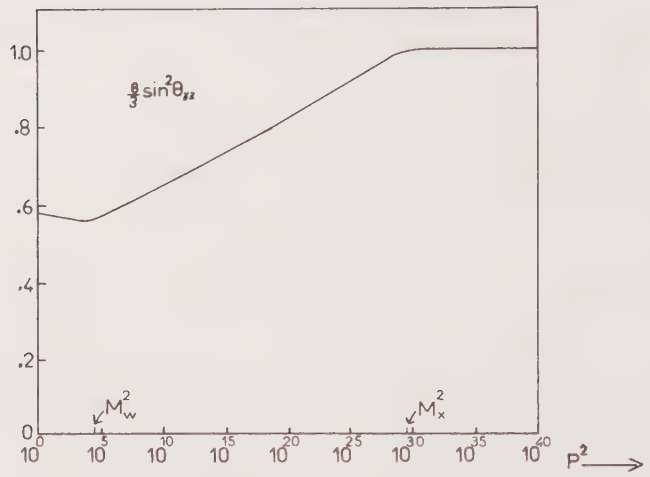

Fig. 9. Plot of $\sin^2\theta_{\gamma Z}$ in the region $p^2 = 10^0 - 10^{40}\,(\mathrm{GeV})^2$.

which is plotted in fig. 9. $R_{\gamma Z}(\mu)$ is the one-loop contribution

$$R_{\gamma Z}(\mu) = -\tfrac{5}{4}[\Phi(\lambda_X, \lambda_X) + A(\lambda_X, 0) - \Phi(\lambda_W, \lambda_W) - A(\lambda_W, 0) + \tfrac{1}{5}B(\lambda_3, \lambda_3) - B(\lambda_4, \lambda_4) + \tfrac{8}{5}\tilde{A}(\lambda_W, 0)]\,, \tag{8.4b}$$

with*

$$\tilde{A}(\lambda_W, 0) = -\tfrac{5}{6}\lambda + \tfrac{1}{6}\lambda^2 + \tfrac{1}{12}\log\lambda + \tfrac{1}{12}(1 + 9\lambda^2 - 2\lambda^3)\log\frac{1+\lambda}{\lambda}\,. \tag{8.4c}$$

$F_{\gamma Z}(\mu)$ is the two-loop correction. As explained in sect. 6 we calculate the two-loop correction using the decoupling theorem and the step approximation. Because of the definition of $g_{\gamma Z}^2 = e^2/\sin^2\theta_{\gamma Z}$ it is straightforward to see that the decoupling properties of the $\beta_{\gamma Z}(g_{\gamma Z}) \equiv \mu^2(\partial/\partial\mu^2)g_{\gamma Z}$ are the same as β_W in the limit where $M_W < \mu < M_X$ [see eq. (5.2b)]. This implies that

$$F_{\gamma Z}(\mu) = F_W(\mu) - F_A(\mu)\,. \tag{8.5}$$

For this reason one expects that the quantity s_W^2 defined as

$$s_W^2(\mu) = s_0^2\frac{g_A^2(\mu)}{g_W^2(\mu)} = \frac{e^2(\mu)}{g_W^2(\mu)} \tag{8.6}$$

* In fact in the case of $\sin^2\theta_{\gamma Z}$, instead of $\Gamma_T(p^2)$, to define $\Pi(p^2)$ [see eq. (3.9b)] we use the sum $(\Gamma_T(p^2) + \Gamma_L(p^2))$ that appears in the decomposition of the two-point irreducible function $\Gamma^{\mu\nu} = (g^{\mu\nu} - p^\mu p^\nu/p^2)(\Gamma_T + \Gamma_L) + (p^\mu p^\nu/p^2)\Gamma_L$. This is more natural for the definition of $\sin^2\theta_W$ because both Γ_T and Γ_L contribute to the three-point reducible Green function. For this reason, instead of $A(\lambda, 0)$ a different threshold function appears, i.e. the $\tilde{A}(\lambda, 0)$ given in eq. (8.4c).

For the definition of the electromagnetic and the strong coupling constants the two possible decompositions become the same because, as expected from the Ward identities, we find $\Gamma_L^{GG}(p^2) = \Gamma_L^{AA}(p^2) = 0$. This is not the case for $\Gamma_L^{ZZ}(p^2)$ and $\Gamma_L^{WW}(p^2)$.

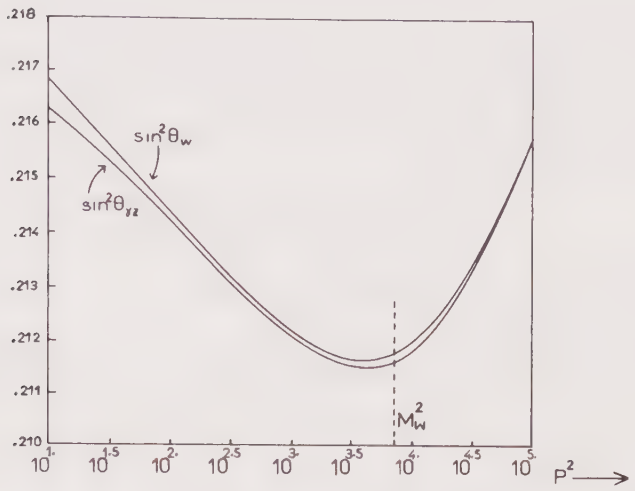

Fig. 10. Comparison of two definitions of the weak angle $\sin^2\theta_w = \alpha_e/\alpha_w$ and $\sin^2\theta_{\gamma Z}$ in the region $p^2 = 10^1 - 10^5\ (\mathrm{GeV})^2$.

will be very similar to $s^2_{\gamma Z}(\mu)$. In fact if one neglects the SU(2) × U(1) breaking then

$$s^2_{\gamma Z}(\mu) = s^2_W(\mu)\,, \qquad M_W/\mu \to 0\,. \tag{8.7}$$

For μ smaller than M_W, $s^2_{\gamma Z}(\mu)$ and $s^2_W(\mu)$ are slightly different (5‰ at $\mu \simeq 3$ GeV) as can be seen in fig. 10. The difference $\Delta(\mu) = s^2_W(\mu) - s^2_{\gamma Z}(\mu)$ gives an order of magnitude of variation when we use several possible definitions of $\sin^2\theta_w$:

$$\begin{aligned}\Delta(\mu) = \frac{\alpha_e(\mu)}{4\pi}\big[&2\Phi(\lambda_W,\lambda_W) - \tfrac{5}{4}\Phi(\lambda_W,\lambda_Z) - \tfrac{3}{4}\Phi(\lambda_W,0) - \tfrac{1}{2}B(\lambda_5,0)\\ &- A(\lambda_W,\lambda_5) + 2A(\lambda_W,0) + \tfrac{9}{64}A(\lambda_Z,0) - 2\tilde{A}(\lambda_W,0)\\ &+ \tfrac{1}{2}C(\lambda_t,\lambda_t) + \tfrac{1}{2}C(\lambda_b,\lambda_b) - \tfrac{3}{4}C(\lambda_t,\lambda_b)\\ &- \tfrac{1}{4}C(\lambda_b,0)\big]\,.\end{aligned} \tag{8.8}$$

For $\mu \ll M_X$ one can use the asymptotic behaviour of the threshold functions and one can write eq. (8.4a) in the following form:

$$\begin{aligned}s^2_{\gamma Z}(\mu) \underset{\mu \ll M_X}{=} \frac{3}{8} - \frac{\alpha_e(\mu)}{4\pi}\Big[&\tfrac{109}{12}\log\frac{M_X}{\mu} + \tfrac{295}{144} - \tfrac{1}{12}\log\frac{a_4^5}{a_3}\\ &+ \tfrac{5}{4}(\Phi(\lambda_W,\lambda_W) + A(\lambda_W,0)) - 2\tilde{A}(\lambda_W,0) - F_{\gamma Z}(\mu)\Big]\end{aligned} \tag{8.9a}$$

and in the case where $\mu \ll M_W$

$$s^2_{\gamma Z}(\mu) \underset{\mu \ll M_W}{=} \frac{3}{8} - \frac{\alpha_e(\mu)}{4\pi}\Big[\tfrac{109}{12}\log\frac{M_X}{M_W} + \tfrac{41}{36} - \tfrac{1}{12}\log\frac{a_4^5}{a_3} - F_{\gamma Z}(\mu)\Big]\,. \tag{8.9b}$$

Observe that in the case where $\mu \ll M_W$ the μ dependence of $s^2_{\gamma Z}(\mu)$ comes only from $\alpha_e(\mu)$. (We will see that the second-order correction $F_{\gamma Z}(\mu)$ is very small). There is no logarithmic behaviour for the small values of μ. As in the case of the M_X determination, $\alpha_e(\mu)$ is not directly the quantity taken from the experimental data, $\alpha_e^D(\mu)$:

$$\frac{\alpha_e(\mu_0)}{4\pi} = \frac{\alpha_e^D(\mu_0)}{4\pi} - \frac{8}{3}\left(\frac{\alpha_e^D(\mu_0)}{4\pi}\right)^2 [A_1(\mu_0)], \tag{8.10}$$

where $A_1(\mu_0)$ is given by eq. (7.12a) and

$$F_{\gamma Z}(\mu_0) = \log \Pi_{\gamma Z}(\mu_0, 2m_b)\Pi_{\gamma Z}(2m_b, 2m_t)\Pi_{\gamma Z}(2m_t, 2M_W)\Pi_{\gamma Z}(2M_W, 2M_X), \tag{8.11}$$

where

$$\Pi_{\gamma Z}(m_1, m_2) = \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right)^{\xi_s(m_1,m_2)} \left(\frac{\alpha_e(m_2)}{\alpha_e(m_1)}\right)^{\xi_A(m_1,m_2)}, \qquad m_1, m_2 < 2M_W, \tag{8.12a}$$

$$\Pi_{\gamma Z}(m_1, m_2) = \left(\frac{\alpha_3(m_2)}{\alpha_3(m_1)}\right)^{\xi_3(m_1,m_2)} \left(\frac{\alpha_2(m_2)}{\alpha_2(m_1)}\right)^{\xi_2(m_1,m_2)} \left(\frac{\alpha_1(m_2)}{\alpha_1(m_1)}\right)^{\xi_1(m_1,m_2)},$$

$$2M_W \leqslant m_1, m_2 < 2M_X, \tag{8.12b}$$

with

$$\xi_s(m_1, m_2) = -\frac{b^1_{AG}}{b^0_G}, \tag{8.13a}$$

$$\xi_A(m_1, m_2) = -\frac{b^1_{AA}}{b^0_A}, \tag{8.13b}$$

$$\xi_i(m_1, m_2) = \frac{b^1_{Wi} - b^1_{Ai}}{b^0_i}, \qquad i = 1, 2, 3. \tag{8.13c}$$

b^0_A, b^0_G, b^0_i, b^1_{AG}, b^1_{AA}, b^1_{Wi}, b^1_{Ai} are given by eqs. (6.13)–(6.18) and are evaluated at an energy scale between m_1, m_2.

Choosing $\mu_0 = 3$ GeV and $\alpha_e^D(\mu_0) = 1/133.3$ and $M_X = 3.1 \times 10^{14}$ one obtains

$$s^2_{\gamma Z}(3\text{ GeV}) = 0.2169, \qquad \Lambda_{\overline{MS}} = 0.2\text{ GeV}. \tag{8.14a}$$

Comparing with the value of $s^2_W(3\text{ GeV})$ in ref. [14],

$$s^2_W(3\text{ GeV}) = 0.217, \qquad \Lambda_{\overline{MS}} = 0.22\text{ GeV}, \tag{8.14b}$$

we conclude that different definitions of $\sin^2\theta_w$ do not change the SU(5) prediction too much. The second-order explicit contribution coming from $F_{\gamma Z}(\mu_0)$ is very small,

$$\Delta s^2_{\gamma Z} = -0.0002. \tag{8.15}$$

One can then write an effective formula for $s^2_{\gamma Z}(\mu)$, $\mu \ll M_W$, using the M_X expression given by eq. (7.25) where Λ uncertainties appear explicitly:

$$s^2_{\gamma Z}(\mu) = \tfrac{3}{8} - \alpha_e^D(\mu) \times 21.076 - 0.0056 \log \frac{\Lambda_{\overline{MS}}}{0.2}. \tag{8.16}$$

The other uncertainties are the following:

(i) α_e uncertainties, $\alpha_e^{-1}(\mu) = \pm 0.5$:

$$\delta s^2_{\gamma Z}(\mu) = \mp 0.006 . \tag{8.17}$$

(ii) m_t uncertainties:

$$\delta s^2_{\gamma Z}(\mu) = 0.00002 \log \frac{m_t}{20}. \tag{8.18}$$

(iii) M_W uncertainties:

$$\delta s^2_{\gamma Z}(\mu) = 0.007 \log \frac{M_W}{85}. \tag{8.19}$$

(iv) M_5 uncertainties:

$$\delta s^2_{\gamma Z}(\mu) = -0.000005 \log \frac{M_5}{85}. \tag{8.20}$$

(v) Superheavy Higgs uncertainties:

$$\delta s^2_{\gamma Z}(\mu) = 0.0003 \log \frac{M_3}{M_X} + 0.0005 \log \frac{M_8}{M_X}. \tag{8.21}$$

(vi) Second-order step approximation (width K):

$$\delta s^2_{\gamma Z}(\mu) = 0.0001 \log K . \tag{8.22}$$

(vii) 4th fermion family with mass scales $M_T = M_B = M_L$:

$$\delta s^2_{\gamma Z}(\mu) = 0.0009 \log \frac{\alpha_s(M_X)}{\alpha_s(M_T)}. \tag{8.23}$$

If there is a 4th fermion family, $s^2_{\gamma Z}(\mu)$ decreases and the variation can be as much as 0.0014 if $M_T \approx M_W$.

In recent experiments, experimentalists parametrized their results with the tree approximation formulas. It is well known that the radiative corrections depend on the process. For this reason one must be careful before comparing their values with the SU(5) predicted value given by eq. (8.14a). Recently Marciano and Sirlin [23] have calculated these radiative corrections and they derive the mean $\sin^2 \theta_w$ which is directly comparable with the SU(5) prediction. This value is equal to

$$\sin^2 \theta_w^{exp} = 0.216 \pm 0.010 , \quad \text{at } \mu = \sqrt{20}\ \text{GeV} . \tag{8.24}$$

This value is exactly that of eq. (8.16)

$$s^2_{\gamma Z}(\sqrt{20}) = 0.2163 \pm 0.006 - 0.0056 \log \frac{\Lambda_{\overline{MS}}}{0.2}. \tag{8.25}$$

Before we close this section we give the value of our calculation at $\mu = M_W$, a value that is usually chosen in other calculations:

$$s^2_{\gamma Z}(M_W) = 0.2132 \pm 0.001 - 0.0056 \log \frac{\Lambda_{\overline{MS}}}{0.2}. \tag{8.26}$$

9. Conclusion

In this work we have performed the crossing of thresholds in a spontaneously broken gauge theory in an exact way. We have studied the threshold functions at the one-loop level and found that the decoupling of heavy particles could be approximated well enough with θ-functions. As an application we have calculated M_X and $\sin^2 \theta_w$ in SU(5) and we get the following numerical values:

$$M_X = 3.1 \left(\frac{\Lambda_{\overline{MS}}}{0.2} (\text{GeV}) \right)^{1.03} \times 10^{14} \text{ GeV}, \tag{9.1}$$

$$\sin^2 \theta_w(\mu) = \tfrac{3}{8} - 21.076 \alpha^D_{em}(\mu) - 0.0056 \log \frac{\Lambda_{\overline{MS}}}{0.2}. \tag{9.2}$$

$\Lambda_{\overline{MS}}$ is the scale parameter of QCD with 4 flavours whose present favoured value is $\Lambda_{\overline{MS}} = 0.16^{+0.1}_{-0.08}$ GeV [27]. $\alpha^D_{em}(\mu)$ is the value of the electromagnetic coupling constant at the scale μ which can be directly obtained from experimental data on the total cross section $e^+e^- \to \gamma \to$ hadrons using a dispersion relation [6, 28]. The weak angle $\sin \theta_w(\mu)$ defined by the above formula is directly related to γ-Z mixing. But eq. (9.2) is strictly valid only in the limit $\mu \ll M_W$. For $\mu = M_W = 85$ GeV we get $\sin^2 \theta_w(M_W) = 0.213 \pm 0.001$ for $\Lambda_{\overline{MS}} = 0.2$ GeV. Our results for M_X and $\sin^2 \theta_w(M_W)$ agree with other calculations [6–13, 19, 22, 29]. Nothing prevents us from using the formula for $\sin^2 \theta_w(\mu)$ in the zero momentum limit where we get $\sin^2 \theta_w(0) = 0.221$ for $\Lambda_{\overline{MS}} = 0.2$ GeV. But before any comparison to atomic physics experiments, further radiative corrections must be taken into account. These radiative corrections have been calculated by Marciano and Sirlin [23] for the ratio of total neutral and total charged current cross sections. They have extracted from all neutral current experiments an effective value $\sin^2 \theta_w$ at $\mu \approx \sqrt{20}$ GeV which can be compared with the SU(5) renormalization group prediction. Their value $\sin^2 \theta_w(20 \text{ GeV}) = 0.216 \pm 0.010$ is in complete agreement with the SU(5) prediction when $\Lambda_{\overline{MS}} = 0.2$ GeV.

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Appendix A

We here explain why the definition of the effective charges [Eq. (3.18)],

$$\langle f_1 W^a f_2\rangle = \sum_b S^{-1/2}_{f_1 f_1} S^{-1/2}_{f_2 f_2} (\Pi^{-1/2})_{ab} V_{f_1 w^b f_2}\,, \tag{A.1}$$

does not depend, at the one-loop level, on which fermions f_1, f_2 we choose if we take the whole fermion multiplet to be massless.

Indeed, since all fermion Higgs couplings are zero in the limit of massless fermions, at one-loop order only graphs 4f and 4g contribute to $V_{f_1 w^a f_2}(p^2)$ defined in eq. (3.11) and only graph 4e contributes to $S_{f_1 f_2}(p^2)$ defined in eq. (3.10). Sorting out the group factors, these contributions read, when subtracting the contribution of the symmetric massless theory [see sect. 5 and eqs. (5.9) for notations]

$$S_{ff}(p^2) = 1 + \frac{1}{2}\frac{g^2}{(4\pi)^2}\sum_b (T^b)^2_{ff} E(\lambda_b) \tag{A.2}$$

$$\begin{aligned} -iV_{f_1 w^a f_2}(p^2) = {} & (T^a)_{f_1 f_2} + \frac{1}{2}\frac{g^2}{(4\pi)^2}\sum_b (T^b T^a T^b)_{f_1 f_2} F(\lambda_b) \\ & + \frac{1}{2}\frac{g^2}{(4\pi)^2}\sum_{b,c} i f_{abc} (T^b T^c)_{f_1 f_2} G(\lambda_b, \lambda_c)\,, \end{aligned} \tag{A.3}$$

where T^a, $a = 1, \ldots, 24$, are the generators of the group G in the fermion representation, f_{abc} are the structure constants of the group $[T^a, T^b] = i f^{abc} T_c$ and as usual $\lambda_a = M_a^2/(-p^2)$. We made use of the fact that the functions E, F and G are independent of the massless fermions to sum over fermions inside the loop.

By standard manipulations we can put eq. (A.3) into the form

$$\begin{aligned} -iV_{f_1 w^a f_2}(p^2) = {} & (T^a)_{f_1 f_2} - \frac{1}{2}\frac{g^2}{(4\pi)^2}\sum_{b,c} f_{abc} f_{bcd} (T^d)_{f_1 f_2} [\tfrac{1}{2} G(\lambda_b, \lambda_c) \\ & + \tfrac{1}{4} F(\lambda_b) + \tfrac{1}{4} F(\lambda_c)] + \frac{1}{4}\frac{g^2}{(4\pi)^2}\sum_b F(\lambda_b)[T^a (T^b)^2 + (T^b)^2 T^a]_{f_1 f_2}\,. \end{aligned} \tag{A.4}$$

But at zero momentum transfer, in the Landau gauge, we have the QED type relation mentioned in eq. (5.19),

$$E(\lambda) = F(\lambda)\,. \tag{A.5}$$

Furthermore, in the SU(5) model, we have $[(T^b)^2 T^a]_{f_1 f_2} = [T^a (T^b)^2]_{f_1 f_2} = (T^a)_{f_1 f_2} (T^b)^2_{f_i f_i}$ for any components a, f_i $(i = 1, 2)$. It follows that, to one-loop order,

$$-iS^{-1/2}_{f_1 f_1} S^{-1/2}_{f_2 f_2} V(f_1 W^a f_2) = (T^a)_{f_1 f_2} - \frac{1}{4}\frac{g^2}{(4\pi)^2}\sum_{b,c,d} f_{abc} f_{bcd} (T^d)_{f_1 f_2} V(\lambda_b, \lambda_c)\,, \tag{A.6}$$

where the function $V(\lambda_b, \lambda_c)$ is defined as

$$V(\lambda_b, \lambda_c) = G(\lambda_b, \lambda_c) + \tfrac{1}{2}E(\lambda_b) + \tfrac{1}{2}E(\lambda_c)\,. \tag{A.7}$$

Let us first consider the components a such that there is no mixing, i.e., satisfying $\Pi_{ad} = 0$ and $\sum_{b,c} f_{abc} f_{bcd} V(\lambda_b, \lambda_c) = 0$ for any $d \neq a$. This is the case when $a = \mathrm{W}$ and G for instance. Then the effective charge in eq. (A.1) can be written

$$\langle f_1 W^a f_2 \rangle = i R^{aa}(p^2)(T^a)_{f_1 f_2}\,, \tag{A.8}$$

where $R^{ad}(p^2)$ is fermion independent and defined, to one-loop order, as

$$R_{ad}(p^2) = (\Pi^{-1/2})_{ad}(p^2) - \frac{1}{4}\frac{g^2}{(4\pi)^2}\sum_{b,c} f_{abc} f_{bcd} V(\lambda_b, \lambda_c)\,. \tag{A.9}$$

This is the result stated in eqs. (3.23c) and (3.23d) for $a = \mathrm{W}$ and G, respectively.

Now we consider two components (a, d) which can mix together such as the (Z, A) system in the SU(5) model. We assume that (a, d) are orthogonal combinations at the tree level.

Then $(\Pi^{-1/2})_{ad}$ is of order g^2 and we get, to this order,

$$\begin{bmatrix} f_1 W^a f_2 \\ f_1 W^d f_2 \end{bmatrix} = i \begin{bmatrix} R_{aa}(p^2) & R_{ad}(p^2) \\ R_{ad}(p^2) & R_{dd}(p^2) \end{bmatrix} \begin{bmatrix} (T^a)_{f_1 f_2} \\ (T^d)_{f_1 f_2} \end{bmatrix}, \tag{A.10}$$

with the notations of eq. (A.9).

For the (Z, A) system in SU(5), $W^a \equiv \mathrm{Z}$, $W^d \equiv \mathrm{A}$, and the normalized charges at the tree level are given by

$$(T^{\mathrm{Z}})_{ff} = \sqrt{\tfrac{8}{5}}(T_{3f} - \tfrac{3}{8}Q_f)\,, \qquad (T^{\mathrm{A}})_{ff} = \sqrt{\tfrac{3}{8}}Q_f\,, \tag{A.11}$$

where T_3 is the weak isospin charge and Q the electric charge. As explained in sect. 3, because the photon corresponds to the generators of an unbroken subgroup of SU(5), the electric charge is conserved to all orders and we must rotate Z and A in such a way that eq. (3.23a) holds:

$$\langle f A f \rangle = i R_{\mathrm{AA}}(p^2)\sqrt{\tfrac{3}{8}}Q_f\,. \tag{A.12}$$

Let

$$R_{(\theta - \theta_0)} = \begin{pmatrix} 1 & \xi \\ -\xi & 1 \end{pmatrix}$$

be this rotation to one-loop order ($\xi \sim \mathrm{O}(g^2)$). Then from eq. (3.19a) we get

$$\xi = R_{\mathrm{AZ}}(p^2) \tag{A.13}$$

and the effective charge for the neutral current becomes

$$\langle f Z f \rangle = \sqrt{\tfrac{8}{5}} R_{\mathrm{ZZ}}(p^2)\left[T_{3f} - \left(\frac{3}{8} - \frac{\sqrt{15}}{4} R_{\mathrm{AZ}}(p^2)\right) Q_f \right]. \tag{A.14}$$

We recover eq. (3.23b) with

$$\sin^2 \theta_{AZ}(p^2) = \frac{3}{8} - \frac{\sqrt{15}}{4} R_{AZ}(p^2) . \tag{A.15}$$

Let us come back to eq. (A.9). The graphs which contribute to $\Pi^{ad}(p^2)$ at one-loop order are figs. 4a–d. The contribution of ghost particles can be dropped in the Landau gauge because it is the same as in the symmetric case. The contribution of fig. 4d can be added to the vertex contribution and we get the combination of eq. (5.20),

$$\Phi(\lambda_1, \lambda_2) = D(\lambda_1, \lambda_2) - G(\lambda_1, \lambda_2) - \tfrac{1}{2}E(\lambda_1) - \tfrac{1}{2}E(\lambda_2) . \tag{A.16}$$

Appendix B

We present in this appendix, the explicit results of all the one-loop Feynman diagrams which have been used for the calculation of the different β-functions presented in sects. 5 and 6. We give all these results without the corresponding group factors and so they do not depend on the particular theoretical lagrangian model. They depend only on the spontaneously broken non-abelian gauge structure of the theory. Finally, we give some interesting details of the calculation concerning the way we did the integrations on the different Feynman x parameters.

We first want to notice the QED-type relation, which links the fermion-boson-fermion vertex to the fermion propagator, so reducing the number of independent function parameters. This relation is based on the one-loop observation that the derivative with respect to p^μ of the fermion propagator, gives the γ^μ vertex with zero transfer momentum:

(B.1)

Following the notations of sects. 4, 5 and using these diagrammatic identities, one can easily extract the following relation between the contributions to the various effective charges:

(B.2)

In regularized form, all the graph functions can be expressed in terms of only one generating integral $\int_0^1 dx\, I^{m-\varepsilon}(\lambda_1, \lambda_2)$, where m is an integer number and $I(\lambda_1, \lambda_2) = x(1-x) + x\lambda_1 + (1-x)\lambda_2$. After the ε-expansion we did all the integrations on the Feynman parameters explicitly and we found relatively simple expressions for all

the threshold functions in terms of polynomials, "log" and "square root" of the different λ_i. In order to do that, one can use the following recursion procedure.

We first need to express all the results only in terms of the generating integral defined above. For this we use the following recursion relations (for simplicity in the following we drop the integration symbol $\int_0^1 dx$):

$$(1-2x)I^a = \frac{1}{1+a}(\lambda_1^{1+a} - \lambda_2^{1+a}), \tag{B.3}$$

$$x^2 I^a = \frac{2+a}{1+a} I^a - \frac{1}{1+a}\lambda_1^{1+a} - \lambda_2 I^a, \tag{B.4}$$

$$2\frac{3+2a}{1+a} I^{1+a} = [1+2(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)^2]I^a + \frac{1}{1+a}[\lambda_1^{1+a} + \lambda_2^{1+a} - (\lambda_1-\lambda_2)(\lambda_1^{1+a} - \lambda_2^{1+a})], \tag{B.5}$$

where a is any real number and in particular we can take $a = n - \varepsilon$ with n any integer number.

We then need to do the integrations on the Feynman x parameters. For that we give two basic integrations:

$$\ln I = \tfrac{1}{2}\ln\lambda_1\lambda_2 - \tfrac{1}{2}(\lambda_1-\lambda_2)\ln\frac{\lambda_1}{\lambda_2} - 2 + a\ln\frac{1+\lambda_1+\lambda_2+2a}{1+\lambda_1+\lambda_2-2a}, \tag{B.6}$$

$$I^{-1} = \frac{1}{2a}\ln\frac{1+\lambda_1+\lambda_2+2a}{1+\lambda_1+\lambda_2-2a}, \tag{B.7}$$

where

$$2a = [1+2(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)^2]^{1/2}.$$

For all the other combinations $I^n \ln^m I$ one must use the recursion relations (B.3)–(B.5). For example one can find the following integration recursion relations:

$$I^n = \frac{n}{2(2n+1)}[1+2(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)^2]I^{n-1} + \frac{1}{2(2n+1)}[\lambda_1^n + \lambda_2^n - (\lambda_1-\lambda_2)(\lambda_1^n - \lambda_2^n)], \tag{B.8}$$

$$I^n \ln I = \frac{n}{2(2n+1)}[1+2(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)^2]I^{n-1}\ln I + \frac{1}{n(2n+1)}I^n + \frac{1}{2(2n+1)}[\lambda_1^n\ln\lambda_1 + \lambda_2^n\ln\lambda_2 - (\lambda_1-\lambda_2)(\lambda_1^n\ln\lambda - \lambda_2^n\ln\lambda_2)] - \frac{1}{2n(2n+1)}[\lambda_1^n + \lambda_2^n - (\lambda_1-\lambda_2)(\lambda_1^n - \lambda_2^n)], \tag{B.9}$$

where n is any integer number, and so on.

We finish this appendix with the presentation of the explicit results. We will give the results only for the graphs which have been used for the different β-function calculations in the Landau gauge. For this we put the fermion masses in the fermion propagator and in the vertex function equal to zero. In this approximation all the Yukawa couplings are zero.

So, in the following we give [in units of $g^2/(4\pi)^2$ and without the group factors]:

(i) The results graph by graph which contribute to the wave-function renormalization of the vector boson propagator ($C(\lambda_1, \lambda_2)$, $B(\lambda_1, \lambda_2)$, $A(\lambda_1, \lambda_2)$, $D(\lambda_1, \lambda_2)$).

(ii) The results for the fermion self-energy $E(\lambda)$.

(iii) The results for the fermion–boson–fermion vertex $G(\lambda_1, \lambda_2)$, $F(\lambda)$ being equal to $E(\lambda)$ following (B.2a).

(iv) The results for the combination $\Phi(\lambda_1, \lambda_2)$ defined in eq. (5.20).

$$= -\frac{4}{3}\left[\frac{1}{\varepsilon} - \log\frac{-p^2}{\mu^2}\right] - \tfrac{8}{9} + C(\lambda_1, \lambda_2)\,, \tag{B.10a}$$

$$\begin{aligned} C(\lambda_1, \lambda_2) = \tfrac{1}{12}\{&-16 s_1\lambda - 64(s_2-2)\lambda^2 - [48 s_1\lambda^2 + 64(s_2-2)\lambda^3] \\ &\times (a-b)\log a + 16\log\lambda + 16[1 - s_1\lambda + 4(s_2-2)\lambda^2 \\ &+ 12\lambda^2/(1 + 2s_1\lambda + (s_2-2)\lambda^2)]\mathcal{D}(\lambda_1, \lambda_2)\}\,, \end{aligned} \tag{B.10b}$$

$$C(\lambda_1, 0) = -\tfrac{4}{3}\lambda - \tfrac{16}{3}\lambda^2 + \tfrac{4}{3}\log\lambda + \tfrac{4}{3}(1 + 3\lambda^2 + 4\lambda^3)\log\frac{1+\lambda}{\lambda}\,, \tag{B.10c}$$

$$C(\lambda, \lambda) = -\tfrac{8}{3}\lambda + \tfrac{4}{3}\log\lambda + \frac{4}{3}\frac{1 + 2\lambda + 4\lambda^2}{1 + 4\lambda}\mathcal{D}(\lambda, \lambda) \equiv C(\lambda)\,, \tag{B.10d}$$

where $\mu^2 \equiv \mu_0^2 4\pi\, e^{\gamma}$, with γ the Euler constant, $\lambda_i = M_i^2/(-p^2)$, $\lambda = (\lambda_1\lambda_2)^{1/2} = M_1M_2/(-p^2)$, $a = M_1/M_2$, $b = 1/a$, $s_n = a^n + b^n$ and the threshold function $\mathcal{D}$ is given by

$$\mathcal{D}(\lambda_1, \lambda_2) = r\log\frac{1 + \lambda_1 + \lambda_2 + 2r}{1 + \lambda_1 + \lambda_2 - 2r}\,, \tag{B.11}$$

with

$$2r = [1 + 2(\lambda_1 + \lambda_2) + (\lambda_1 - \lambda_2)^2]^{1/2}\,.$$

$$= \frac{1}{6}\left[\frac{1}{\varepsilon} - \log\frac{-p^2}{\mu^2}\right] - \tfrac{5}{18} + B(\lambda_1, \lambda_2)\,, \tag{B.12a}$$

$$B(\lambda, 0) = \tfrac{1}{3}\lambda + \tfrac{4}{3}\lambda^2 + \tfrac{1}{6}\log\lambda + (\tfrac{1}{6} - \lambda^2 - \tfrac{4}{3}\lambda^3)\log\frac{1+\lambda}{\lambda}\,, \tag{B.12b}$$

$$B(\lambda, \lambda) = \tfrac{2}{3}\lambda + \tfrac{1}{6}\log\lambda + (\tfrac{1}{6} - \tfrac{1}{3}\lambda)\mathcal{D}(\lambda, \lambda)\,. \tag{B.12c}$$

$$= A(\lambda_1, \lambda_2)\,, \tag{B.13a}$$

$$A(\lambda_1,\lambda_2)=\frac{1}{12}\Big\{-4a\lambda+(8a^2-16)\lambda^2+[4+12(1-b^2)\lambda^2+8(a-b)^3\lambda^3]\log a$$
$$-(4-4b\lambda-8b^2\lambda^2)(1+b\lambda)\log\frac{1+b\lambda}{b\lambda}+\Big[4-4s_1\lambda$$
$$-8(s_2-2)\lambda^2+12a\lambda-12\,\frac{a\lambda+(1+a^2)\lambda^2}{1+2s_1\lambda+(s_2-2)\lambda^2}\Big]$$
$$\times\mathscr{D}(\lambda_1,\lambda_2)\Big\}, \tag{B.13b}$$

$$A(\lambda,0)=-\tfrac{1}{3}\lambda+\tfrac{2}{3}\lambda^2+\tfrac{1}{3}\log\lambda+(\tfrac{1}{3}-\tfrac{2}{3}\lambda)\log\frac{1+\lambda}{\lambda}, \tag{B.13c}$$

$$A(0,\lambda)=0. \tag{B.13d}$$

$$=\frac{7}{3}\Big[\frac{1}{\varepsilon}-\log\frac{-p^2}{\mu^2}\Big]+\tfrac{23}{36}+D(\lambda_1,\lambda_2), \tag{B.14a}$$

$$D(\lambda_1,\lambda_2)=\tfrac{17}{12}+\tfrac{13}{12}s_1\lambda+(\tfrac{8}{3}s_2-6)\lambda^2-\Big(\frac{7}{3}+\frac{1}{8\lambda^2}\Big)\log\lambda$$
$$+\Big[\frac{(1+a\lambda)^2}{\lambda^2}(-\tfrac{1}{8}+\tfrac{5}{6}a\lambda-\tfrac{7}{12}a^2\lambda^2+\tfrac{1}{3}a^3\lambda^3)\log\frac{1+a\lambda}{a\lambda}+a\Leftrightarrow b\Big]$$
$$+2\Big[(\tfrac{1}{6}s_4+\tfrac{4}{3}s_2-3)\lambda^3+(\tfrac{1}{24}s_3+\tfrac{35}{24}s_1)\lambda^2+\tfrac{23}{48}s_1+\frac{7}{24}\,\frac{1}{\lambda}\Big]$$
$$\times(a-b)\log a+\frac{1}{12}\Big[\frac{3}{2\lambda^2}-\frac{17}{2\lambda}s_1-(3s_2+26)+(3s_3+9s_1)\lambda$$
$$-(4s_4+32s_2-72)\lambda^2+\frac{18(1+s_1\lambda-8\lambda_2)}{1+2s_1\lambda+(s_2-2)\lambda^2}\Big]\mathscr{D}(\lambda_1,\lambda_2), \tag{B.14b}$$

$$D(\lambda,0)=\tfrac{7}{12}+\tfrac{5}{3}\lambda+\tfrac{7}{3}\lambda^2-\Big(\frac{7}{3}+\frac{7}{12}\,\frac{1}{\lambda}\Big)\log\lambda-\Big(\frac{7}{12}\,\frac{1}{\lambda}+\tfrac{7}{3}$$
$$+\tfrac{17}{6}\lambda^2+\tfrac{7}{3}\lambda^3\Big)\log\frac{1+\lambda}{\lambda}, \tag{B.14c}$$

$$D(\lambda,\lambda)=\tfrac{17}{12}+\tfrac{13}{6}\lambda-\tfrac{2}{3}\lambda^2-\Big(\frac{7}{3}+\frac{1}{8\lambda^2}\Big)\log\lambda+(1+\lambda)^2\Big(\tfrac{1}{6}\lambda^2-\tfrac{5}{4}\lambda$$
$$+\frac{1}{2}+\frac{17}{12}\,\frac{1}{\lambda}-\frac{1}{4\lambda^2}\Big)\log\frac{1+\lambda}{\lambda}+\frac{1}{12}\Big(\frac{3}{2\lambda^2}-\frac{17}{\lambda}-14-12\lambda\Big)\mathscr{D}(\lambda,\lambda). \tag{B.14d}$$

$$=-\frac{1}{6}\Big[\frac{1}{\varepsilon}-\log\frac{-p^2}{\mu^2}\Big]-\frac{1}{9}. \tag{B.15}$$

$$= E(\lambda)\,, \tag{B.16a}$$

$$E(\lambda) = -\tfrac{1}{2} + \lambda + \frac{1}{2\lambda}\log\lambda + \left(\frac{1}{2\lambda} - \tfrac{3}{2}\lambda - \lambda^2\right)\log\frac{1+\lambda}{\lambda}\,. \tag{B.16b}$$

$$= -\frac{3}{2}\left[\frac{1}{\varepsilon} - \log\frac{-p^2}{\mu^2}\right] - \tfrac{3}{2} + G(\lambda_1, \lambda_2)\,, \tag{B.17a}$$

$$\begin{aligned} G(\lambda_1, \lambda_2) = {} & -\tfrac{3}{4} - \tfrac{1}{3}s_1\lambda + \left[-\frac{1}{6\lambda^2} + \frac{3}{2}\right]\log\lambda + \left[\frac{s_1}{6\lambda^2} + \frac{5}{3\lambda}\right. \\ & \left. + \tfrac{3}{2}s_1\right]\frac{1}{a-b}\log a + \left[\frac{1}{6(a-b)}\left(\frac{1}{a\lambda^2} + \frac{2}{\lambda}\right)(1+a\lambda)^3\log\frac{1+a\lambda}{a\lambda}\right. \\ & \left. + a \Leftrightarrow b\right], \end{aligned} \tag{B.17b}$$

$$\begin{aligned} G(\lambda, 0) = {} & -\tfrac{3}{4} - \tfrac{1}{3}\lambda - \frac{1}{6\lambda} + \frac{1}{6}\left(\frac{1}{\lambda^2} + \frac{5}{\lambda} + 9\right)\log\lambda \\ & + \frac{1}{6}\left(\frac{1}{\lambda^2} + \frac{2}{\lambda}\right)(1+\lambda)^3\log\frac{1+\lambda}{\lambda}\,, \end{aligned} \tag{B.17c}$$

$$G(\lambda, \lambda) = -\frac{1}{12} + \frac{1}{6\lambda} + \lambda + \left(\frac{3}{2} - \frac{1}{6\lambda^2}\right)\log\lambda + \left(-\frac{1}{6\lambda^2} + \tfrac{3}{2} + \tfrac{7}{3}\lambda + \lambda^2\right)\log\frac{1+\lambda}{\lambda}\,. \tag{B.17d}$$

Finally the combination Φ is

$$\begin{aligned} \Phi(\lambda_1, \lambda_2) = {} & \frac{1}{12}\left\{32 + 11s_1\lambda + (32s_2 - 72)\lambda^2 - \left(46 + \frac{3s_1}{\lambda} - \frac{1}{2\lambda^2}\right)\log\lambda\right. \\ & + \left[\frac{10}{\lambda} + \tfrac{23}{2}s_1 + (s_3 + 35s_1)\lambda^2 + (4s_4 + 32s_2 - 72)\lambda^3 - \frac{1}{s_2 - 2}\right. \\ & \left. \times\left(\frac{2}{\lambda^2}s_1 + \frac{20}{\lambda} + 18s_1\right)\right](a-b)\log a + \left(-\left[(1+a\lambda)\left(\frac{3}{2\lambda^2} - \frac{17}{2\lambda}a\right.\right.\right. \\ & \left. -3a^2 + 3a^3\lambda - 4a^4\lambda^2\right) + \frac{3}{a\lambda} - 9a\lambda - 6a^2\lambda^2\Big]\log\frac{1+a\lambda}{a\lambda} \\ & + \frac{1}{a-b}\left(\frac{2}{a\lambda^2} + \frac{4}{\lambda}\right)(1+a\lambda)^3\log\frac{1+a\lambda}{a\lambda} + a \Leftrightarrow b\Big) + \Big[\frac{3}{2\lambda^2} \\ & - \frac{17}{2\lambda}s_1 - (3s_2 + 26) + (3s_3 + 9s_1)\lambda - (4s_4 + 32s_2 - 72)\lambda^2 \\ & \left. + \frac{18(1 + s_1\lambda - 8\lambda^2)}{1 + 2s_1\lambda + (s_2 - 2)\lambda^2}\Big]\mathcal{D}(\lambda_1, \lambda_2)\right\}, \end{aligned} \tag{B.18a}$$

$$\Phi(\lambda,0)=\tfrac{19}{12}+\tfrac{3}{2}\lambda+\tfrac{7}{3}\lambda^2+\frac{1}{6\lambda}-\left(\frac{23}{6}+\frac{5}{3\lambda}+\frac{1}{6\lambda^2}\right)\log\lambda$$
$$-\left(\tfrac{7}{3}\lambda^3+\tfrac{8}{3}\lambda^2+\tfrac{5}{12}\lambda+\frac{23}{6}+\frac{5}{3\lambda}+\frac{1}{6\lambda^2}\right)\log\frac{1+\lambda}{\lambda}, \tag{B.18b}$$

$$\Phi(\lambda,\lambda)=2+\tfrac{13}{6}\lambda-\tfrac{2}{3}\lambda^2-\frac{1}{6\lambda}+\left(-\frac{23}{6}-\frac{1}{2\lambda}+\frac{1}{24\lambda^2}\right)\log\lambda$$
$$+\left(\tfrac{2}{3}\lambda^3+\tfrac{1}{6}\lambda^2-\tfrac{5}{6}\lambda+\frac{5}{12}+\frac{2}{3\lambda}-\frac{1}{12\lambda^2}\right)\log\frac{1+\lambda}{\lambda}$$
$$-\left(\lambda+\frac{7}{6}+\frac{17}{12\lambda}-\frac{1}{8\lambda^2}\right)\mathscr{D}(\lambda,\lambda)\,. \tag{B.18c}$$

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CHAPITRE V

FORTE VIOLATION DE LA SYMETRIE CP ET SOLUTIONS POSSIBLES

V. 1. Strong CP Violation and axion mechanism

V. 2. Lower bound for branching ratio of $K^+ \to \pi^+$ axion and non-existence of Peccei-Quinn axion

STRONG CP VIOLATION AND AXION MECHANISM*

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ABSTRACT : We discuss the strong CP violation problem and its possible solutions. We describe the Peccei-Quinn mechanism and we review the principle properties of the axion. We show why the standard Peccei-Quinn axion cannot exist because of its incosistency with several experiments and we examine other possibilities to solve the θ - problem in QCD.

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1. - CP VIOLATION IN QCD

1.1. - Pure gauge theory

It is well known that because of the instanton effects, the QCD Lagrangian has an extra additional piece which although it is a total divergence, it cannot be neglected. [1]

$$\mathcal{L}_{QCD} = -\frac{1}{2g^2} \mathrm{Tr}\, F_{\mu\nu}^2 + \frac{\theta}{32\pi^2} \mathrm{Tr}\, F_{\mu\nu} \tilde{F}_{\mu\nu} \quad , \tag{1.1}$$

where the strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu]$ and $A_\mu = \sum_a A_\mu^a \frac{\lambda^a}{2}$ with $\lambda^a, a = 1, \ldots, 8$ the usual SU(3) Gell-Mann matrices; the dual tensor $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho} F_{\lambda\rho}$, g is the strong coupling constant and θ is an extra parameter which characterizes the theory. Using the Bardeen's identity [2] the second term can be written as a total divergence :

$$\mathrm{Tr}\, F_{\mu\nu} \tilde{F}_{\mu\nu} = \partial_\mu K_\mu$$

$$\text{with} \quad K_\mu = \epsilon_{\mu\nu\lambda\rho} \mathrm{Tr}\, A_\nu \left[\partial_\lambda A_\rho - \frac{2}{3} A_\lambda A_\rho \right] \quad . \tag{1.2}$$

Because $\tilde{F}_{\mu\nu}$ violates parity but conserves charge conjugation the second term of eq. 1.1. violates CP. To illustrate the fact that this term cannot be neglected, let us take as an example the vacuum to vacuum amplitude and show that it depends explicitely on the parameter-θ [3]. The vacuum to vacuum amplitude is given by the functional integration :

$$\langle \mathrm{vac}\ t=-\infty \,|\, \mathrm{vac}\ t=+\infty \rangle = \int [dA_\mu] \exp i \int d^4x\, \mathcal{L}_{QCD} \quad , \tag{1.3}$$

where one must integrate over all field configurations A_μ which at $t = -\infty$ and $t = +\infty$ are pure gauge. By generalizing the SU(2) example of Callan, Dashen and Gross [4] in SU(3) let us work in the temporal gauge $A_0 = 0$ and consider the following expression for the form of a pure gauge configuration at $t = -\infty$ or $t = +\infty$:

$$A_i(\vec{r}) = \omega^{-1}(\vec{r})\, \partial_i\, \omega(\vec{r}) \quad , \tag{1.4a}$$

$$\text{with } \omega(\vec{r}) = e^{i\lambda^a \theta^a(\hat{r}) f(r)} \quad \text{and} \quad \theta^a(\hat{r}) = \frac{i}{2} \epsilon_{ijk} \lambda^a_{jk} \hat{r}_i \quad , \tag{1.4b}$$

where ϵ_{ijk} is the antisymmetric tensor in 3 dimensions and f(r) is an arbitrary function of $r=|\vec{r}|$. From the definition (1.4b) the parameters $\theta^a(\hat{r})$ belong to an SU(2) subspace of SU(3) which corresponds to the three antisymmetric Gell-Mann matrices. Using the properties of Gell-Mann matrices λ^a:

$$\sum_a \lambda^a_{\alpha\beta}\lambda^a_{\gamma\varepsilon} = 2\delta_{\alpha\varepsilon}\delta_{\beta\gamma} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\gamma\varepsilon} \quad , \tag{1.5}$$

one can easily show that θ^a are real and $\sum_a (\theta^a)^2 = 1$. Finally we can rewrite $\omega(\vec{r})$ in the form :

$$\left[\omega(\vec{r})\right]_{\alpha\beta} = \cos f(r)\,\delta_{\alpha\beta} + \left(1-\cos f(r)\right)\hat{r}_\alpha\hat{r}_\beta + \sin f(r)\,\epsilon_{i\alpha\beta}\hat{r}_i \quad . \tag{1.6}$$

By imposing now the extra limiting condition in the space direction that $\omega \to 1$ as $r \to \infty$, we see that $f(r)$ must be characterized by an integer index n which is determined from its asymptotic behavior:

$$f_n(r) \xrightarrow[r\to\infty]{} 2n\pi \quad . \tag{1.7}$$

So we made a mapping of S_3 onto the group space (which is well known from the instanton analysis, n being the winding number [1,5]).
We can now construct a gauge invariant θ-vacuum which is a linear superposition of n-state vacua: [4]

$$|\theta\rangle = \sum_{n=-\infty}^{+\infty} e^{in\theta}\,|n\rangle \quad . \tag{1.8}$$

Then the vacuum-θ to the vacuum-θ amplitude is given by:

$$\langle \text{vac-}\theta,\, t=-\infty \,|\, \text{vac-}\theta,\, t=+\infty\rangle = \sum_{n_+,n_-} e^{i(n_+-n_-)\theta}\,\langle n_-|n_+\rangle$$

$$= \sum_n e^{in\theta}\left(\sum_{n_-}\langle n_-|n_-+n\rangle\right) \quad , \tag{1.9}$$

where $n = n_+ - n_-$.

Take now the extra term of eq. (1.1):

$$\frac{\theta}{32\pi^2}\int d^4x\ \mathrm{Tr}\, F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{\theta}{32\pi^2}\int d^4x\ \partial_\mu K_\mu$$

$$= \frac{\theta}{32\pi^2}\int d\sigma_\mu K_\mu = \frac{\theta}{32\pi^2}\int d^3x\ K_0\Big|_{t=-\infty}^{t=+\infty} , \tag{1.10}$$

where we have used eq. (1.2) and the Gauchy's theorem. From eq. (1.4) and the particular form (1.6) we get:

$$A^i_{\alpha\beta} = \left(\omega^{-1}\partial^i\omega\right)_{\alpha\beta}$$

$$= \frac{1}{r}\left(1-\cos f(r)\right)\left(\hat{r}_\alpha\delta_{i\beta} - \hat{r}_\beta\delta_{i\alpha}\right) + \frac{1}{r}\sin f(r)\cos f(r)\,\epsilon_{i\alpha\beta} + \tag{1.11}$$

$$+ \frac{1}{r}\sin f(r)\left(1-\cos f(r)\right)\left(\hat{r}_\alpha\epsilon_{ij\beta} - \hat{r}_\beta\epsilon_{ij\alpha}\right)\hat{r}_j + \left[\frac{\partial}{\partial r}f(r) - \frac{1}{r}\sin f(r)\cos f(r)\right]\hat{r}_i\epsilon_{\alpha\beta j}\hat{r}_j .$$

Now it is trivial to verify that :

$$\epsilon^{ijk}\,\mathrm{Tr}\,A^iA^jA^k = \frac{12}{r^2}\left(1-\cos f(r)\right)\frac{\partial f(r)}{\partial r} \tag{1.12}$$

and then by combining (1.10) and (1.12)

$$\frac{\theta}{32\pi^2}\int d^4x\ \mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{\theta}{2\pi}\int_0^\infty dr\,\frac{\partial}{\partial r}\left(f(r)-\sin f(r)\right)\Big|_{t=-\infty}^{t=+\infty}$$

$$= \theta\,(n_+ - n_-) = \theta n \tag{1.13}$$

From eqs. (1.3), (1.9) and (1.13) we conclude that the vacuum to vacuum amplitude depends explicitely on the parameter $-\theta$ and so the additional term in the QCD Lagrangian can not be neglected although it is a total divergence. So, when $\theta \neq 0$ or n we have strong P and CP violation.

1.2. - Inclusion of fermions

Let us perform a chiral rotation of a quark field:

$$q \longrightarrow e^{i\gamma_5\eta}\,q \quad ; \tag{1.14}$$

then the variation of the action is given by the divergence of the corresponding axial current $J_\mu^5 = \bar{q}\gamma_\mu\gamma_5 q$ which is equal to the strong anomaly [6] up to mass terms which cancel in the zero quark mass limit :

$$\delta S = -i\eta \int d^4x\, \partial_\mu J_\mu^5 = -\frac{i\eta}{16\pi^2}\int d^4x\, \mathrm{Tr}\, F_{\mu\nu}\tilde{F}_{\mu\nu} \,. \qquad (1.15)$$

Therefore a chiral rotation of a quark field by an angle η changes the parameter θ by $\theta \to \theta - 2\eta$ [7] (see eq. 1.1).
To see the physical consequence of this phenomenon, let us consider the effect of weak interactions which lead to a quark mass matrix of the form :

$$\mathcal{L}_{mass} = -\bar{q}'_L M q'_R - \bar{q}'_R M^+ q'_L \,, \qquad (1.16)$$

where $q'_{L(R)}$ is a column vector of left (right) handed quark fields $q'_{L(R)} = \frac{1 \mp \gamma_5}{2} q'$ and M is the quark mass matrix remaining after the Higgs mechanism.
Then we diagonalize M as $M = U^+ \bar{M} U$, where $\bar{M}$ is diagonal and U is a unitary matrix, and we redefine the quark field by $q = Uq'$. After diagonalization the quark mass piece of the Lagrangian (1.16) may be written in the form :

$$\mathcal{L}_{mass} = -\frac{1}{2}\left(\bar{M}_i^+ + \bar{M}_i\right)\bar{q}_i q_i - \frac{1}{2}\left(\bar{M}_i^+ - \bar{M}_i\right)\bar{q}_i\gamma_5 q_i \,, \qquad (1.17)$$

where the elements $\bar{M}_i$ of the diagonal mass matrix $\bar{M}$ can be put in the form $\bar{M}_i = \frac{m_i}{\cos\theta_i} e^{i\theta_i}$. In order to cancel the second term in eq. (1.17) we must make a chiral rotation of the quark field $q_i \to e^{i\frac{\theta_i}{2}\gamma_5} q_i$ which leads to

$$\mathcal{L}_{mass} = -m_i \bar{q}_i q_i \qquad (1.18)$$

and the angle θ of eq. (1.1) changes into

$$\theta \to \theta + \sum_i \theta_i = \theta + \mathrm{Arg\,det}\, M \,. \qquad (1.19)$$

So we see that the total contribution to the parameter θ comes from pure QCD without quarks plus the argument of the determinant of the quark mass matrix :

$$\theta = \theta_{QCD} + \mathrm{Arg\,det}\, M \quad . \tag{1.20}$$

The parameter θ contributes to the neutron electric dipole moment. But experimentaly [8] we know that the latter is smaller than 10^{-24} (e)cm which leads [9] to $\theta \lesssim 10^{-8}$. The question which arises is to explain this small number. In other words to undrestand why there is no strong P and CP violation in QCD.

1.3. - Possible proposed solutions to the θ - problem

(i) If the mass of a quark is zero the angle θ could be rotated away (see eqs. (1.14), (1.15)) and so no strong P and CP violation problem could arised. Using the ordinary quarks we could think to put $m_u = 0$, but this is in contradiction with the current algebra mass ratio :

$$m_u/m_d \simeq .56 \qquad [10]$$

(ii) Suppose that we put $\theta = 0$ by hand. Then θ could become different from zero by the radiative corrections due to the weak interactions in the renormalization of ArgdetM. To impose $\theta \lesssim 10^{-8}$ or zero order by order, by adjusting the different parameters of the theory, is an unnatural procedure, so the only hope is to construct a realistic model which predicts ArgdetM $< 10^{-8}$ (in such a model CP would be softly broken) [11, 12].

(iii) An other plausible possibility to solve the strong CP violation problem is to impose extra approximate global symmetries on the weak interactions and to use them in order

to rotate away the angle θ [7]. The price to pay for this procedure is the appearance of extra pseudo-Goldstone bosons (axions) [11,13]. In the next sections we will examine in details this mechanism which is due to Peccei and Quinn [7] and we will study the principal properties of the axions. We will show why the standard Peccei-Quinn axion can not exist because it is already rulled out by existing experimental data and we will present briefly some other open possibilities like the invisible axion [14].

(iv) Finally in supersymmetric theories there are new possibilities to solve the strong CP violation problem [15] : By using the R-invariance (there is no realistic model for the moment), or if the gluinos (color partner of gluons) were massless.

2. - PECCEI-QUINN MECHANISM [7]

This model is an extension of the standard $SU(3)\times SU(2)_L\times U(1)$ theory with two Higgs doublets:

$$\Phi_u = \begin{pmatrix} \Phi_u^0 \\ \Phi_u^- \end{pmatrix} \quad , \quad \Phi_d = \begin{pmatrix} \Phi_d^+ \\ \Phi_d^0 \end{pmatrix} \quad . \tag{2.1}$$

The first doublet Φ_u gives masses to the $+\frac{2}{3}$ charged quarks (u) and the second one Φ_d to the $-\frac{1}{3}$ charged quarks (d) and the leptons (l). Then the Higgs Lagrangian is

$$\mathcal{L}_\Phi = |D_\mu \Phi_u|^2 + |D_\mu \Phi_d|^2 - V(\Phi_u,\Phi_d) - \sum_{i=1}^{N} G_{u_i}\left(\bar{q}_{L_i}\Phi_u u_{R_i} + h.c.\right) - \sum_{i=1}^{N} G_{d_i}\left(\bar{q}_{L_i}\Phi_d d_{R_i} + h.c.\right) - \sum_{i=1}^{N} G_{\ell_i}\left(\bar{\ell}_{L_i}\Phi_d \ell_{R_i} + h.c.\right) \quad , \tag{2.2}$$

where D_μ is the covariant derivative, $G_{u_i}, G_{d_i}, G_{\ell_i}$ are the different Yukawa coupling constants, $V(\Phi_u,\Phi_d)$ is the Higgs potential, q_L and ℓ_L are the quark and lepton SU(2) doublets $q = \begin{pmatrix} u \\ d \end{pmatrix}$, $\ell = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$ and N is the number of doublets.

The $U(1)_{P.Q.}$ axial symmetry is the following:

$$u_i \to e^{i\eta\gamma_5} u_i \qquad \Phi_u \to e^{-2i\eta} \Phi_u$$
$$d_i \to e^{i\eta\gamma_5} d_i \qquad \Phi_d \to e^{-2i\eta} \Phi_d \tag{2.3}$$

The Yukawa part of the Lagrangian (2.2) remains invariant under the transformations (2.3) which define an extra axial U(1) symmetry of the theory. (The Higgs potential $V(\Phi_u, \Phi_d)$ must have the most general form which is also invariant under the above transformations). From eq. (1.15) we see that the above transformations change also the angle θ by $\theta - 4\eta N$ and so they can be used to rate θ away. Therefore the above model does not suffer from the strong CP violation.

But, by the Higgs mechanism, $SU(2)_L \times U(1)$ is spontaneously broken down to $U(1)_{em}$. The Higgs doublets Φ_u and Φ_d acquire non zero vacuum expectation values:

$$\langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ 0 \end{pmatrix} \quad , \quad \langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d \end{pmatrix} \tag{2.4}$$

and hence the SU(2) gauge bosons $W_\mu^\pm$ and Z_μ^0, the quarks and the leptons acquire masses. The $U(1)_{P.Q.}$ axial symmetry is also spontaneously broken and four Goldstone bosons appear. Three of them are absorbed by the Higgs mechanism giving masses to the $W_\mu^\pm$ and Z_μ^0, and the remaining one is the axion [11,13].

Following Weinberg's method [10] we shall treat the light quark masses m_u, m_d and m_s as small perturbative parameters. So in the zero light quark mass limit (which is obtained by putting the corresponding Yukawa couplings G_u, G_d and G_s equal to zero) we have an invariance under four

independent axial phase transformations on u, d, s and $\Phi_{u,d}$. But because of the strong anomaly we have only three conserved currents. Thus, after spontaneously symmetry breaking, they will appear 3 massless pseudoscalar Goldstone bosons: the pion (π°), the η° and the axion (a). But because the quark masses m_u, m_d and m_s are different from zero there is a mixing of the axion with π° and η° and all of them acquire small masses.

3. - AXION MIXING WITH π° AND η° [16, 17]

The form of the axial currents we have to deal with is :

$$J^5_{\mu_i} = \bar{q}\,\gamma_\mu\gamma_5\,\frac{\lambda_i}{2}\,q \quad , \tag{3.1.}$$

where q denotes a column of u, d, s quark states, $i = 0, 3, 8$ and $\lambda_0 = \sqrt{\frac{2}{3}}\,\mathbb{1}$. In the absence of quark masses $J^5_{\mu_3}$ and $J^5_{\mu_8}$ are conserved but $J^5_{\mu_0}$ has a QCD anomaly :

$$\partial_\mu J^5_{\mu_0} = \sqrt{\frac{3}{2}}\,\frac{1}{16\pi^2}\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu} \quad . \tag{3.2}$$

The extra axial current which corresponds to the $U(1)_{P.Q.}$ symmetry reads (see eqs. (2.2) and (2.3))

$$J^5_{\mu\,P.Q.} = f\,\partial_\mu h + \frac{x}{2}\sum_i \bar{u}_i\gamma_\mu\gamma_5 u_i + \frac{1}{2x}\sum_i\left(\bar{d}_i\gamma_\mu\gamma_5 d_i + \bar{\ell}_i\gamma_\mu\gamma_5\ell_i\right), \tag{3.3}$$

where $f = \sqrt{v_u^2+v_d^2} = \left(\sqrt{2}\,G_F\right)^{-1/2} \simeq 250\,GeV$ and x is on unknown positive quantity wich represents the ratio of the two expectation values of the two Higgs doublets $x = \frac{v_d}{v_u}$; h is a linear combination of $\mathrm{Im}\,\Phi^\circ_u$ and $\mathrm{Im}\,\Phi^\circ_d$ given by $h = -\frac{1}{\sqrt{2}}\left(\cos\varphi\,\mathrm{Im}\,\Phi^\circ_u + \sin\varphi\,\mathrm{Im}\,\Phi^\circ_d\right)$ with $\sin\varphi = \frac{v_u}{\sqrt{v_u^2+v_d^2}}$. This current is conserved except for the QCD anomaly:

$$\partial_\mu J^5_{\mu\,P.Q.} = \frac{N}{2}\left(x + \frac{1}{x}\right)\frac{1}{16\pi^2}\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu} \quad . \tag{3.4}$$

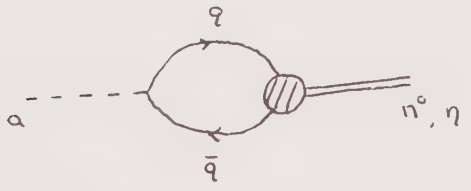
Because of the instanton effects we will only have true $U(1)_{P.Q.}$ symmetry if we eliminate the anomaly in $J^5_{\mu\,P.Q.}$.

In the absence of the light quark masses, the pseudoscalar mesons π_3 and π_8 associated with the currents $J^5_{\mu_3}$ and $J^5_{\mu_8}$ are true Goldstone bosons, namely the pion and the η , and so is the π_h field, called axion, associated with the modified $J^5_{\mu\,P.Q.}$ current which has no anomaly. To find this current we must construct a modified $J^5_{\mu\,P.Q.}$ which is not only anomaly free but its charge also commutes with the corresponding charges of $J^5_{\mu_3}$ and $J^5_{\mu_8}$. Such a current is given by:

$$J^5_{\mu_h} = J^5_{\mu\,P.Q.} - \frac{1}{\sqrt{6}}\left(x+\frac{1}{x}\right)N\,J^5_{\mu_0} - \frac{1}{2}\left(x-\frac{1}{x}\right)J^5_{\mu_3} - \frac{1}{2\sqrt{3}}\left(x-\frac{1}{x}\right)J^5_{\mu_8}$$

$$= f\,\partial_\mu h + \frac{1}{\sqrt{6}}\left[\left(x+\frac{2}{x}\right)-\left(x+\frac{1}{x}\right)N\right]J^5_{\mu_0} + \frac{x}{2}\,\bar{c}\,\gamma_\mu\gamma_5 c + \cdots \qquad (3.5)$$

But because the light quark masses are different from zero the pion, η and axion are not true Goldstone bosons. The axion obtains a small mass through its mixing with π° and η° (see fig.).



To carry out the perturbation calculation we must construct a 3 x 3 mixing mass matrix whose eigenvalues correspond to the masses of the physical particles π, η and a. This mass matrix can be calculated by the standard current algebra method and is given by:

$$m_{ij} = \frac{i}{f_i f_j}\langle 0|\left[Q^5_i, \partial_\mu J^5_{\mu j}\right]|0\rangle \quad , \qquad (3.6)$$

with $Q_i^5 = \int d^3x\, J^5_{o_i}$ and $\partial_\mu \bar{q}\gamma_\mu\gamma_5 q = 2im_q \bar{q}\gamma_5 q$ + anomaly. In the following we set $f_3 = f_8 = f_\eta = 94$ MeV . The mass matrix m_{ij} reads:

$$m = \frac{1}{f_\eta^2}\begin{bmatrix} u+d & -\lambda u+\rho d & \frac{1}{\sqrt{3}}(u-d) \\ -\lambda u+\rho d & \lambda^2 u+\rho^2(d+s) & -\frac{1}{\sqrt{3}}\left[\lambda u+\rho(d-2s)\right] \\ \frac{1}{\sqrt{3}}(u-d) & -\frac{1}{\sqrt{3}}\left[\lambda u+\rho(d-2s)\right] & \frac{1}{3}(4s+u+d) \end{bmatrix}, \quad (3.7)$$

where $\lambda = \frac{1}{3}\frac{f_\eta}{f}\left[(x+\frac{1}{x})N+2(x-\frac{1}{x})\right]$, $\rho = \frac{1}{3}\frac{f_\eta}{f}\left[(x+\frac{1}{x})N-(x-\frac{1}{x})\right]$ and $u \simeq m_u\langle\bar{u}u\rangle$, $d \simeq m_d\langle\bar{d}d\rangle$, $s \simeq m_s\langle\bar{s}s\rangle$ [17]. Assuming SU(3) invariance we have $\langle\bar{u}u\rangle \simeq \langle\bar{d}d\rangle \simeq \langle\bar{s}s\rangle$.

After diagonalising the mass matrix, we get the following values for η, η' and axion masses [17]:

$$m_\eta = \sqrt{\frac{2}{3}}\,\frac{1}{f_\eta}\left[(s+u+d)-(s^2+u^2+d^2-ud-us-ds)^{1/2}\right]^{1/2} + O\left(\frac{s^{1/2}}{f}\right), \quad (3.7a)$$

$$m_{\eta'} = \sqrt{\frac{2}{3}}\,\frac{1}{f_\eta}\left[(s+u+d)+(s^2+u^2+d^2-ud-us-ds)^{1/2}\right]^{1/2} + O\left(\frac{s^{1/2}}{f}\right), \quad (3.7b)$$

$$m_a = m_\eta\,\frac{f_\eta}{f}\left(x+\frac{1}{x}\right)N\left\{\frac{3}{2}\,\frac{uds}{(ud+us+ds)\left[(s+u+d)-(s^2+u^2+d^2-us-ds-ud)^{1/2}\right]}\right\}^{1/2} + \quad (3.7c)$$
$$+ O\left(m_\eta\frac{f_\eta^2}{f^2}\right)$$

and the following mixing angles for the axion:

$$a = \eta_h + \xi_\eta\,\eta_3 + \xi_{\eta'}\,\eta_8 \quad (3.8a)$$

with

$$\xi_\eta = -\frac{1}{2}\,\frac{f_\eta}{f}\left[\left(x+\frac{1}{x}\right)N\,\frac{s(d-u)}{ud+us+ds} - x + \frac{1}{x}\right] + O\left(\frac{f_\eta^2}{f^2}\right), \quad (3.8b)$$

$$\xi_{\eta'} = -\frac{1}{2\sqrt{3}}\,\frac{f_\eta}{f}\left[\left(x+\frac{1}{x}\right)N\,\frac{us+ds-2ud}{ud+us+ds} - x + \frac{1}{x}\right] + O\left(\frac{f_\eta^2}{f^2}\right). \quad (3.8c)$$

We note finally that the heavy quark (q) contribution to the above results is of the order of m_d/m_q .

4. - PRINCIPAL PROPERETIES OF THE AXION

4.1. - Axion mass

In the limit $m_s >> m_u, m_d$ eq. (3.7c) gives:

$$m_a \simeq m_\pi f_\pi (\sqrt{2} G_F)^{1/2} N \frac{z^{1/2}}{1+z} \left(x + \frac{1}{x}\right) , \qquad (4.1a)$$

where $z = \frac{m_u}{m_d} \simeq .56$. Then, because $x + \frac{1}{x} \geq 2$ and $N \geq 3$ the axion mass m_a must be greater than $\sim$ 150 kev. In this approximation, the mixing angles ξ_π and ξ_η become (see eqs. (3.8)).

$$\xi_\pi \simeq -\frac{1}{2} \frac{f_\pi}{f} \left[\left(x + \frac{1}{x}\right) N \frac{1-z}{1+z} - x + \frac{1}{x} \right] \quad \text{and} \qquad (4.1b)$$

$$\xi_\eta \simeq -\frac{1}{2\sqrt{3}} \frac{f_\pi}{f} \left[\left(x + \frac{1}{x}\right) N - x + \frac{1}{x} \right] . \qquad (4.1c)$$

4.2. - $a \longrightarrow e^+ e^-$

If $m_a \geq 2 m_e$ the $a \to e^+ e^-$ would be its dominant decay with a transition rate given by the expression:

$$\Gamma(a \to e^+ e^-) = \frac{G_F \sqrt{2}}{8\pi} m_a m_e^2 \frac{1}{x^2} \left(1 - \frac{4 m_e^2}{m_a^2}\right)^{1/2} \qquad (4.2)$$

$$\Rightarrow \tau(a \to e^+ e^-) \lesssim 10^{-8} \text{ sec} \quad (x \sim \mathcal{O}(1)) ,$$

which is excluded from beam dump experiments by several orders of magnitude. By combining this result with (4.1) we can conclude that the axion mass is bounded as [15]:

$$150 \text{ keV} \lesssim m_a \lesssim 1 \text{ MeV} . \qquad (4.3)$$

4.3. - $a \to \gamma\gamma$

Because of eq. (4.3) the dominant decay of the axion is $a \to \gamma\gamma$ given easily by the electromagnetic anomaly of the

divergence of the axion current [16]:

$$\frac{A(a\to 2\gamma)}{A(\pi^0\to 2\gamma)} = \frac{f_\pi}{f}\,\frac{T_r Q_a Q^2}{T_r Q_3 Q^2} \simeq \frac{f_\pi}{f} N\left(x+\frac{1}{x}\right)\frac{m_u}{m_u+m_d} \qquad (4.4a)$$

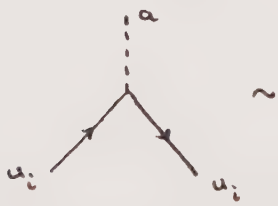
$$\Rightarrow \tau(a\to 2\gamma) = \tau(\pi^0\to 2\gamma)\left(\frac{m_\pi}{m_a}\right)^5 \frac{m_d}{m_e} = .7\,\text{sec}\left(\frac{100\,\text{kev}}{m_a}\right)^5 . \qquad (4.4b)$$

Using now eq. (4.3) one finds:

$$7\cdot 10^{-6}\,\text{sec} \lesssim \tau(a\to 2\gamma) \lesssim .091\,\text{sec} \qquad (4.4c)$$

4.4. - Heavy quarks decay experiments [18-20]

The axion coupling with the quarks or the leptons is (see eqs. (2.2), (2.4) and (3.3))



$$\sim \frac{m_{u_i}}{f}\,x \qquad (4.5a)$$

$$\sim \frac{m_{d_i(\ell_i)}}{f}\,\frac{1}{x} \qquad (4.5b)$$

Then the branching ratios of $\psi\to a\gamma$ and $\Upsilon\to a\gamma$ are given by:

$$\frac{\Gamma(\psi\to a\gamma)}{\Gamma(\psi\to\mu^+\mu^-)} = \frac{G_F}{\sqrt{2}\,\pi\alpha_e}\,m_c^2\,x^2 \qquad \text{and} \qquad (4.6a)$$

$$\frac{\Gamma(\Upsilon\to a\gamma)}{\Gamma(\Upsilon\to\mu^+\mu^-)} = \frac{G_F}{\sqrt{2}\,\pi\alpha_e}\,m_b^2\,\frac{1}{x^2} . \qquad (4.6b)$$

Therefore, from the heavy quark systems decay experiments one can restrict the upper and lower bounds of the unknown parameter x [18]. Experimentaly the upper limits for these branching ratios are $B(\psi \to \gamma a) < 1.4 \times 10^{-5}$ (90% C.L) [19] and $B(\Upsilon \to \gamma a) < 3.5 \times 10^{-4}$ (90% C.L.) [20]. But from eqs (4.6) we see that the product $B(\psi \to \gamma a) \cdot B(\Upsilon \to \gamma a)$ does not depend on x and it gives a theoretical prediction 1.6×10^{-8} which is incosistent with the experimental upper limit being 0.6×10^{-9} (smaller by almost 2 orders of magnitude!)

4.5 - Lower limit for BR ($K^+ \to \pi^+ a$) [17,21]

Using standard current algebra techniques, one can find a strict parameter-free lower bound for the branching ratio of $K^+ \to \pi^+ a$. We use an effective weak Hamiltonian in the form of current x current interaction, $H_w = \frac{G_F}{\sqrt{2}} \times$ octet part of $J_\mu^+ J_\mu$, where J_μ is the usual charged current (we take only the octet part in order to satisfy the phenomenological $\Delta I = \frac{1}{2}$ rule). Using eqs. (3.8), we have:

$$A(K^+ \to \pi^+ a(k)) = \langle \pi^+ \pi_h(k) | H_w | K^+ \rangle + \xi_\pi \langle \pi^+ \pi_3(k) | H_w | K^+ \rangle + \xi_\eta \langle \pi^+ \pi_8(k) | H_w | K^+ \rangle \quad (4.7)$$

The first term on the r.h.s. of eq.(4.7) vanishes in the current algebra limit

$$\lim_{k_\mu \to 0} \langle \pi^+ \pi_h(k) | H_w | K^+ \rangle = \frac{1}{f} \langle \pi^+ | [Q_h, H_w] | K^+ \rangle , \quad (4.8)$$

because H_w commutes with Q_h^5 (see eq. (3.5)). Let us also note that the matrix element $\langle \pi^+ \pi_3(k) | H_w | K^+ \rangle$ has an off mass-shell momentum dependent term [22] which is absent in the $\langle \pi^+ \pi_8(k) | H_w | K^+ \rangle$. Hence we can assume that the last one is constant while there is a strong momentum dependence in the first one:

$$\langle \pi^+(k_+) \pi_3(k_3) | H_w | K^+ \rangle \simeq \frac{1}{2 f_\pi} \frac{k_+^2 - k_3^2}{(k_+ + k_3)^2} \langle \pi^+ | H_w | K^+ \rangle \quad \text{and} \quad (4.9a)$$

$$\lim_{k\to 0} \langle \pi^+ \eta_c(k) | H_w | K^+ \rangle = \frac{1}{f_\eta} \langle \pi^+ | [Q_8^5, H_w] | K^+ \rangle \quad . \tag{4.9b}$$

The last commutator satisfies the following relation:

$$[Q_8^5, H_w]_{\Delta S=1} = -\frac{1}{\sqrt{3}} [Q_3^5, H_w]_{\Delta S=1} \quad , \tag{4.10}$$

which combining with the former eqs. implies:

$$A(K^+ \to \pi^+ a) = \left(\frac{m_\eta^2 - m_a^2}{m_K^2} \xi_\pi - \frac{1}{\sqrt{3}} \xi_\eta \right) \frac{1}{2 f_\pi} \langle \pi^+ | H_w | K^+ \rangle \quad . \tag{4.11}$$

It is now crucial to realize that the matrix element $\langle \pi^+ | H_w | K^+ \rangle$ is related to that of $K_s^\circ \to \pi^+ \pi^-$ as:

$$\langle \pi^+ \pi^- | H_w | K_s^\circ \rangle = \frac{1}{2 f_\pi} \langle \pi^+ | H_w | K^+ \rangle \quad . \tag{4.12}$$

Using the expressions (3.8) one finds the bounds:

$$\xi_\pi - \sqrt{3}\,\xi_\eta = \frac{f_\pi}{f} \left(2 + \frac{1}{2}\right) N \frac{u(s-d)}{ud+us+ds} \geqslant 2N \frac{f_\pi}{f} \frac{u(s-d)}{ud+us+ds} \qquad \text{and} \tag{4.13a}$$

$$-\sqrt{3}\,\xi_\eta \geqslant \frac{f_\pi}{f} \left[N \frac{us+ds-2ud}{us+ds+ud} - 1 \right] \quad , \tag{4.13b}$$

which imply:

$$A(K^+ \to \pi^+ a) \geqslant \frac{f_\pi}{f} \left\{ \frac{m_\pi^2}{m_K^2} N \frac{2u(s-d)}{ud+us+ds} + \frac{m_K^2 - 3m_\pi^2}{3m_K^2} \left[N \frac{us+ds-2ud}{us+ds+ud} - 1 \right] \right\} \times \tag{4.14}$$

$$\times A(K_s^\circ \to \pi^+ \pi^-)$$

After corrected the above result by taking into account the pion final state interaction we find [17]

$$BR(K^+ \to \pi^+ a) \geqslant 8.5 \times 10^{-6} \tag{4.15}$$

which is incosistent with the experimental upper limit which is [23] $\lesssim 4.8 \times 10^{-8}$ (the theoretical lower bound is violated by 2 orders of magnitude!)

By combining the incosistency with experiment of the lower limit for $BR(K^+ \to \pi^+ a)$ together with that of $BR(\psi \to a\gamma) \times BR(\gamma \to a\gamma)$ we can conclude that the standard Peccei-Quinn axion does not exist!

5. - INVISIBLE AXION

In section 4 we saw that the axion mass and its coupling with matter is proportional to $1/f$, where f is the scale of breaking of $U(1)_{P.Q.}$ symmetry. So if f is very large, the axion would be almost invisible. There are two such possibilities:

(i) Couple the axion with some superheavy quarks (Kim; Shiffman et al.) [24]. But in this way one constructs rather complicated models with many extra particles and parameters and one creates a new hierarchy problem because of the new large scale which is the scale of breaking of $U(1)_{P.Q.}$ symmetry or the mass of the superheavy quarks.

(ii) P.Q. symmetries in Grand-Unified-Theories (Dine, Fischler, Srednicki, Wise, Georgi, Glashow, Hall) [14]. In the model discussed in section 2 let us introduce an extra complex scalar Higgs Φ singlet under $SU(2)_L \times U(1)$. In the Higgs Lagrangian (2.2) let us add the kinetic term $\frac{1}{2}(\partial_\mu \Phi)^2$, leave unchange the Yukawa couplings and modify only the Higgs potential $V(\phi_u, \phi_d, \Phi)$ in such a way that the theory is invariant under the phase transformations:

$$u_i \to e^{i\eta\gamma_5} u_i \qquad \phi_u \to e^{-2i\eta} \phi_u$$
$$d_i \to e^{i\eta\gamma_5} d_i \qquad \phi_d \to e^{-2i\eta} \phi_d \qquad (6.1)$$
$$\Phi \to e^{2i\eta} \Phi$$

The above transformations are a trivial extension of the $U(1)_{P.Q.}$ symmetry (2.3). A Higgs potential invariant under the new

$U(1)_{P.Q.}$ symmetry transformations (6.1) is [14]:

$$V(\phi_u,\phi_d,\Phi) = \lambda_u|\phi_u|^4 + \lambda_d|\phi_d|^4 + \lambda|\Phi|^4 + \rho|\phi_u^+\phi_d|^2 + \left(a|\phi_u|^2 + b|\phi_d|^2\right)|\Phi|^2 + \\ + \left(c\phi_u^i \epsilon_{ij}\phi_d^j \Phi^2 + h.c.\right) + d|\phi_u^i \epsilon_{ij}\phi_d^j|^2 , \qquad (6.2)$$

where ϵ_{ij} is the complete antisymmetric tensor of SU(2) with $i,j = 1,2$.

Then Φ obtains also a non zero vacuum expectation value such that $\langle\Phi\rangle = v >> \sqrt{v_u^2 + v_d^2}$ (see eq. (2.4)). Because Φ also participates to the $U(1)_{P.Q.}$ symmetry (see eq. (6.1)) the corresponding current $J^5_{\mu\,P.Q.}$ contains Φ. It is straight forward to verify that

$$J^5_{\mu\,P.Q.} = f\,\partial_\mu h + \cdots , \qquad (6.3)$$

where $h \simeq \mathrm{Im}\,\Phi$ and $f \sim \frac{v}{2}\left(\lambda + \frac{1}{\lambda}\right)$.

But a very light axion would have astrophysical consequences [14]. In fact if the axion decay time τ is $10^5\,\mathrm{yrs} \lesssim \tau \lesssim 10^{10}\,\mathrm{yrs}$ it might give an observable distorsion of the cosmic microwave radiation background (especialy from the existence of red giant stars). It would be copiously produced and rapidly carry off all of their thermal energy. Then τ must be greater than 10^{10} yrs (or smaller than 10^5 yrs) which gives an axion mass $m_a \lesssim .01$ eV and so $\langle\Phi\rangle = v \gtrsim 10^9$ GeV (see eqs. 4.4).

Therefore an invisible axion without astrophysical consequences could be easily implemented in GUTs with v being of the order of the Grand-Unification mass scale ($\sim 10^{14}$ GeV)

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LOWER BOUND FOR BRANCHING RATIO OF $K^+ \to \pi^+$AXION AND NON-EXISTENCE OF PECCEI–QUINN AXION

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A strict parameter-free lower bound for the branching ratio of $K^+ \to \pi^+ a$ is obtained: $BR(K^+ \to \pi^+ a) \geqslant 3.5 \times 10^{-5}$ which rules out the existence of the Peccei–Quinn axion.

A few years ago, Weinberg [1] and Wilczek [2] showed that there must exist a very light pseudoscalar boson, the axion, as a consequence of the Peccei–Quinn mechanism to solve the problem of *P* and *CP* violation in Quantum Chromodynamics [3]. Experimental results showed the existence of the Peccei–Quinn axion which we simply call the axion in the following to be unlikely, although there were considerable uncertainties in the interpretation of the experimental data [4]. There is now, however, a claim of an observation of a light particle of a mass less than 1 MeV and decaying into 2γ [5].

The purpose of this note is to point out that if this experiment was correct, this light particle could not be an axion whose existence is ruled out by the present calculation on the lower bound of the branching ratio of $K^+ \to \pi^+ a$: $BR(K^+ \to \pi^+ a) \geqslant 3.5 \times 10^{-5}$ which is in conflict with the experimental data: $BR(K^+ \to \pi^+ a) \leqslant 1.4 \times 10^{-6}$. (It has been argued that this lower bound can be pushed down to 2.7×10^{-7} (90% CL [6].) This calculated lower bound obtained here has no free parameter dependence and has the usual reliability of a typical current algebra calculation. It is of the same type of the known lower bound for the axion mass and its two photon decay rate [1, 2,7]. To achieve this result, the current algebra technique is used in a consistent manner throughout in contrast with earlier calculations [6,8] which gave wildly different results. Care is given in the determination of the ratio of up and down quark masses and also in the inclusion of the effect of strong final state interaction in $K_s^0 \to \pi^+\pi^-$ on which the amplitude $K^+ \to \pi^+ a$ depends indirectly.

To carry out our program, it is useful to write down explicitly various currents involved in the calculation of the masses of the pseudoscalar mesons and axion and their mixing parameters which will be used in the subsequent current algebra calculation of $K^+ \to \pi^+ a$. We shall modify the method of Bardeen and Tye [7] and include in our calculation the contribution of the s-quark (this method is also implicitly given by Weinberg [1]). The form of the axial currents which we have to deal with is:

$$J^5_{\mu_i} = \bar{q}\gamma_\mu\gamma_5(\lambda_i/2)q, \tag{1}$$

where q denotes a column of u, d, s quark states and λ_i are the usual Gell-Mann matrices with $i = 0, 3, 8$; $\lambda_0 = (2/3)^{1/2}\mathbf{1}$. In the absence of quark masses $J^5_{\mu_3}$ and $J^5_{\mu_8}$ are conserved but $J^5_{\mu_0}$ has a QCD anomaly [9]:

$$\partial_\mu J^5_{\mu_0} = (3/2)^{1/2}[g^2/(4\pi)^2]\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}\,. \tag{2}$$

Let us introduce an axial current $J^5_{\mu_{P.Q.}}$ having an extra $U(1)_{P.Q.}$ chiral symmetry which was introduced by Peccei and Quinn to eliminate the *P*, *CP* violation in QCD. This model is an extension of the standard $SU(2) \times U(1)$

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theory with two Higgs doublets which acquire, both of them, non-zero vacuum expectation values. The first doublet gives masses to the 2/3 charged quarks and the second one to the $-1/3$ charged quarks and the leptons. The $U(1)_{P.Q.}$ axial symmetry is spontaneously broken and four Goldstone bosons appear. Three of them are absorbed by the Higgs mechanism giving masses to the $W_\mu^\pm$ and Z_μ^0 gauge bosons. The remaining one is the axion. Following the notations of ref. [7] this extra axial current which corresponds to the $U(1)_{P.Q.}$ symmetry reads:

$$J^5_{\mu P.Q.} = f\partial_\mu h + \frac{x}{2}\sum_i \bar{u}_i\gamma_\mu\gamma_5 u_i + \frac{1}{2x}\sum_i \bar{d}_i\gamma_\mu\gamma_5 d_i + \frac{1}{2x}\sum_i \bar{\ell}_i\gamma_\mu\gamma_5 \ell_i, \tag{3}$$

where u_i, d_i and ℓ_i denote, respectively, 2/3, $-1/3$ charged quarks and leptons, $\sqrt{2}\,G_F f^2 = 1$ and x is an unknown positive quantity which represents the ratio of the two expectation values of the two Higgs doublets. This current is conserved except for the QCD anomaly:

$$\partial_\mu J^5_{\mu P.Q.} = \tfrac{1}{2}(x + 1/x)N[g^2/(4\pi)^2]G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}, \tag{4}$$

where N is the number of doublets. Because of the instanton effects, we will only have true $U(1)_{P.Q.}$ symmetry if we eliminate the anomaly in $J^5_{\mu P.Q.}$.

In the absence of the quark masses, the pseudoscalar mesons ϕ_3 and ϕ_8 associated with the currents $J^5_{\mu_3}$ and $J^5_{\mu_8}$ are true Goldstone bosons, namely the pion and the η, and so is the ϕ_h field, called axion, associated with the modified $J^5_{\mu P.Q.}$ current which has no anomaly. To find this current we must construct a modified $J^5_{\mu P.Q.}$ which is not only anomaly free but also commute with $J^5_{\mu_3}$ and $J^5_{\mu_8}$. Such a current is given by:

$$J^5_{\mu_h} = J^5_{\mu P.Q.} - (\sqrt{6})^{-1}(x + 1/x)NJ^5_{\mu_0} - \tfrac{1}{2}(x - 1/x)J^5_{\mu_3} - (2\sqrt{3})^{-1}(x - 1/x)J^5_{\mu_8} \tag{5a}$$

$$= f\partial_\mu h + (\sqrt{6})^{-1}[(x + 2/x) - (x + 1/x)N]\,J^5_{\mu_0} + \tfrac{1}{2}x\bar{c}\gamma_\mu\gamma_5 c + \dots . \tag{5b}$$

But, because the quark masses are different from zero the pion, η and axion are not true Goldstone bosons. To carry out the perturbation calculation we must construct a 3×3 mixing mass matrix whose eigenvalues correspond to the masses of the physical particles π, η and the axion a. This mass matrix can be calculated by the standard current algebra method and is given by

$$M_{ij} = (i/f_i f_j)\langle 0|[Q_i^5, \partial_\mu J^5_{\mu_i}]\,|0\rangle .$$

In the following we set $f_3 = f_8 = f_\pi = 94$ MeV.

After diagonalising the mass matrix, we get the following values for π, η and axion masses:

$$m_\pi = \sqrt{2/3}\,f_\pi^{-1}[(s + u + d) - (s^2 + u^2 + d^2 - ud - us - ds)^{1/2}]^{1/2} + O(s^{1/2}/f), \tag{6a}$$

$$m_\eta = \sqrt{2/3}\,f_\pi^{-1}[(s + u + d) + (s^2 + u^2 + d^2 - ud - us - ds)^{1/2}]^{1/2} + O(s^{1/2}/f), \tag{6b}$$

$$m_a = m_\pi(f_\pi/f)(x + 1/x)N$$

$$\times \left(\tfrac{3}{2}uds/\{(ud + us + ds)[(s + u + d) - (s^2 + u^2 + d^2 - ud - us - ds)^{1/2}]\}\right)^{1/2} + O(m_\pi f_\pi^2/f^2) \tag{6c}$$

and the following mixing angles for the axion:

$$a = \phi_h + \xi_\pi\phi_3 + \xi_\eta\phi_8 , \tag{7a}$$

with

$$\xi_\pi = -\tfrac{1}{2}(f_\pi/f)[(x + 1/x)Ns(d - u)/(ud + us + ds) - x + 1/x] + O(f_\pi^2/f^2), \tag{7b}$$

$$\xi_\eta = -(2\sqrt{3})^{-1}(f_\pi/f)[(x+1/x)N(us+ds-2ud)/(ud+us+ds) - x + 1/x] + O(f_\pi^2/f^2), \tag{7c}$$

where $u \equiv m_u v_u$, $d \equiv m_d v_d$ with $v_u = \langle 0|\bar{u}u \cos(xh/f) + i\bar{u}\gamma_5 u \sin(xh/f)|0\rangle$, $v_d = \langle 0|\bar{d}d \cos(h/xf) + i\bar{d}\gamma_5 d \sin(h/xf)|0\rangle$, where u and d denote, respectively, 2/3 and $-1/3$ charged quarks. Assuming SU(3) invariance we have $v_u \simeq v_d \simeq v_s$ and in the approximation $m_s \gg m_u, m_d$ we recover the Weinberg results of ref. [1] for masses and mixing angles (putting $N = 2$). (Note that we actually need only SU(2) invariance; SU(3) invariance is only needed to show $m_s \gg m_u, m_d$). Our results also agree, in the approximation $m_s \gg m_u, m_d$, with those of Bardeen and Tye [7] except the ξ_η coefficient. The reason for this discrepancy is due to their neglect of s quark in η-current which is not realistic.

We note finally that the heavy quark (q) contribution to the above results is of the order of m_d/m_q.

We are now in a position to calculate the matrix element for the process $K^+ \to \pi^+ a$. To be consistent with the approach used by Peccei–Quinn [3] which consists in generalising the Weinberg–Salam model, we must use the effective weak hamiltonian H_w in the form of current × current interaction, $H_w = (G_F/\sqrt{2})J_\mu^\dagger J_\mu$, where J_μ is the usual charged current. Using eqs. (7) and standard current algebra technique, we have

$$\lim_{k_\mu \to 0} f_\pi M(K^+ \to \pi^+ a(k)) = (f_\pi/f)\langle\pi^+|[Q_h^5, H_w]|K^+\rangle + \xi_\pi\langle\pi^+|[Q_3^5, H_w]|K^+\rangle + \xi_\eta\langle\pi^+|[Q_8^5, H_w]|K^+\rangle. \tag{8}$$

The first commutator on the r.h.s. of eq. (8) vanishes because H_w commutes with Q_h^5 [see eqs. (5)]. The last two commutators do not vanish but they satisfy the following relation:

$$[Q_8^5, H_w]_{\Delta S=1} = -\sqrt{3}\,[Q_3^5, H_w]_{\Delta S=1}, \tag{9}$$

which implies:

$$\lim_{k_\mu \to 0} M(K^+ \to \pi^+ a(k)) = (1/2f_\pi)(\xi_\pi - \sqrt{3}\,\xi_\eta)\langle\pi^+|H_w^{p.c.}|K^+\rangle. \tag{10}$$

Using eqs. (7) one finds

$$\xi_\pi - \sqrt{3}\,\xi_\eta = \frac{f_\pi}{f}(x+1/x)N\frac{u(s-d)}{ud+us+ds} \geqslant 2N\frac{f_\pi}{f}\frac{u(s-d)}{ud+us+ds} \tag{11a}$$

and then

$$\lim_{k_\mu \to 0} |M(K^+ \to \pi^+ a(k))| \geqslant N\frac{f_\pi}{f}\frac{u(s-d)}{ud+us+ds}\frac{1}{f_\pi}|\langle\pi^+|H_w^{p.c.}|K^+\rangle|, \tag{11b}$$

where we have used the fact $(x + 1/x) \geqslant 2$ and hence obtained a lower bound which is independent of x. It is interesting to note that the same type of lower bounds exists also for the mass of the axion, eq. (6c), and for its 2γ decay rate [7].

We now have to correct eq. (8) to use it in the physical region, where $k_\mu \neq 0$, but $k_\mu^2 = m_a^2 \sim 0$. For this purpose, let us note that the matrix element $\langle\pi^+\pi_3(k)|H_w|K^+\rangle$ has an off mass-shell momentum dependence term [10] which is absent in the $\langle\pi^+\pi_8(k)|H_w|K^+\rangle$.

Hence we can assume that the last one is constant while there is a strong momentum dependence in the first one:

$$\langle\pi^+(k_+)\pi_3(k_3)|H_w|K^+\rangle \approx (1/2f_\pi)[(k_+^2 - k_3^2)/(k_+ + k_3)^2]\langle\pi^+|H_w|K^+\rangle. \tag{12}$$

It is now crucial to realize that the matrix element $\langle\pi^+|H_w|K^+\rangle$ is related to that of $K_s^0 \to \pi^+\pi^-$ and not to $K^+ \to \pi^+\pi^0$ as it was erroneously stated in the ref. [6]. For a discussion on this point see for example ref. [10]. Because of the absence of the off mass-shell momentum dependence term, we have in fact (see below):

$$\langle\pi^+\pi^-|H_w^{p.v.}|K_s^0\rangle = (1/2f_\pi)\langle\pi^+|H_w^{p.c.}|K^+\rangle, \tag{13}$$

where we have decomposed H_w into a parity violating part ($H_w^{p.v.}$) and a parity conserving part ($H_w^{p.c.}$). Instead of eq. (10), with the axion on its mass shell we now have

$$M(K^+ \to \pi^+ a) \simeq \{[m_\pi^2 - m_a^2)/m_K^2]\,\xi_\pi - \sqrt{3}\,\xi_\eta\} M(K_s^0 \to \pi^+\pi^-). \tag{10'}$$

As before, we can bound the r.h.s. of the eq. (10') using eq. (11a) together with the inequality [see eq. (7c)]

$$-\sqrt{3}\,\xi_\eta \geqslant \frac{f_\pi}{f}\left[N^2\left(\frac{us + ds - 2ud}{us + ds + ud}\right)^2 - 1\right]^{1/2}. \tag{14}$$

We finally arrive at

$$\frac{\Gamma(K^+ \to \pi^+ a)}{\Gamma(K_s^0 \to \pi^+\pi^-)} \geqslant \frac{f_\pi^2}{f^2}\left\{\frac{m_\pi^2}{m_K^2} N \frac{2u(s-d)}{ud + us + ds} + \frac{m_K^2 - m_\pi^2}{m_K^2}\left[N^2\left(\frac{us + ds - 2ud}{us + ds + ud}\right)^2 - 1\right]^{1/2}\right\}^2 \frac{(m_K^2 - m_\pi^2)}{m_K (m_K^2 - 4m_\pi^2)^{1/2}} \tag{15a}$$

$$\geqslant 1.1 \times 10^{-6}, \tag{15b}$$

where the last factor of the r.h.s. of eq. (15a) is the phase space contribution. Finally, in terms of the $K^+ \to \pi^+ a$ branching ratio

$$\mathrm{BR}(K^+ \to \pi^+ a) \geqslant 1.1 \times 10^{-4}, \tag{16}$$

where we have used $N \geqslant 3$ and $m_u/m_d = 0.56$ [11]. This is to be compared with the experimental limit

$$\mathrm{BR}(K^+ \to \pi^+ a) \leqslant 1.4 \times 10^{-6},$$

which can be pushed further down to 2.7×10^{-7} as argued by Goldman and Hoffman [6]. We can see thus the theoretical lower bound [eq. (16)] is violated by two orders of magnitude which leads us to the conclusion of the *non existence of the axion.*

Our result on the BR is one order of magnitude larger than that given in ref. [8]. If we used, however, H_w in the form of quark density (which was not the effective hamiltonian obtained from the standard model) we would get a closer agreement.

It is now reasonable to ask if there were any loophole in our calculation. Our final result depends on the reliability of the calculation of m_u/m_d and also the neglect of the strong pion–pion final state interaction which was found to be important in K_{ℓ_4} [12] decay and $\eta \to 3\eta$ decay [13] and which exists in the $K_s^0 \to \pi^+\pi^-$ amplitude but not in $K^+ \to \pi^+ a$ amplitude.

We deal first with the determination of the ratio of m_u/m_d by Weinberg [11] and whose value is used in the present calculation. There was some doubt associated with this determination because when this value was used to calculate the $\eta \to 3\pi$ rate, it was found that the $\eta \to \pi^+\pi^-\pi^0$ rate was a factor of 4 too small [13,14]. This discrepancy is recently removed [13] (just the same as in the K_{ℓ_4} problem [12]) by taking into account of the strong pion–pion final state interaction which was previously unjustifiably neglected. Because the new calculation of $\eta \to 3\pi$ agrees with the experimental rate within an uncertainty of the order of 20%, we can conclude that the ratio $m_u/m_d = 0.56$ has the same uncertainty which is quite adequate for our purpose.

Because the strong final state interaction cannot be neglected we have to correct the r.h.s. of eq. (15a) for this effect. This can be straightforwardly done by noting that the $K_s^0 \to \pi^+\pi^-$ amplitude must satisfy:

(i) The final state theorem which states that it must have the phase of δ, where δ is the s-wave $I = 0$ pion–pion phase shift.

(ii) The analytic property of the $K_s^0 \to \pi^+\pi^-$ amplitude.

(i) and (ii) lead us to the usual factor

$$D^{-1}(s, m_\pi^2) = \exp \frac{s - m_\pi^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds' \delta_0(s')}{(s' - m_\pi^2)(s' - s)},$$

where we normalize to unity at the current algebra point $s = p_K^2 = m_\pi^2$. The physical amplitude $K_s^0 \to \pi^0\pi^-$ with both pions on their mass shell, instead of eq. (13) is now given by

$$\langle\pi^+\pi^-|H_w^{p.v.}|K_s^0\rangle = (2f_\pi)^{-1}\langle\pi^+|H_w^{p.c.}|K^+\rangle D^{-1}(m_K^2, m_\pi^2)\,, \tag{13'}$$

which modify eq. (15) by an extra factor $|D(m_K^2, m_\pi^2)|^2$ on its r.h.s. Using the expression for D^{-1} given in ref. [12] and which is the same one to solve the $\eta \to 3\pi$ problem, the r.h.s. of eq. (16) is reduced by a factor of 3 to give 3.5×10^{-5} which is still at least one order of magnitude larger than the experimental upper bound for the branching ratio of $K^+ \to \pi^+ a$.

Similar lower bound exists also for the branching ratio of $K_L^0 \to \pi^+\pi^- a$, although there is no yet reported measurement for this ratio. As we shall see below if the axion existed, it is much more difficult to detect it in the decay of K_L^0. Using the same method to derive eq. (10), we have on one hand

$$\lim_{k_\mu \to 0} \langle\pi^+\pi^- a(k)|H_w^{p.c.}|K_L^0\rangle = (2f_\pi)^{-1}(\xi_\pi - \sqrt{3}\,\xi_\eta)\langle\pi^+\pi^-|H_w^{p.v.}|K_s^0\rangle \tag{17}$$

and on the other hand

$$\lim_{p_{\pi^-} \to 0} \langle\pi^+\pi^- a|H_w^{p.c.}|K_L^0\rangle = -(\sqrt{3}/2f_\pi)\,\xi_\eta\langle\pi^+\pi^-|H_w^{p.v.}|K_s^0\rangle\,. \tag{18}$$

But one observes that

$$\frac{ud+us+ds}{2u(s-d)}\left(\frac{us+ds-2ud}{us+ds+ud} - \frac{1}{N}\right) \leqslant \frac{-\sqrt{3}\,\xi_\eta}{\xi_\pi - \sqrt{3}\,\xi_\eta} \leqslant \frac{ud+us+ds}{2u(s-d)}\left(\frac{us+ds-2ud}{us+ds+ud} + \frac{1}{N}\right). \tag{19}$$

Eqs. (17)–(19) show that the matrix element depends slightly on axion energy. Assuming a linear dependence in the axion energy (due to the bremsstrahlung term) we get the following bound which is valid throughout the physical region:

$$|M(K_L^0 \to \pi^+\pi^- a)| \geqslant \frac{ud+us+ds}{2u(s-d)}\left(\frac{us+ds-2ud}{ud+us+ds} - \frac{1}{N}\right)(\xi_\pi - \sqrt{3}\,\xi_\eta)\,\frac{1}{2f_\pi}\,|\langle\pi^+\pi^-|H_w^{p.v.}|K_s^0\rangle|\,. \tag{20}$$

Using this inequality, together with eq. (11a) we find

$$\mathrm{BR}(K_L^0 \to \pi^+\pi^- a) \geqslant 2 \times 10^{-6}\,. \tag{21}$$

We end this note by commenting on various alternatives to the P and CP (θ-problem) in QCD:

(i) If $m_u = 0$, the angle θ could be rotated away, i.e. there would be no P, CP violation problem in QCD. This possibility is excluded because of the current algebra mass ratio $m_u/m_d = 0.56$.

(ii) Instead of the Peccei–Quinn model, we could invent a more complicated model with many parameters, or more simply the model proposed by Shifman et al. [15], where the axion was coupled only to superheavy quarks. In this case the axion could not be experimentally observed and in particular there is no lower bound on $K^+ \to \pi^+ a$ and no production of the axion in the beam dump experiment.

(iii) We might set $\theta = 0$ by hand, without introducing the extra $U(1)_{P.Q.}$ symmetry, then, the radiative corrections could change θ by a finite amount which could bring us back again to the θ-problem.

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CHAPITRE VI

SUPERSYMETRIE

VI. 3. A study of a possible quark-gluon symmetry in QCD

VI. 4. Auxiliary field-free supersymmetric gauges

VI - SUPERSYMETRIE

VI.1 - Aspect général et définitions

Dans toutes les théories de jauge, comme le modèle standard et les théories grandement unifiées, il y a toujours trois secteurs indépendants : les bosons vectoriels de jauge, les fermions et les scalaires de Higgs. En fixant le groupe de jauge G, les premiers sont complètement déterminés, parcequ'ils appartiennent à la représentation adjointe tandis que les deux derniers restent théoriquement arbitraires. En effet, les fermions appartiennent à des représentations arbitraires du groupe de jauge. Leur couplage avec les bosons de jauge est fixé par l'invariance de jauge. Pour le secteur de scalaires de Higgs, l'aribitraire est plus grand. Non seulement leurs représentations sont indéterminées, mais on doit introduire un grand nombre de paramètres supplémentaires, à priori inconnus, les constantes de couplages entre eux et avec les fermions. Il apparaît en plus le phénomène de divergences quadratiques qui n'existait pas dans les autres secteurs, et qui induit le problème de la "hiérarchie de jauge".

Comment peut-on réduire tous ces arbitraires ? Une possibilité consiste à introduire une symétrie qui relirait ces différents secteurs. Par conséquent, la connaissance du secteur de jauge pourrait aider à déterminer également les autres. De plus, l'absence de divergences quadratiques pour les masses des fermions (en relation avec l'invariance chirale) pourrait empêcher leur existence pour les masses des scalaires.

Cette symétrie devrait nécessairement relier des particules de spins différents. Par conséquent, ses paramètres infinitésimaux devaient se transformer d'une façon non triviale sous le groupe de Lorentz. S'ils portaient un spin entier non nul, on pourrait par exemple, relier des particules scalaires avec des bosons vectoriels. Si on veut relier des spineurs avec des bosons, ces paramètres infinitésimaux doivent être des variables anticommutantes et porter un spin 1/2 afin de pouvoir changer le spin aussi bien que la statistique.

La supersymétrie[12] permet la réalisation de ce programme pour échanger les fermions et les bosons. Elle contient nécessairement des

générateurs spinoriels. Plus exactement ses générateurs forment une "algèbre de Lie graduée"[39] définie par D générateurs d'un groupe de Lie ordinaire t^a , a= 1,...,D et N éléments anticommutants Q^m ,m = 1,...,N qui satisfont aux relations suivantes :

$$[t^a,\ t^b] = if^{ab}{}_c t^c$$

$$[t^a,\ Q^m] = iS^{am}{}_n Q^n \qquad \text{(VI.1)}$$

$$\{Q^m,\ Q^n\} = iF^{mm}{}_a t^a$$

On note t^a les éléments pairs et Q^m les éléments impairs de l'algèbre de Lie graduée de dimension D+N. L'algèbre de Lie utilisée pour les théories supersymétriques est celle du groupe Poincaré et dans le cas le plus simple (supersymétrie N=1) on utilise un seul générateur spinoriel qui se transforme comme un spineur de Majorana Q_α ($Q^c = Q$)[12] :

$$[P_\mu\ ,\ P_\nu\ ] = 0$$

$$[M_{\mu\nu}\ ,\ P_\rho\ ] = i(g_{\mu\rho}\ P_\nu - g_{\nu\rho}\ P_\mu)$$

$$[M_{\mu\nu},\ M_{\rho\sigma}] = i(\ g_{\mu\rho}\ M_{\nu\sigma} + g_{\nu\sigma}\ M_{\mu\rho} - g_{\mu\sigma}\ M_{\nu\rho} - g_{\nu\rho}\ M_{\mu\sigma}\ ) \qquad \text{(VI.2)}$$

$$[Q_\alpha\ ,\ P_\mu\ ] = 0 \quad ,\ [Q_\alpha\ ,\ M_{\mu\nu}\ ] = \frac{i}{4}\ [\gamma^\mu\ ,\ \gamma^\nu]_{\alpha\beta}\ Q_\beta$$

$$\{Q_\alpha,\ \bar{Q}_\beta\} = \gamma^\mu_{\alpha\beta}\ P_\mu$$

Ces relations définissent donc l'algèbre de la supersymétrie N = 1 (avec un générateur spinoriel).La généralisation des eqs. (VI.2) pour définir la supersymétrie avec N générateurs spinoriels formant une représentation d'un groupe de symétrie interne G est donnée par les relations supplémentaires [40,41] :

$$[t^a,\ t^b] = if^{ab}{}_c t^c$$

$$[t^a,\ Q^m_\alpha] = iS^{am}{}_n Q^n_\alpha \qquad \text{(VI.3)}$$

$$\{Q^m_\alpha,\ \bar{Q}^n_\beta\} = \delta^{mn}\gamma^\mu_{\alpha\beta} P_\mu$$

Les eqs. (VI.3) restent invariantes si les N générateurs spinoriels Q^m_α se transforment suivant la représentation fondamentale du groupe unitaire U(N), qui est la symétrie maximale qu'on peut associer à ces générateurs spinoriels.

Cherchons maintenant le contenu en particules des différentes représentations d'une supersymétrie[40,42]. D'après les relations (VI.2), (VI.3) les générateurs spinoriels Q^m commutent avec P_μ mais pas avec $M_{\mu\nu}$. C'est pourquoi les supermultiplets contiennent des particules de quadri-impulsion fixée et de même masse, mais de spins différents. Etant donné que P^2 est un opérateur de Casimir de l'algèbre de la supersymétrie, on peut classer les représentations en termes d'états à une particule suivant la valeur de sa masse. Pour $P^2 \neq 0$, on peut choisir le référentiel au repos de la particule. Alors chaque anticommutateur de (VI.2) et (VI.3) devient un nombre et les Q^m, $\bar{Q}^m$ peuvent être considérés comme les opérateurs de création et d'annihilation d'une algèbre de Clifford. Du fait que Q^m porte un spin 1/2, l'action de Q^m et $\bar{Q}^m$ sur un état à une particule change son spin par une demi-unité ($\pm$ 1/2). On définit aussi un opérateur de parité transformant le spineur de Majorana Q_α en $(\gamma^\circ Q)_\alpha$.

On peut alors construire directement un espace de Fock de dimension 2^{2N}. En effet, prenons un état d'une particule $|J, J_z\rangle$ de spin J et de projection J_z de son spin sur l'axe z, et considérons-le comme le vide de Clifford, $\bar{Q}^m_\alpha |J, J_z\rangle = 0$; puisque le spineur de Majorana Q_α contient deux composantes indépendantes (appelons-les Q_1 et Q_2), on peut construire quatre combinaisons possibles (4^N pour N générateurs), $|J, J_z, (n_1, n_2)\rangle = Q_1^{n_1} Q_2^{n_2} |J, J_z\rangle$ avec n_1, n_2 = 0 ou 1. On peut voir aussi que toute représentation irréductible contient autant d'états (de spin ou d'hélicité) bosoniques que fermioniques.

Quand la masse des particules est nulle ($P_\mu P^\mu = 0$) il faut travailler avec les états d'hélicité. Dans ce cas, il y a une façon très simple de compter les états d'hélicité de chaque représentation, basée sur l'observation qu'on peut obtenir l'une des deux composantes indépendantes (Q^m_1 et Q^m_2) du générateur spinoriel de Majorana Q^m par une transformation CPT de l'autre. Pour construire une représentation dans laquelle l'hélicité la plus grande des étants contenues est λ, nous choisissons l'état d'hélicité $|\lambda\rangle$

comme le vide. En faisant agir successivement toutes les combinaisons indépendantes $(Q_i^1)^{m_1}\ldots(Q_i^N)^{m_N}$, avec $m_i = 0$ ou 1, on trouve la moitié des états, l'autre moitié étant sa transformée par CPT.

Prenons par exemple la supersymétrie N = 1 et commençons par l'état d'hélicité $|1/2\rangle$. Dans ce cas, nous avons un générateur Q_1. Par action sur $|1/2\rangle$ il donne $|0\rangle^+$. En faisant agir CPT, on obtient $|-1/2\rangle$ et $|0\rangle^-$. Si on les met ensemble, on obtient une représentation qui contient un scalaire, un pseudoscalaire et un spineur de Majorana. Comme autre exemple, considérons la supersymétrie N = 4. Cette fois, il n'est pas possible de commencer avec l'hélicité $|1/2\rangle$ parce que l'action de ses générateurs peut créer des états d'hélicité plus grande. Commençons donc avec l'état $|1\rangle$ et appliquons successivement les combinaisons indépendantes $(Q_i^1)^{m_1}\ldots(Q_i^1)^{m_4}$ avec $m_i = 0$ ou 1. Il est facile de vérifier qu'on trouve 4 états $|1/2\rangle$, 3 états $|0\rangle^+$, 3 $|0\rangle^-$, 4 états $|-1/2\rangle$ et un $|-1\rangle$. On a alors un vecteur, 4 spineurs de Majorana, 3 scalaires et 3 pseudoscalaires. Dans ce qui suit, on s'intéresse aux représentations qui ne contiennent pas de particules de spin plus grand que un, parce que nous voulons par la suite utiliser ces représentations dans la théorie des champs et construire des théories renormalisables. (Les supermultiplets contenant des états de spin 2 et 3/2 servent en supergravité).

Commençons par le cas de masse nulle. Pour la supersymétrie N = 1, on considère deux multiplets : un multiplet scalaire (Φ) qui contient un scalaire, un pseudoscalaire et un spineur de Majorana, et un multiplet vectoriel (V) avec un spineur de Majorana et un vecteur (ou pseudovecteur) de masse nulle. Nous avons aussi besoin de quelques représentations simples des supersymétries avec $N > 1$. Leur contenu en termes des multiplets scalaire Φ et vectoriel V de N = 1 est le suivant : pour la supersymétrie N=2, on a un multiplet scalaire qui est le même que Φ et un multiplet vectoriel qui contient un V et un Φ. Finalement, les supersymétries N = 3 et N = 4 ont seulement des multiplets vectoriels qui se décomposent en un V et trois Φ. Pour $N > 4$, il n'y a pas de multipletsqui ne contiennent pas des particules de spin supérieur à un.

Dans le cas de masse non nulle, en plus des multiplets scalaires qu'on a considéré auparavant (rendus massifs), pour la supersymétrie N = 2, on

peut aussi en avoir un autre qui contient un spineur de Dirac (quatre états de spin indépendants), deux scalaires et deux pseudoscalaires. En ce qui concerne les multiplets vectoriels massifs, il y en a un dans la supersymétrie N =1 qui contient un boson vectoriel (ou pseudovectoriel), deux spineurs de Majorana, et un pseudoscalaire (ou scalaire), tous ayant la même masse ; il y en a aussi un autre dans le cas N = 2 avec un vecteur, deux spineurs de Dirac, et cinq scalaires. On observe que le nombre total des états de spin d'une représentation vectorielle massive est le même que celui d'une combinaison d'un multiplet vectoriel de masse nulle avec un multiplet scalaire. Cette propriété permet le supermécanisme de Higgs, où un multiplet scalaire s'élimine en donnant une masse au multiplet vectoriel (voir chapitres suivants).

Terminons ce chapitre en faisant une dernière remarque. Pour les supersymétries avec $N > 1$, il est possible d'obtenir un autre type de multiplets vectoriels massifs en combinant ensemble deux multiplets vectoriels de masse nulle[42]. On obtient un multiplet pour N = 2 avec un vecteur complexe, 2 spineurs de Dirac, et un scalaire complexe, et un multiplet pour N = 4 avec un vecteur complexe, 4 spineurs de Dirac et 5 scalaires complexes. Finalement pour N = 2, il y a un troisième type de multiplet vectoriel massif en combinant deux multiplets vectoriels et un multiplet scalaire ; on obtient un vecteur complexe, 4 spineurs de Dirac et 10 scalaires. Dans tous ces derniers cas, l'algèbre de la supersymétrie est légèrement modifiée en ajoutant au troisième anticommutateur des eqs. (VI.3) un terme du type $\varepsilon_{\alpha\beta} Z^{mn}$ où Z^{mn} commutant avec tous les éléments de l'algèbre "graduée" et appelé charge centrale.

VI.2 - Formalisme de superchamps et exemples de théories supersymétriques

Pour construire des théories des champs supersymétriques, on doit étudier les représentations de la supersymétrie en termes de champs. Une simplification énorme a été l'introduction du formalisme de superchamps[43] pour la supersymétrie avec un générateur spinoriel. On élargit d'abord l'espace ordinaire de Minkowski en introduisant quatre dimensions supplémentaires dont les coordonnées θ et $\bar{\theta}$ sont des éléments dans une algèbre de Grassmann, et correspondent aux opérateurs Q et $\bar{Q}$ (chaque opérateur spinoriel a deux composantes indépendantes).Le superchamp est une fonction définie sur cet espace. Les transformations de supersymétrie

agissent sur les superchamps comme des translations généralisées dans le superespace. En particulier, les générateurs spinoriels Q et $\bar{Q}$ peuvent se représenter en termes d'opérateurs différentiels par rapport à x, θ et $\bar{\theta}$, appelés "dérivées covariantes". Evidemment, le produit de deux superchamps est encore un superchamp. En conséquence, toute action invariante par la supersymétrie est donnée par l'intégrale sur x, θ et $\bar{\theta}$ d'un Lagrangien qui est une fonction des superchamps et de leurs dérivées covariantes.

Les superchamps, comme toute fonction des variables anticommutantes, sont des polynômes de θ et $\bar{\theta}$. Les coefficients des différents termes correspondent aux champs ordinaires. Nous allons voir que parmi ces champs il y en a quelques-uns qui sont auxiliaires et qui servent à linéariser les lois de transformation des composantes des superchamps par la supersymétrie. Une fois ces champs auxiliaires éliminés, les champs restants se transforment de façon non linéaire. Aux deux supermultiplets de la supersymétrie N = 1, introduits au chapitre précédent, correspondent deux types de superchamps :

- le superchamp chiral[44] complexe Φ (ou antichiral $\bar{\Phi}$) qui correspond au multiplet scalaire et le superchamp réel V qui corrspond au multiplet vectoriel.

Ces superchamps se développent en termes de leurs composantes :

$$\Phi(x,\theta) = \frac{1}{2}(S - iP) + \theta\cdot\psi + \frac{1}{2}\theta\cdot\theta\,(F + iG) \qquad \text{(VI.4a)}$$

$$V(x,\theta,\bar{\theta}) = C + i\,\theta\cdot\chi - i\,\bar{\theta}\cdot\bar{\chi} + \frac{i}{2}\theta\cdot\theta\,(M + iN) - \frac{i}{2}\bar{\theta}\cdot\bar{\theta}\,(M - iN) - \theta\cdot\sigma_\mu\cdot\theta\,A^\mu$$
$$+ i(\theta\cdot\theta)\,\bar{\theta}\cdot\left(\bar{\lambda} - \frac{i}{2}\sigma_\mu^{\dagger}\partial^\mu\chi\right) - i(\bar{\theta}\cdot\bar{\theta})\,\theta\cdot\left(\lambda + \frac{i}{2}\sigma_\mu\partial^\mu\bar{\chi}\right) \qquad \text{(VI.4b)}$$
$$+ \frac{1}{2}(\theta\cdot\theta)(\bar{\theta}\cdot\bar{\theta})\left(D + \frac{1}{2}\Box C\right)$$

$$\bar{\Phi}(x,\theta,\bar{\theta}) = e^{2i\theta\cdot\sigma_\mu\cdot\bar{\theta}\partial^\mu}\ \Phi^*(x,\theta) \qquad \text{(VI.4c)}$$

où nous avons utilisé le formalisme de Weyl[45] à deux composantes pour les spineurs de Majorana ; les σ_μ sont les matrices de Pauli de SU(2), σ_0 étant la matrice unité. Nous avons choisi une représentation où le superchamp chiral Φ ne dépend pas de $\bar{\theta}$. S, P, et ψ sont les champs scalaire, pseudoscalaire et spineur de Majorana du multiplet chiral. A_μ, χ, λ, et C

sont un champ vectoriel (ou pseudovectoriel), deux spineurs de Majorana, et un pseudoscalaire (ou scalaire), du multiplet vectoriel massif, tandis que F, G, D, M et N sont des champs auxiliaires. Dans le cas du multiplet vectoriel de masse nulle, les champs χ et C sont aussi auxiliaires. Les dérivées covariantes D et $\bar{D}$ agissant sur Φ ou V donnent :

$$DV = \left(\frac{\partial}{\partial\theta} + i\sigma_\mu \bar{\theta} \partial^\mu \right) V \quad , \quad \bar{D} V = \left(- \frac{\partial}{\partial\bar{\theta}} - i\theta\cdot\sigma_\mu \partial^\mu \right) V \qquad \text{(VI.5a)}$$

$$\text{et } D\Phi = \left(\frac{\partial}{\partial\theta} + 2i\sigma_\mu \cdot \bar{\theta} \partial^\mu \right) \Phi \quad , \quad \bar{D}\Phi = - \frac{\partial}{\partial\bar{\theta}} \Phi = 0 \quad . \qquad \text{(VI.5b)}$$

On arrive maintenant au problème de la construction d'actions supersymétriques. D'après les relations (VI.4), la dimension canonique de θ et $\bar{\theta}$ est -1/2 et, celle des superchamps - 1. On en conclut que la seule théorie supersymétrique et renormalisable que l'on peut écrire en utilisant un superchamp chiral est celle de Wess et Zumino[46] :

$$S = \int d^4x \left\{ \int d^4\theta \, \bar{\Phi} \Phi + m \int d^2\theta \, \Phi^2 + \frac{\lambda}{3!} \int d^2\theta \, \Phi^3 \right\} \quad . \qquad \text{(VI.6)}$$

Pour écrire des théories qui contiennent des multiplets vecteurs, il faut imposer l'invariance de jauge si l'on veut garder la renormalisabilité et l'unitarité. Les transformations de jauge généralisées des superchamps font intervenir, à la place des paramètres infinitésimaux ordinaires $\Lambda(x)$, un multiplet chiral $\Lambda(x,\theta)$[47] :

$$\delta V = i(\Lambda - \bar{\Lambda}) - \frac{i}{2} g [\Lambda + \bar{\Lambda}, V] \quad , \qquad \text{(VI.7)}$$

où V est maintenant une matrice dans la représentation adjointe du groupe de jauge. On peut montrer que la quantité

$$W = \bar{D}^2 \, (e^{-gV} \, D \, e^{gV}) \qquad \text{(VI.8a)}$$

définit un superchamp chiral qui se transforme par le groupe de jauge comme

$$W \longrightarrow e^{-ig\Lambda} W \, e^{ig\Lambda} \qquad \text{(VI.8b)}$$

et par conséquent W correspond au $F_{\mu\nu}$ de théories de jauge ordinaires. L'action la plus générale qui possède à la fois la supersymétrie N = 1 et l'invariance de jauge et qui contient un supermultiplet vecteur de jauge (de masse-nulle) et un autre superchamp chiral appartenant à une certaine représentation du groupe de jauge, est donnée par[47,48] :

$$S = \int d^4x \left\{ \int d^4\theta \, \mathrm{Tr} W^2 + \int d^4\theta \, \bar{\Phi} e^{gV} \Phi + m \int d^2\theta \Phi^2 + \frac{\lambda}{3!} \int d^2\theta \quad h^{ijk} \Phi_i \Phi_j \Phi_k \right\} \tag{VI.9}$$

Essayons maintenant d'écrire quelques exemples en termes des composantes des superchamps. En utilisant la décomposition (III.4), il est facile de trouver le Lagrangien du modèle de Wess-Zumino (VI.6). Le premier terme donne les énergies cinétiques, la deuxième les masses et la troisième les interactions[46] :

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu S)^2 + \tfrac{1}{2}(\partial_\mu P)^2 + \tfrac{i}{2} \bar{\psi} \partial\!\!\!/ \psi + \tfrac{1}{2} F^2 + \tfrac{1}{2} G^2 + m(FS + GP - \tfrac{1}{2} \bar{\psi}\psi)$$
$$+ \lambda \left[F(S^2 - P^2) + 2GSP - \bar{\psi}(S - \gamma_5 P)\psi \right] \tag{VI.10}$$

Les champs auxiliaires F et G peuvent s'éliminer en utilisant leurs équations de mouvement :

$$F = -\,[mS + \lambda(S^2 - P^2)] \quad , \quad G = -\,[mP + 2\lambda SP] \tag{VI.11}$$

et la densité de Lagrangien (VI.10) devient :

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu S)^2 + \tfrac{1}{2}(\partial_\mu P)^2 + \tfrac{i}{2} \bar{\psi} \partial\!\!\!/ \psi - \tfrac{1}{2} m^2 (S^2 + P^2) - \tfrac{m}{2} \bar{\psi}\psi \tag{VI.12}$$
$$- \lambda \bar{\psi}(S - \gamma_5 P)\psi - m\lambda S(S^2 + P^2) - \tfrac{1}{2} \lambda^2 (S^2 + P^2)^2$$

Pour les théories supersymétriques invariantes de jauge, la situation se complique à cause du problème du choix de la jauge, pour pouvoir définir la théorie. Il y a une première approche pour fixer la jauge en introduisant ensuite des fantômes en termes des superchamps ayant l'inconvénient d'avoir une densité de Lagrangien non polynomiale qui conduit à des interactions compliquées[49]. Une autre approche est de choisir une jauge particulière introduite par Wess et Zumino, qui donne une densité de Lagrangien polynomiale[47] :

$$\mathcal{L} = -\tfrac{1}{4} \mathrm{Tr}\, F_{\mu\nu}^2 + \tfrac{i}{2} \bar{\lambda} \not{D} \lambda + \tfrac{1}{2} \mathrm{Tr} D^2 \quad . \tag{VI.13}$$

La densité de Lagrangien (VI.13) correspond au premier terme de (VI.9) et contient simplement un supermultiplet vecteur de jauge (de masse-nulle). $\mathcal{D}_\mu$ est la dérivée covariante habituelle des théories de jauge, λ est le spineur de Majorana du supermultiplet vectoriel, et D est un champ auxiliaire qui sert seulement à fermer l'algèbre des transformations de supersymétrie exprimées en fonction des composantes du superchamp. Ces transformations deviennent non linéaires[50] :

$$\begin{aligned}
\delta_s A_\mu^a &= \bar{\varepsilon}\, \gamma_\mu \gamma_5 \lambda^a \\
\delta_s \lambda^a &= \frac{i}{2} F_{\mu\nu}^a \gamma^\mu \gamma^\nu \gamma_5 \varepsilon - D^a \gamma_5 \varepsilon \\
\delta_s D^a &= i \bar{\varepsilon} (\not{\partial} \lambda)^a
\end{aligned} \qquad \text{(VI.14)}$$

où le paramètre infinitésimal ε est un spineur de Majorana constant. L'invariance par les transformations supersymétriques définit toujours un courant vecteur-spineur (de spin 3/2) conservé[46,51] dont le charge est exactement le générateur Q des rels. (VI.2). Le courant conservé de spin 3/2 est :

$$J^\mu = \frac{1}{2} F_{\lambda\rho}^a \gamma^\lambda \gamma^\rho \gamma^\mu \gamma_5 \lambda^a \quad . \qquad \text{(VI.15)}$$

En choisissant la jauge de Wess-Zumino, nous avons laissé un arbitraire par rapport aux transformations de jauge ordinaires. On procède alors comme dans les théories de jauge habituelles. Le problème supplémentaire est que l'introduction du terme permettant de fixer la jauge, et par la suite celui des fantômes, brise la supersymétrie explicitement.

Terminons ce chapitre en discutant brièvement la renormalisation des théories supersymétriques[51-53]. En utilisant le formalisme des superchamps et les règles de Feynman ainsi obtenues[44,53,54], on peut démontrer[49,55] la validité des théorèmes appelés "de non-renormalisation" suivant lesquels il y a beaucoup moins de constantes de renormalisation indépendantes, que le comptage de puissance "naïf" ne le prévoit. Par exemple, on peut démontrer que tout terme de masse du type Φ^2 ou d'interaction du type Φ^3 entre les superchamps n'a pas besoin d'un contre-terme[52,49,55] contrairement au cas d'une théorie ordinaire. Cette propriété exprimant

l'absence des divergences quadratiques est très importante pour une solution éventuelle du problème de hiérarchie de jauge dans les théories grandement unifiées, problème qui a été présenté au chapitre III.2.
D'après ce théorème de "non renormalisation", le modèle de Wess et Zumino (VI.6) ne possède qu'une seule constante de renormalisation, celle de la fonction d'onde du superchamp chiral Φ, et en général toute théorie de jauge supersymétrique (N = 1) se renormalise en utilisant deux contre-termes pour la fonction d'onde et la constante de couplage du multiplet vecteur de jauge et un pour la fonction d'onde de chaque multiplet chiral de la matière et celui des fantômes [53]. Il y a aussi des exemples, comme la supersymétrie N = 4 qui semble être une théorie complètement finie ! (la fonction $-\beta$ s'annule jusqu'au niveau de trois boucles[56] et des arguments généraux semblent montrer la finitude à tous les ordres [57]).
Dans cet esprit, on espère que les théories de la supergravité sont des théories finies, bien qu'elles ne soient pas renormalisables par comptage de puissance.

Dans les deux chapitres suivants, nous étudions quelques propriétés particulières de la théorie de Yang-Mills supersymétrique (N = 1) avec un seul multiplet de jauge. D'après l'eq. (VI.13) on voit que cette théorie n'est rien d'autre qu'une théorie de jauge ordinaire couplée avec des fermions de Majorana dans la représentation adjointe du groupe de jauge. Au chapitre (VI.3), nous étudions comment la symétrie entre les quarks et les gluons se manifeste à travers une relation entre les opérateurs de Wilson singulet de saveur, et, au chapitre (VI.4), nous examinons le problème du choix de la jauge. Nous allons démontrer que la jauge axiale $n_\mu A^r = 0$ ($n^2 \neq 0$) préserve la supersymétrie N = 1 qui reste après avoir imposé la jauge de Wess et Zumino pour rendre le Lagrangien polynomial (voir VI.13, VI.14). Par conséquent, sans avoir de termes non polynomiaux, les identités de Ward et toutes les propriétés supersymétriques s'expriment sous une forme très simple en termes des composantes physiques des superchamps. Dans cette jauge, nous démontrons aussi qu'on a besoin seulement d'un seul contre-terme de renormalisation de la fonction d'onde.

Une dernière remarque : dans ce qui suit, nous utilisons un schéma de renormalisation qui semble ne pas briser la supersymétrie tout en gardant en même temps l'invariance de jauge. Ce schéma s'appelle "réduction dimensionnelle"[58] et consiste à garder la dimension des champs (et par conséquent l'algèbre de tous les indices) à 4 dimensions alors que la dimension d'espace-temps est changée de 4 à $4-2\varepsilon$ dimensions.

CHAPITRE VII

PREDICTIONS DES THEORIES SUPERSYMETRIQUES

VII. 3. Light gluinos in deep inelastic scattering

VII. 4. Supersymmetric enhancement factor for the 1-Jet cross-section in p-p reactions.

VII. 5. Polarization effects of supersymmetric QCD in large-P_T direct photon production.

VII. 7. Simple treatment of threshold effects

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A STUDY OF A POSSIBLE QUARK-GLUON SYMMETRY IN QCD

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We study the quark-gluon symmetry manifest in a simple relation for the anomalous dimensions of the Wilson, flavour singlet, operators of deep inelastic scattering. We find that, in the supersymmetry preserving dimensional reduction renormalization scheme, the relation is true at the 2nd order of perturbation expansion in the strong coupling constant.

1. Introduction

It is well-known that supersymmetric lagrangian field theories enjoy a number of interesting symmetry properties which reduce the number of independent parameters such as coupling constants and field normalizations [1]. These symmetry properties remain valid at any order of renormalized perturbation expansion if one uses supersymmetry preserving renormalization schemes. In this paper we are going to discuss an interesting symmetry property of supersymmetric Yang-Mills lagrangians of the type $\mathrm{SU}(N)_{\mathrm{colour}} \times \mathrm{SU}(n)_{\mathrm{flavour}}$.

The flavour singlet Wilson operators of spin s are defined as follows [2]:

$$O_{\mathrm{q}}^{s} = \tfrac{1}{2} i^{s-1} \bar{q} \gamma^{\mu_1} D^{\mu_2} \cdots D^{\mu_s} q - \text{traces}, \tag{1.1a}$$

$$O_{\mathrm{g}}^{s} = i^{s-2} \mathrm{Tr}\, F^{\mu_1 a} D^{\mu_2} \cdots D^{\mu_{s-1}} F_{a}^{\mu_s} - \text{traces}, \tag{1.1b}$$

where q is the quark field operator, $D^{\mu} \equiv \partial^{\mu} + igA^{\mu}$ the covariant derivative, and $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} + g[A^{\mu}, A^{\nu}]$, the gauge field strength tensor. The anomalous dimension matrix, γ, of these operators satisfies the following relation, at lowest

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order in $\alpha_s \equiv g^2/4\pi$ [3]:

$$\gamma_{qq}^s + \gamma_{qg}^s = \gamma_{gq}^s + \gamma_{gg}^s, \qquad s = 1, 2, 3 \cdots . \tag{1.2}$$

This relation is correct in the case when the quark fields transform under the adjoint representation of the colour group and they are described by Majorana spinors. These conditions imply the following relation for the Casimir operators of the fermion representation, $c_F(R) = \sum_a t^a t^a$, $\mathrm{Tr}\, t^a t^b = T(R)\delta^{ab}$:

$$c_F(R) = c_A = 2T(R), \tag{1.3}$$

where c_A is the Casimir operator for the gauge boson fields which belong to the adjoint representation.

It is known that this condition is also sufficient to ensure that the Yang-Mills lagrangians are supersymmetric [1]. Let us write down the expressions of γ_{ij}^s, $i, j = q, g$, for later use [2]:

$$\gamma_{qq}^s = \frac{2c_F\alpha_s}{4\pi}\left[1 - \frac{2}{s(s+1)} + 4\sum_{j=2}^{s}\frac{1}{j}\right], \tag{1.4a}$$

$$\gamma_{qg}^s = \frac{4T(R)\alpha_s}{4\pi}\left[\frac{2}{s} - \frac{4}{s+1} + \frac{4}{s+2}\right], \tag{1.4b}$$

$$\gamma_{gq}^s = \frac{2c_F\alpha_s}{4\pi}\left[\frac{4}{s-1} - \frac{4}{s} + \frac{2}{s+1}\right], \tag{1.4c}$$

$$\gamma_{gg}^s = \frac{2c_A\alpha_s}{4\pi}\left[\frac{1}{3} - \frac{4}{s-1} + \frac{4}{s} - \frac{4}{s+1} + \frac{4}{s+2} + 4\sum_{j=2}^{s}\frac{1}{j}\right]\frac{2T(R)\alpha_s}{3\pi}. \tag{1.4d}$$

Using condition (1.3) one easily verifies (1.2). Let us note that up to now no simple explanation for the infinite number of relations (1.2) is known.

If we take the inverse Mellin transform of relation (1.2) we obtain

$$P_{qq}(x) + P_{qg}(x) = P_{gq}(x) + P_{gg}(x), \tag{1.2'}$$

where

$$\int_0^1 \mathrm{d}x\, x^{s-1} P_{ij}(x) = -\gamma_{ij}^s, \qquad i, j = q, g.$$

If we recall that $P_{ij}(x)$ is the variation, per unit of x and unit of the off-mass-shell momentum $p^2 = -\mu^2$, of the probability for a parton i to decay inclusively into a parton j, [4], then relation (1.2′) guarantees that the variation of the decay probability for a quark is equal to that of a gluon for supersymmetric representations of the quarks.

In the next sections we want to check if relation (1.2) is true at the next order in $\alpha_s \equiv g^2/4\pi$. Our reasoning goes as follows. Suppose that the origin of relation (1.2) is some supersymmetric Ward identity for the Wilson operators in the supersymmetric Yang-Mills case, then using a supersymmetry preserving renormalization procedure we should be able to verify (1.4d) even at order α_s^2. To do that we need the results of order α_s^2 for γ_{ij}^s in some scheme, say the minimal subtraction scheme, together with some transformation relation from one scheme to another.

In the next section we establish the transformation rule between any two schemes, looking for the invariant quantities under renormalization scheme change.

2. Renormalization prescription invariants

It is obvious that physical (measurable) quantities must be invariant under changes of renormalization prescription. In perturbative expansions this is true only if we do the expansion in terms of the same coupling constants. A suitable physical quantity for our purpose of connecting the anomalous dimension matrix, γ^5, in two different schemes is the corresponding moments of the structure functions [2].

If a, b are the two different schemes and

$$M_a^s(Q^2, g_a) \equiv \int_0^1 \mathrm{d}x\, x^{s-1} F_a^{\text{singlet}}(x, Q^2, g_a)$$

$$= O_{q,a}^s C_{q,a}^s(Q^2, g_a) + O_{g,a}^s C_{g,a}^s(Q^2, g_a), \tag{2.1}$$

are moments of the singlet structure functions, the above-mentioned conditions for these physical quantities are

$$M_a^s(Q^2, g_a) = M_b^s(Q^2, g_b(g_a)), \tag{2.2}$$

where the right-hand side must be expanded in g_a. In perturbative Yang-Mills theories with fermions we have the possibility of taking longitudinal or transverse (L, T) singlet structure functions of quark or gluon targets, so the moments (2.1) are really matrices M_{lm}, $l = \mathrm{L}, \mathrm{T}$, $m = \mathrm{q}, \mathrm{g}$.

Eq. (2.1) can be written in matrix form,

$$\begin{pmatrix} M_{Tq}^s & M_{Tg}^s \\ M_{Lq}^s & M_{Lg}^s \end{pmatrix} = \begin{pmatrix} C_{Tq}^s & C_{Tg}^s \\ C_{Lq}^s & C_{Lg}^s \end{pmatrix} \cdot \begin{pmatrix} O_{qq}^s & O_{qg}^s \\ O_{gq}^s & O_{gg}^s \end{pmatrix} \tag{2.3}$$

or in short-hand notation

$$M^s = C^s \cdot O^s. \tag{2.4}$$

Now using the renormalization group method for improved perturbation theory [2], we can derive the following evolution equation [4]:

$$\dot{M}^s \equiv Q\frac{\partial}{\partial Q}M^s$$

$$= \left[\dot{C}^s \cdot C^{s^{-1}} - C^s \cdot \gamma^s \cdot C^{s^{-1}}\right] \cdot M^s. \tag{2.5}$$

Now eq. (2.5) provides the invariant expression we look for. Since $\dot{M}^s$ and M^s are in principle measurable (in the way we treat renormalized perturbation theory) so is the evolution kernel $E^s(\bar{\alpha}_s(Q))$,

$$E^s \equiv \dot{C}^s \cdot C^{s^{-1}} - C^s \cdot \gamma^s \cdot C^{s^{-1}}, \tag{2.6}$$

where $\bar{\alpha}_s(Q)$ is the running coupling constant. So this kernel should satisfy relation (2.2). Using eq. (2.4) to determine $C^s(Q^2 = \mu^2, \bar{g}(Q))$, we find (omitting indices)

$$E^s(\alpha) = b(\alpha)\frac{\partial M^s}{\partial \alpha}M^{s^{-1}} - M^s \cdot \left[b(\alpha)O^{s^{-1}}\frac{\partial O^s}{\partial \alpha} + O^{s^{-1}}\gamma^s O^s\right]M^{s^{-1}}, \tag{2.7}$$

where

$$b(\alpha) \equiv \mu\frac{\partial}{\partial \mu}\alpha.$$

We easily see that the first term on the right-hand side of (2.7) is invariant under a change of renormalization scheme, while, for the second term, (2.2) implies for two schemes "1" and "2",

$$b_1(\alpha_1)O_1^{s^{-1}}\frac{\partial O_1^s}{\partial \alpha_1} + O_1^{s^{-1}}\gamma_1^s O_1^s = b_2(\alpha_2)O_2^{s^{-1}}\frac{\partial O_2^s}{\partial \alpha_2} + O_2^{s^{-1}}\gamma_2^s O_2^s. \tag{2.8}$$

If we use the relation

$$\alpha_2 = \alpha_1 + \lambda\alpha_1^2 + O(\alpha_1^3) \tag{2.9}$$

and expand up to order α_1^2 in both sides of (2.8), we find that the coefficients in the expansions,

$$b(\alpha) = b_0\alpha^2 + b_1\alpha^3 + \cdots,$$

$$\gamma(\alpha) = \gamma_{(0)}\alpha + \gamma_{(1)}\alpha^2 + \cdots, \tag{2.10a}$$

$$O(\alpha) = 1 + O_{(1)}\alpha + \cdots, \tag{2.10b}$$

in the two schemes are related as follows:

$$\gamma_{(0)2} = \gamma_{(0)1}, \tag{2.11}$$

$$\gamma_{(1)2} + b_0 O_{(1)2} + \left[\gamma_{(0)2}, O_{(1)2}\right] = \gamma_{(1)1} + b_0 O_{(1)1} + \left[\gamma_{(0)1}, O_{(1)1}\right] - \lambda\gamma_{(0)1}. \tag{2.12}$$

Let us recall the relation we want to check. If we denote by ΔA, the linear combination,

$$\Delta A = A_{11} + A_{12} - A_{21} - A_{22}, \tag{2.13}$$

for a 2×2 matrix, then due to the property (1.2) for $\gamma_{(0)}$ we obtain from (2.12),

$$\Delta\gamma_{(1)1} + (b_0 - \gamma_{(0)qg} - \gamma_{(0)gq})\Delta O_{(1)1} = \Delta\gamma_{(1)2} + (b_0 - \gamma_{(0)qg} - \gamma_{(0)gq})\Delta O_{(1)2}. \tag{2.14}$$

If for a supersymmetry preserving renormalization scheme eq. (1.2) was true at α^2 order $\Delta\gamma_{(1)2} = 0$, then, knowing $\Delta O_{(1)1}$, $\Delta O_{(1)2}$ and three elements of the matrix $\gamma_{(1)}$ say $\gamma_{(1)qq}, \gamma_{(1)qg}, \gamma_{(1)gq}$ we could immediately calculate from (2.14) the 4th element $\gamma_{(1)gg}$, which, in fact, is the most difficult part of the calculation [5,6]. In sect. 3 we calculate the matrices in the minimal subtraction [7] and in the dimensional reduction [8] renormalization schemes.

3. The calculation of Wilson operators

Recently a scheme called the dimensional reduction renormalization scheme was proposed with the interesting property of preserving both gauge invariance and supersymmetry invariance of supersymmetric Yang-Mills lagrangians [8]. This prescription consists in doing all the Lorentz index algebra in four dimensions and evaluating the resulting scalar Feynman integrals in $4 - 2\varepsilon$ dimensions. Then the renormalized integral is defined as the Taylor part in the ε expansion of the result. This prescription can be applied to any renormalizable lagrangian, not necessarily a supersymmetric one [9].

The diagrams whose finite parts we had to calculate are presented in fig. 1. We give the results diagram by diagram in the appendix and we present here the results, for the matrix elements $O_{(1)qq}, O_{(1)qg}, O_{(1)gq}, O_{(1)gg}$ in the modified minimal subtraction ($\overline{\text{MS}}$) [10] and dimensional reduction schemes ($\overline{\text{DR}}$) for $p^2 = -\mu^2$ with external off-shell quarks or gluons.

In what follows, the result in $\overline{\text{MS}}$ appears first, then appears the piece which must be added to obtain the result in $\overline{\text{DR}}$ (no $\log 4\pi - \gamma$ terms as in $\overline{\text{MS}}$). If there is no

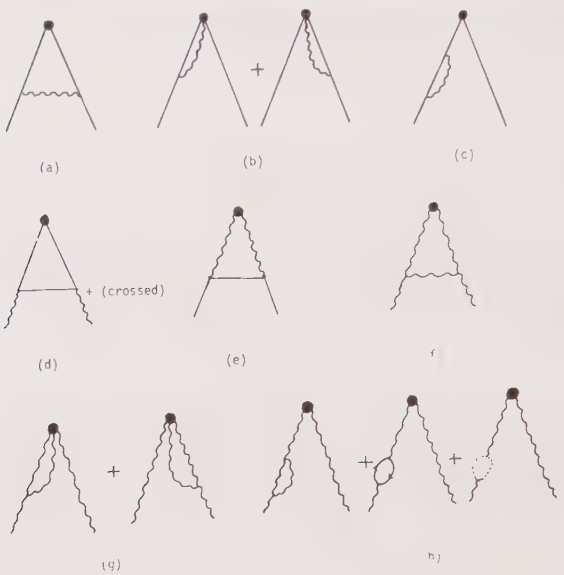

Fig. 1. Diagrams (a,b,c) for $O_{(1)qq}$, (d) for $O_{(1)qg}$, (e) for $O_{(1)gq}$, (f,g,h) for $O_{(1)gg}$.

term to add in $\overline{\text{MS}}$ this means that this term is zero:

$$O^N_{(1)qq} = \frac{\alpha_s c_F}{4\pi}\left[\frac{2}{N}\left(\frac{S_1(N)}{N+1} + \frac{2}{(N+1)^2} - \frac{3}{N+1} + \frac{1}{N} - 1\right)\right. \tag{3.1a}$$

$$\left. -6S_2(N) - 2S_1^2(N) + 7\right]$$

$$= \frac{\alpha_s c_F}{4\pi}\left(\frac{2}{N(N+1)} - 1\right), \tag{3.1b}$$

$$O^N_{(1)qg} = \frac{4\alpha_s T(R)}{4\pi}\left[\frac{4}{(N+1)^2} - \frac{4}{(N+2)^2} - \frac{2}{(N+1)(N+2)} - \frac{1}{N^2}\right.$$

$$\left. + \frac{1}{N} + \left(\frac{2}{(N+1)(N+2)} - 1\right)S_1(N)\right], \tag{3.2}$$

$$O^N_{(1)gq} = \frac{-2c_F\alpha_s}{4\pi}\left[\frac{2}{(N+1)^2} - \frac{2}{N^2} + \frac{2}{(N-1)^2} + \frac{3}{N}\right.$$

$$\left. - \frac{4}{N-1} - \frac{1}{N+1} + \left(\frac{1}{N+1} - \frac{2}{N} + \frac{2}{N-1}\right)S_1(N)\right] \tag{3.3a}$$

$$= \frac{-2c_F\alpha_s}{4\pi}\frac{1}{N+1}, \tag{3.3b}$$

$$O^N_{(1)gg} = \frac{\alpha_s}{4\pi}\left\{c_A\left[\left(-\frac{4}{N+2}+\frac{4}{N+1}-\frac{4}{N}+\frac{4}{N-1}\right)S_1(N)\right.\right.$$

$$-\frac{8}{(N+2)^2}+\frac{8}{(N+1)^2}-\frac{4}{N^2}+\frac{4}{(N-1)^2}$$

$$+\frac{10}{N}-\frac{8}{N-1}-\frac{8}{N+1}+\frac{8}{N+2}$$

$$\left.\left.-2S_1^2(N)-6S_2(N)+\tfrac{67}{9}\right]-\tfrac{20}{9}T(R)\right\}, \tag{3.4a}$$

$$\frac{\alpha_s c_A}{4\pi}\left[\frac{4}{N+1}-\frac{4}{N+2}-\tfrac{1}{3}\right], \qquad N=1,2,3,\cdots. \tag{3.4b}$$

Notice that the results (3.1a) and (3.2) agree with those of ref. [11] (see also [5]) while the others are new.

The functions $S_1(N)$ and $S_2(N)$ are the following:

$$S_m(N) \equiv \sum_{j=1}^{N}\frac{1}{j^m}, \qquad m=1,2.$$

Before we check if relation (2.14) gives (see sect. 2)

$$\Delta\gamma_{(1)\overline{DR}}=0, \tag{3.5}$$

let us note that for $N=2$ operators and for a general representation of fermions the relations (energy-momentum conservation)

$$\gamma_{qq}^{N=2}-\gamma_{gq}^{N=2}=0, \tag{3.6a}$$

$$\gamma_{qg}^{N=2}-\gamma_{gg}^{N=2}=0, \tag{3.6b}$$

are true in the $\overline{MS}$ scheme (or MS) at any order in α_s (see ref. [5]) and they provide a check of the calculation. We can easily check, doing a little algebra, that relation (2.12) and the results (3.1)–(3.4) imply the validity of relations (3.6a,b) in the scheme $\overline{DR}$ to order α_s^2.

Recall now eq. (2.14),

$$\Delta\gamma_{(1)\overline{\mathrm{DR}}} - \Delta\gamma_{(1)\overline{\mathrm{MS}}} = (b_0 - \gamma_{(0)\mathrm{qg}} - \gamma_{(0)\mathrm{gq}})(\Delta O_{(1)\overline{\mathrm{MS}}} - \Delta O_{(1)\overline{\mathrm{DR}}}). \tag{3.7}$$

From the results obtained for $O_{(1)\overline{\mathrm{MS}}}$ and $O_{(1)\overline{\mathrm{DR}}}$ we get $\Delta O_{(1)\overline{\mathrm{MS}}}$ and $\Delta O_{(1)\overline{\mathrm{DR}}}$ for $c_A = c_F = 2T(\mathrm{R})$:

$$\Delta O^N_{(1)\overline{\mathrm{MS}}} = \frac{\alpha_s}{4\pi}\left[\tfrac{2}{3} - \frac{6}{N} + \frac{4}{N+1} - \frac{4}{N+2}\right], \tag{3.8a}$$

$$\Delta O^N_{(1)\overline{\mathrm{DR}}} = -\frac{\alpha_s}{4\pi}\frac{4}{N}. \tag{3.8b}$$

The compactified form for $\gamma_{(1)\overline{\mathrm{MS}}}$ is given in ref. [12] and from eqs. (1.4a–d), (3.1)–(3.4) and (3.8a,b) we find,

$$\begin{aligned}\Delta\gamma^N_{(1)\overline{\mathrm{DR}}} = \frac{1}{2\pi^2}\Bigg\{ & S_1^2(N)\left[\frac{1}{2}\frac{1}{N+1} + \frac{5}{4}\frac{1}{N} - \frac{5}{4}\frac{1}{N-1}\right] \\ & - S_2(N)\left[\frac{3}{2}\frac{1}{N+1} + \frac{3}{4}\frac{1}{N} + \frac{5}{4}\frac{1}{N-1}\right] \\ & - S_1(N)\left[\frac{2}{(N+1)^2} + \frac{3}{N^2} + \frac{13}{4}\frac{1}{N+1} + \frac{6}{N} - \frac{8}{N-1}\right] \\ & + \frac{5}{2}\frac{1}{N^3} - \frac{15}{4}\frac{1}{(N+1)^2} - \frac{1}{2}\frac{1}{N^2} \\ & + \frac{6}{N+1} + \frac{6}{N} - \frac{11}{2}\frac{1}{N-1}\Bigg\}. \end{aligned} \tag{3.9}$$

If we use the recently presented results for $\gamma_{(1)\overline{\mathrm{MS}}}$ (ref. [6]) we find

$$\Delta\gamma^N_{(1)\overline{\mathrm{DR}}} = 0. \tag{3.10}$$

So it seems that the supersymmetric relation is true at order α_s^2, although the discrepancy of the results of refs. [5, 6] makes the above conclusion questionable. For more important reasons a clearing-up third calculation of $\gamma_{(1)\overline{\mathrm{MS}}}$ is urgently needed.

Appendix

We present here, diagram by diagram (fig. 1) the results for the Wilson operator matrix elements $O^N_{(1)qq}, O^N_{(1)qg}, O^N_{(1)gq}, O^N_{(1)gg}$. The results are presented first for $\overline{\text{MS}}$ (a), and for $\overline{\text{DR}}$ one has to add the expression of the next formula (b).

For diagram (a)

$$\frac{2c_F\alpha_s}{4\pi}\frac{1}{N}\left[\frac{S_1(N)}{N+1}+\frac{2}{(N+1)^2}-\frac{3}{N+1}+\frac{1}{N}-1\right], \tag{A.1a}$$

$$\frac{2c_F\alpha_s}{4\pi}\frac{1}{N(N+1)}, \tag{A.1b}$$

for diagram (b),

$$-\frac{4c_F\alpha_s}{4\pi}\left[\tfrac{3}{2}S_2(N)+\tfrac{1}{2}S_1^2(N)-2\right], \tag{A.2}$$

for diagram (c),

$$-\frac{c_F\alpha_s}{4\pi}, \tag{A.3a}$$

$$-\frac{c_F\alpha_s}{4\pi}, \tag{A.3b}$$

for diagram (d),

$$\frac{4T(R)\alpha_s}{4\pi}\left[\frac{4}{(N+1)^2}-\frac{4}{(N+2)^2}-\frac{2}{(N+1)(N+2)}-\frac{1}{N^2}+\frac{1}{N}\right.$$

$$\left.+\left(\frac{2}{(N+1)(N+2)}-\frac{1}{N}\right)S_1(N)\right], \tag{A.4}$$

for diagram (e),

$$-\frac{2c_F\alpha_s}{4\pi}\left[\frac{2}{(N+1)^2}-\frac{2}{N^2}+\frac{2}{(N-1)^2}+\frac{3}{N}-\frac{4}{N-1}\right.$$

$$\left.-\frac{1}{N+1}+\left(\frac{1}{N+1}-\frac{2}{N}+\frac{2}{N-1}\right)S_1(N)\right], \tag{A.5a}$$

$$-\frac{2c_F\alpha_s}{4\pi}\frac{1}{N+1}, \tag{A.5b}$$

for diagram (f),

$$\frac{c_A\alpha_s}{4\pi}\left\{\left(-\frac{4}{N+2}+\frac{6}{N+1}-\frac{4}{N}+\frac{4}{N-1}\right)S_1(N)-\frac{8}{(N+2)^2}\right.$$

$$\left.+\frac{12}{(N+1)^2}-\frac{4}{N^2}+\frac{4}{(N-1)^2}+\frac{10}{N}-\frac{8}{N-1}-\frac{8}{N+1}+\frac{8}{N+2}\right\}, \quad \text{(A.6a)}$$

$$\frac{c_A\alpha_s}{4\pi}\,\frac{4}{(N+1)(N+2)}\,, \quad \text{(A.6b)}$$

for diagram (g),

$$-\frac{2c_A\alpha_s}{4\pi}\left\{3S_2(N)+S_1^2(N)+\frac{1}{N+1}S_1(N)+\frac{2}{(N+1)^2}-2\right\}, \quad \text{(A.7)}$$

for diagram (h),

$$\frac{\alpha_s}{4\pi}\left[\tfrac{31}{9}c_A-\tfrac{20}{9}T(R)\right], \quad \text{(A.8a)}$$

$$-\frac{\alpha_s}{4\pi}\tfrac{1}{3}c_A. \quad \text{(A.8b)}$$

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AUXILIARY FIELD-FREE SUPERSYMMETRIC GAUGES*

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We find that, in perturbation theory, non-light-cone axial gauges, $N \circ A^a(x) = 0$, preserve the supersymmetry remaining in $N = 1$ supersymmetric YM theories, after imposing the Wess-Zumino gauge.

Recently there has been a lot of interest in supersymmetric grand unified theories, because of their claimed ability to preserve the fine tuning of their low and high mass scales imposed at the tree level [1, 2]. This property comes from the validity, in these theories, of a set of non-renormalization theorems which are established in the language of superfields.

On the other hand, to make the physics as clear as possible, one should keep only the physical degrees of freedom. To do that one can use the Wess-Zumino gauge [3], in which the auxiliary fields are eliminated from the gauge multiplet, and an effective supersymmetry is left for the classical action [4]. In order to quantize these theories the invariance under the usual gauge transformations must be broken by a gauge-fixing term. Covariant gauges are known to break the effective supersymmetry. So the question comes, what happens to the non-renormalization theorems in the Wess-Zumino gauge? In this work we present a proof that non-light-like axial gauges, $N \cdot A = 0$, preserve at the level of the counter terms the effective supersymmetry and thus the non-renormalization theorems remain true. In the following, we assume that there exists a supersymmetric regularization scheme [5, 6].

Consider the gauge multiplet part of the YM lagrangian in the Wess-Zumino gauge [3, 4]:

$$\mathcal{L}_{\mathrm{cl}} = -\tfrac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \tfrac{1}{2} i\, \bar{\lambda}^a (\not{\mathcal{D}}\lambda)^a + \tfrac{1}{2}\left(D^a\right)^2, \tag{1}$$

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where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + igf^{abc}A^b_\mu A^c_\nu,$$

$$\not{\mathcal{D}} = \gamma^\mu \mathcal{D}_\mu, \qquad \mathcal{D}^{ab}_\mu = \partial_\mu \delta^{ab} - gf^{abc}A^c_\mu, \tag{2}$$

and D^a is a non-interacting and non-propagating scalar field of dimension 2.

The action of this lagrangian is invariant under the supersymmetric transformations [4]

$$\delta_\varepsilon A^a_\mu = \bar{\varepsilon}\gamma_\mu\gamma_5\lambda^a, \tag{3a}$$

$$\delta_\varepsilon \lambda^a = \tfrac{1}{2} iF^a_{\mu\nu}\gamma^\mu\gamma^\nu\gamma_5\varepsilon - D^a\gamma_5\varepsilon, \tag{3b}$$

$$\delta_\varepsilon D^a = i\bar{\varepsilon}(\not{\mathcal{D}}\lambda)^a, \tag{3c}$$

where ε is an infinitesimal constant Majorana spinor. In order to quantize the action (1) we add the gauge-fixing term [7]

$$\mathcal{L}_{\mathrm{gf}} = C^a(x)N^\mu A^a_\mu(x), \tag{4}$$

where $N^2 \neq 0$ and $C^a(x)$ is a Lagrange multiplier field. It has been argued that this gauge is renormalizable and unitary [8–10]. The axial gauge (4) simplifies the Ward gauge identities [8]. Let Z_{AA}, Z_{AAA}, Z_{FF}, Z_{FFA} be the two- and three-point function renormalization constants for the gauge field A and the fermion field F which in the axial gauge are known to satisfy the (abelian type) gauge Ward identities:

$$Z^{1/2}_{AA}Z_g = \frac{Z_{AAA}}{Z_{AA}} = \frac{Z_{FFA}}{Z_{FF}} = 1, \tag{5}$$

where Z_g renormalizes the coupling constant g.

From relations (5) it is obvious that if the YM theory considered, is to remain supersymmetric at the counter term level, it is enough to prove the following relation:

$$Z_{FF} = Z_{AA}. \tag{6}$$

For this reason we shall consider the supersymmetric Ward identities for the generating functional of the 1PI Green functions. To linearize these Ward identities we shall introduce a source for the supersymmetric variation of the gauge-fixing term:

$$\delta_\varepsilon \mathcal{L}_{\mathrm{gf}}(x) = C^a(x)\bar{\varepsilon}\not{N}\gamma_5\lambda^a(x) \equiv \bar{\varepsilon}O(x). \tag{7}$$

The generating functional, W, for the connected Green functions is defined as

$$e^{iW(j,n,K,L,M)} \equiv Z_0^{-1} \int [dA\, d\lambda\, dC\, dD] e^{iS(j,n,K,L,M)}, \tag{8}$$

where

$$S(j,n,K,L,M) \equiv \int d^D x \left\{ \mathcal{L}_{cl}(x) + \mathcal{L}_{gf} + j \cdot A + \bar{n} \cdot \lambda + K \cdot C + L \cdot D + \bar{M} O \right\},$$

and Z_0 is defined so that

$$W(j,n,K,L,M)|_{j=n=K=L=M=0} = 0. \tag{9}$$

The generating functional for the 1PI Green functions is the Legendre transform of W with respect to the sources:

$$\begin{aligned} W(j,n,K,L,M) = \Gamma(\hat{A},\hat{\lambda},\hat{C},\hat{D},\hat{O}) \\ + \int d^D x \left\{ \tfrac{1}{2}\hat{D}^2 + \hat{C} \cdot (N \cdot \hat{A}) + j \cdot \hat{A} + \bar{n} \cdot \hat{\lambda} + K \cdot \hat{C} \right. \\ \left. + L \cdot \hat{D} + \bar{M} (\hat{O} + \hat{C} \not{N} \gamma_5 \hat{\lambda}) \right\}, \end{aligned} \tag{10}$$

where the "classical" fields, $\hat{A}, \hat{\lambda}, \ldots$ are defined as functionals of the sources:

$$\hat{A} = \frac{\delta W}{\delta j}, \qquad \hat{\lambda} = \frac{\delta W}{\delta \bar{n}}, \qquad \hat{C} = \frac{\delta W}{\delta K},$$

$$\hat{D} = \frac{\delta W}{\delta L}, \qquad \hat{O} = \frac{\delta W}{\delta \bar{M}} - \hat{C} \not{N} \gamma_5 \hat{\lambda}. \tag{11}$$

The "classical" fields satisfy the following equations of motion:

$$\frac{\delta \Gamma}{\delta \hat{A}_\mu^a} = -j_\mu^a - \hat{C}^a N_\mu, \tag{12a}$$

$$\frac{\delta \Gamma}{\delta \hat{\lambda}^a} = -\bar{n}^a - \bar{M} \hat{C}^a \not{N} \gamma_5, \tag{12b}$$

$$\frac{\delta \Gamma}{\delta \hat{C}^a} = 0, \qquad \frac{\delta \Gamma}{\delta \hat{D}^a} = 0, \tag{12c}$$

$$\frac{\delta \Gamma}{\delta \hat{O}} = -\bar{M}. \tag{12d}$$

Now we proceed to the derivation of the supersymmetric Ward identities. We transform the fields in the functional integral (8) according to (3a, c), supplemented by

$$\delta_\varepsilon C^a = 0 . \tag{13}$$

We find,

$$0 = \int [\mathrm{d}A \ldots] \mathrm{e}^{iS} \int \mathrm{d}^D x \Big\{ \big(j_\mu^a + N_\mu C^a \big) \bar{\varepsilon} \gamma^\mu \gamma_5 \lambda^a + \vec{n}^a \Big(\tfrac{1}{2} i F_{\rho\lambda}^a \gamma^\rho \gamma^\lambda \gamma_5 - D^a \Big) \varepsilon$$

$$+ L^a i \varepsilon (\not{\partial} \lambda)^a + \bar{M} \delta_\varepsilon O \Big\} .$$

Since $\delta\Gamma/\delta\hat{D}^a = 0$ we put everywhere $\hat{D} = L = 0$ and we use eqs. (12a–d) to obtain

$$0 = \int \mathrm{d}^D x \left\{ \frac{\delta\Gamma}{\delta \hat{A}_\mu^a(x)} \delta_\varepsilon \hat{A}_\mu^a(x) + \left(\frac{\delta\Gamma}{\delta \hat{\lambda}^a(x)} - \frac{\delta\Gamma}{\delta \hat{O}(x)} \hat{C}^a(x) \not{N} \gamma_5 \right) \right.$$

$$\times \left(\delta_\varepsilon \hat{\lambda}^a(x) - \tfrac{1}{2} i g f^{abc} \frac{\delta^2 W}{\delta j_\rho^b(x) \delta j_\lambda^c(x)} \gamma^\rho \gamma^\lambda \gamma_5 \varepsilon \right)$$

$$\left. - \frac{\delta\Gamma}{\delta \hat{O}(x)} \mathrm{e}^{-iW} \langle \delta_\varepsilon O(x) \rangle \right\} , \tag{14}$$

where

$$\delta_\varepsilon \hat{A}_\mu^a(x) \equiv \bar{\varepsilon} \gamma_\mu \gamma_5 \hat{\lambda}^a(x) , \tag{14a}$$

$$\delta_\varepsilon \hat{\lambda}^a(x) \equiv \tfrac{1}{2} i \hat{F}_{\rho\lambda}^a(x) \gamma^\rho \gamma^\lambda \gamma_5 \varepsilon , \tag{14b}$$

$$\delta_\varepsilon O(x) \equiv C^a(x) \not{N} \gamma_5 \tfrac{1}{2} i F_{\rho\lambda}^a(x) \gamma^\rho \gamma^\lambda \gamma_5 \varepsilon , \tag{14c}$$

$$\langle \delta_\varepsilon O(x) \rangle \equiv Z_0^{-1} \int [\mathrm{d}A \ldots] \mathrm{e}^{iS} \delta_\varepsilon O(x) . \tag{14d}$$

We want to derive a relation between the fermion and gauge boson propagators, so we take derivatives of eq. (14) with respect to $\hat{\lambda}^a(x)$, $\hat{A}_\mu^b(y)$:

$$\left. \frac{\delta^2}{\delta \hat{\lambda}^b(y) \delta A_\mu^a(x)} \right|_{\hat{\lambda} = \hat{A} = \hat{C} = \hat{O} = 0} .$$

We obtain, keeping only the functional derivatives which survive when $\hat{\lambda} = \hat{A} = \hat{C} = \hat{O} = 0$,

$$0 = \frac{\delta^2\Gamma}{\delta\hat{A}^a_\mu(x)\delta\hat{A}^b_\nu(y)}\gamma_\nu\gamma_5\varepsilon + \tfrac{1}{2}i\left(\partial^x_\lambda g_{\rho\mu} - \partial^x_\rho g_{\lambda\mu}\right)\frac{\delta^2\Gamma}{\delta\hat{\bar{\lambda}}^b(y)\delta\lambda^a(x)}$$

$$\times\gamma^\lambda\gamma^\rho\gamma_5\varepsilon - \tfrac{1}{2}igf^{a'b'c}\int \mathrm{d}^D z\,\frac{\delta^2\Gamma}{\delta\hat{\bar{\lambda}}^b(y)\delta\hat{\lambda}^{a'}(z)}\,\frac{\delta}{\delta\hat{A}^a_\mu(x)}\,\frac{\delta^2 W}{\delta j^{b'}_\rho(z)\delta j^c_\lambda(z)}\gamma^\rho\gamma^\lambda\gamma_5\varepsilon$$

$$-\int \mathrm{d}^D z\,\frac{\delta^2\Gamma}{\delta\hat{\bar{\lambda}}^b(y)\delta\hat{O}(z)}\,\frac{\delta}{\delta\hat{A}^a_\mu(x)}\mathrm{e}^{-iW}\langle\delta_\varepsilon O\rangle\Big|_{\hat{\lambda}=\hat{A}=\hat{C}=\hat{O}=0}. \tag{15}$$

The last term vanishes because the function $\delta^2\Gamma/\delta\hat{\bar{\lambda}}^b(y)\delta O(z)$ propagates a group singlet object $\hat{O}(z)$ to a group non-singlet object $\bar{\lambda}^b(y)$. The last but one term in (15) contains the factor

$$\frac{\delta}{\delta\hat{A}^a_\mu(x)}\,\frac{\delta^2 W}{\delta j^b_\rho(z)\delta j^c_\lambda(z)}f^{abc}\Big|_{\hat{A}=\hat{\lambda}=J=n=M=0}.$$

This term represents the connected diagrams with one insertion of the vertex, $f^{abc}A^b_\rho(z)A^c_\lambda(z)$, which are 1PI in the external leg. We can see that the diagrams contributing to

$$f^{abc}\langle A^a_\mu(x)A^b_\rho(z)A^c_\lambda(z)\rangle_{\text{connected, 1PI in } A(x)}$$

are antisymmetric in ρ, λ and transverse to N_ρ, N_λ. This completely determines the Lorentz structure of the diagrams:

$$f^{abc}\langle A^a_\mu(x)A^b_\rho(z)A^c_\lambda(z)\rangle_{\text{conn. 1-PI in } A(x)} \overset{\text{F.T.}}{=} A_{\mu\rho\lambda}(k,N)F(k^2,k\cdot N), \tag{16}$$

where

$$A_{\mu\rho\lambda}(k,N) = g_{\rho\lambda}\left(k_\mu - \frac{k\cdot N}{N^2}N_\mu\right) - g_{\mu\lambda}\left(k_\rho - \frac{k\cdot N}{N^2}N_\rho\right)$$

$$+\frac{N_\lambda}{N^2}\left(N_\mu k_\rho - N_\rho k_\mu\right), \tag{17}$$

and k is the external momentum, conjugate to $x - z$. We observe that when $k = N$,

$$A_{\mu\rho\lambda}(N,N) = 0. \tag{18}$$

In momentum space and for $k = N$ eq. (15) becomes

$$0 = \Pi_{\mu\nu}(k)\gamma^{\nu} + \tfrac{1}{2}\Sigma(k)(\not{k}\gamma_{\mu} - \gamma_{\mu}\not{k})|_{k=N}. \tag{19}$$

The Lorentz index decomposition of the 1PI propagators for the gauge and the fermion fields are

$$i\Pi_{\mu\nu}(k, N) = k^2\left(g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)\Pi_1(k, N) + k^2\left(N_{\mu} - \frac{N\cdot k}{k^2}k_{\mu}\right)\left(N_{\nu} - \frac{N\cdot k}{k^2}k_{\nu}\right)\Pi_2(k, N), \tag{20a}$$

$$-i\Sigma(k, N) = \Sigma_1(k, N)\not{k} + \Sigma_2(k, N)\frac{k^2\not{N}}{k\cdot N}. \tag{20b}$$

If we substitute into (19) for $k = N$ we find,

$$\Pi_1 = \Sigma_1 + \Sigma_2. \tag{21}$$

This relation is true for the regularized functions, $\Pi_1, \Sigma_1, \Sigma_2$ so it must be true for the pole parts and the finite parts in a dimensional regularization scheme like the one proposed in ref. [5]. Since in such a scheme (PP, pole part; FP, finite part)

$$Z_{FF} \equiv 1 + \mathrm{PP}\Sigma_1, \tag{22a}$$

$$Z_{AA} \equiv 1 + \mathrm{PP}\Pi_1, \tag{22b}$$

$$\Sigma_{1,\mathrm{ren}} = \mathrm{FP}\,\Sigma_1, \tag{22c}$$

$$\Sigma_{2,\mathrm{ren}} = \mathrm{FP}\,\Sigma_2, \tag{22d}$$

$$\Pi_{1,\mathrm{ren}} = \mathrm{FP}\,\Pi_1, \tag{22e}$$

we obtain from eq. (21),

$$Z_{FF} = Z_{AA}, \tag{23a}$$

$$\Pi_{1,\mathrm{ren}} = \Sigma_{1,\mathrm{ren}} + \Sigma_{2,\mathrm{ren}}|_{k=N}. \tag{23b}$$

We have checked, at the one-loop level, relations (23a, b). We used dimensional regularization with different dimensions for the loop momentum integration ($d_1 = 4 - 2\varepsilon_1$) and for the γ-matrix algebra ($d_2 = 4 - 2\varepsilon_2$). For $k = N$ the results are (we

disagree with the result of ref. [11] for the quantity Π_1)

$$\Pi_1 = 1 + \frac{g^2}{16\pi^2} C_A \left[-3\left(\zeta + \log \frac{-k^2}{\mu^2} \right) - 8\log 2 + 2 + \tfrac{1}{3}\varepsilon_2 \right], \tag{24a}$$

$$\Sigma_1 = 1 + \frac{g^2}{16\pi^2} C_A \left[-3\left(\zeta + \log \frac{-k^2}{\mu^2} \right) - \tfrac{16}{3}\log 2 - \tfrac{2}{3} + \varepsilon_2 \right], \tag{24b}$$

$$\Sigma_2 = \frac{g^2}{16\pi^2} C_A \left[\tfrac{8}{3} - \tfrac{8}{3}\log 2 \right], \tag{24c}$$

where

$$\zeta = -\frac{1}{\varepsilon_1} + \gamma - \log(4\pi),$$

and γ is the Euler-Maceroni constant.

We observe that relation (23a) is satisfied but relation (23b) only after imposing $\varepsilon_2 = 0$. This supports the dimensional reduction renormalization scheme [5, 6].

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VII - PREDICTIONS DES THEORIES SUPERSYMETRIQUES

VII.1 - Brisure spontanée de supersymétrie

La supersymétrie n'est pas une symétrie apparente de la nature. Par conséquent, elle doit nécessairement être brisée. L'échelle de masse de la brisure de la supersymétrie doit être beaucoup plus petite que la masse de "grande unification" - probablement de l'ordre de M_W - afin de pouvoir utiliser la supersymétrie au niveau de la "grande unification" comme une symétrie exacte pour résoudre le problème de la "hiérarchie de jauge". D'autre part, même indépendamment de la "grande unification", l'échelle de la brisure de la supersymétrie doit être de l'ordre de M_W, si l'on ne veut pas avoir de problèmes de type "fine tuning" des différents paramètres.

La brisure spontanée de la supersymétrie est très difficile à réaliser[59]. La raison est que l'énergie du vide est toujours positive. En effet, par la dernière relation des eqs. (VI.2), on peut trouver que [13,14]

$$\frac{1}{2}\sum_{\alpha} Q_{\alpha}^{2} = P_{o} \quad . \tag{VII.1}$$

Or, Po est l'Hamiltonien H du système. Par conséquent, si la supersymétrie n'est pas brisée, le vide $|0\rangle$ est annihilé par les charges Q_α et selon (VII.1) l'énergie du vide est nulle ($H|0\rangle = 0$). Par contre si la supersymétrie est brisée spontanément, $Q_\alpha |0\rangle \neq 0$ ce qui implique que l'énergie du vide est positive :

$$\langle 0|H|0\rangle = \frac{1}{2}\sum_{\alpha}\langle 0|Q_{\alpha}^{2}|0\rangle = \frac{1}{2}\sum_{\alpha}\sum_{n}|\langle 0|Q_{\alpha}|n\rangle|^{2} > 0 \quad . \tag{VII.2}$$

Cet argument est en fait heuristique car dans le cas d'une symétrie brisée, la charge Q_α n'est pas un opérateur bien défini, mais la positivité (VII.2) reste valable pour la densité de l'énergie, comme nous allons voir dans quelques exemples.

Pour pouvoir briser la supersymétrie, il faut donc que l'énergie du vide soit strictement positive, situation exactement opposée à celle de la brisure spontanée d'une symétrie ordinaire. Ces arguments deviennent plus clairs sur l'exemple du potentiel des scalaires dans une théorie

supersymétrique. Il est facile de voir, par exemple de (VI.10), que ce potentiel vient toujours des termes :

$$L_V = \tfrac{1}{2}F^2 + \tfrac{1}{2}G^2 + \tfrac{1}{2}D^2 - F\, f(\varphi) - G\, g(\varphi) - Dd(\varphi)\ , \quad \text{(VII.3)}$$

où f, g et d sont des fonctions des différents champs scalaires φ de la théorie dépendant du modèle donné. Après avoir éliminé les champs auxiliaires F, G et D en utilisant leurs équations de mouvement

$$F = f(\varphi)\ ,\quad G = g(\varphi)\ ,\quad D = d(\varphi) \qquad \text{(VII.4)}$$

on obtient le potentiel V :

$$V = \tfrac{1}{2} f^2(\varphi) + \tfrac{1}{2} g^2(\varphi) + \tfrac{1}{2} d^2(\varphi)\ . \qquad \text{(VII.5)}$$

La condition de minimum V = 0 donne $f(\varphi) = g(\varphi) = d(\varphi) = 0$. Si, il y a des solutions qui satisfont cette condition (même si $\langle\varphi\rangle \neq 0$!) la supersymétrie ne peut être brisée, l'énergie du vide étant nulle. Par contre, si il n'y a pas de solutions à ces équations, l'énergie du vide devient strictement positive ($V > 0$) et la supersymétrie est spontanément brisée. Ceci implique que la valeur moyenne dans le vide d'un, au moins, des champs auxiliaires est non nulle[13,14] (voir rels.(VII.4)).

Jusqu'à maintenant, on connaît deux moyens pour réaliser la brisure spontanée de supersymétrie[59,60]. Tous les deux sont basés sur le fait qu'on peut ajouter à la densité de Lagrangien supersymétrique un terme linéaire en un superchamp chiral Φ ou vectoriel V (intégré sur θ et $\bar{\theta}$) sans briser la supersymétrie. Pour ne pas briser l'invariance de jauge, il faut de plus que Φ appartienne à un singulet du groupe de jauge et que V soit associé à un groupe abelien U(1). Après intégration sur θ et $\bar{\theta}$ ces termes donnent un terme proportionnel à F[60] ou à D[59] dont les variations par les transformations de supersymétrie sont des dérivées totales (voir par ex. (VI.14) avec un groupe abélien). Pour briser spontanément la supersymétrie, il faut construire des modèles pour lesquels la condition de minimum du potentiel n'ait aucune solution.

Une conséquence directe de la brisure spontanée de la supersymétrie est l'apparition d'un spineur de Goldstone de masse nulle, appelé Goldstino[59,60], qui a les mêmes nombres quantiques que la divergence du courant supersymétrique (voir par exemple (VI.15)). Une autre conséquence est que, même après la brisure spontanée, il y a des relations qui restent entre les masses des particules[13,61,62]. (Si il n'y a pas de générateurs, qui correspondent à des facteurs U(1), ayant de trace non nulle) :

$$\sum_J (-)^{2J} (2J + 1) m_J^2 = 0 \quad , \qquad \text{(VII.6)}$$

où m_J est la masse de la particule de spin J.

VII.2 - Extensions supersymétriques du modèle standard

Nous avons expliqué au début du chapitre précédent que l'échelle de la brisure de la supersymétrie doit être de l'ordre de 100-1000 GeV. En utilisant la méthode présentée auparavant, il y a plusieurs façons de rendre supersymétrique le modèle standard[63]. Un désavantage commun à tous ces modèles est qu'ils ne relient pas entre elles des particules déjà connues, résultat qui serait remarquable. Ils prédisent des particules nouvelles, en doublant au moins le spectre connu expérimentalement.

Ainsi toute particule ordinaire a un partenaire supersymétrique[13,63]. Il y a d'abord les spineurs de Majorana associés au bosons de jauge : les gluinos qui forment un octet de couleur (associés des gluons), les gauginos qui forment un triplet de $SU(2)_L$ (associés aux bosons W intermédiaires des interactions faibles) et le photino (associé au photon). Il y a ensuite, les quarks et les leptons scalaires qui sont les partenaires des quarks et des leptons (deux champs complexes, scalaire et pseudoscalaire, sont associés à chaque spineur de Dirac) et qui forment des triplets et des singulets de couleur et des doublets de $SU(2)_L$. Il y a enfin d'autres spineurs de Majorana qui sont les partenaires des Higgs. Il y a, en plus, le Goldstino. Deux questions maintenant peuvent être posées :

(i) où le Goldstino, qui a un spin 1/2, se trouve-t-il ? Etant donné que pour des raisons phénoménologiques, il est impossible de l'identifier ni avec un fermion connu (comme par exemple un neutrino)[64,65] ni avec le partenaire fermionique d'un boson de jauge neutre connu,

γ ou Z[61], il y a deux réponses alternatives[13] : ou bien on l'utilise dans un super-mécanisme de Higgs[66] pour donner une masse au gravitino de spin 3/2[67] si bien qu'il s'incorpore aux théories de supergravité, ou bien on suppose qu'il existe comme particule physique réelle, de spin 1/2). Dans ce cas là, il est associé à un boson de jauge qui, pour des raisons phénomènologiques, ne peut être ni le photon ni le Z^0, ce qui implique qu'on doit élargir le modèle standard, et considérer au moins un U(1) supplémentaire[68]: $SU(3)_c \times SU(2)_L \times U(1)_y \times U(1)$. Dans ce modèle, la supersymétrie est spontanément brisée, ainsi que le groupe de jauge $SU(2)_L \times U(1)_y \times U(1) \rightarrow U(1)_{em}$.

(ii) Les gluinos et le photino ont une masse nulle à l'ordre le plus bas et probablement ils acquièrent une masse petite par corrections radiatives (une fois que la supersymétrie est brisée)[13,69]. La question qui se pose alors est de comprendre pourquoi ces fermions légers (ainsi que le goldstino) n'auraient pas été observés jusqu'à maintenant. Une solution est que tous ces fermions supplémentaires portent un nouveau nombre quantique conservé qui correspond à une symétrie globale supplémentaire appelée"invariance-R"[64,70]. Cette symétrie se manifeste par des transformations de phase des coordonnées θ et $\bar{\theta}$ du super-espace et agit sur les superchamps chiraux en les multipliant par des facteurs de phase tandis que les superchamps de jauge restent invariants. Par conséquent, la plupart de particules supplémentaires portent ce nombre quantique $R \neq 0$, et la conservation implique leur production par paires. Les gluinos et les quarks scalaires, qui sont colorés, doivent nécessairement former des états liés hadroniques qui s'appellent R-hadrons[71] parce qu'ils portent une unité du nombre quantique R. On trouve[63] que la masse des gluinos doit être plus grande que 2 GeV et celle des quarks scalaires plus grande que 15 GeV pour rendre compte de l'absence de production de R-hadrons dans les désintégrations des ψ et Υ et dans les expériences e^+e^-, ainsi que par les expériences de calorimètre.

A part la brisure spontanée de la supersymétrie, il y a d'autres moyens de brisure proposés pour construire des modèles phénoménologiquement acceptables, sans avoir les inconvénients du modèle précédent : la brisure explicite mais douce (par des termes de dimension moins que quatre)[72]

ou, peut-être la brisure dynamique[14,73], qui pourrait éventuellement conduire à une théorie où deux échelles de masse apparaissent d'une façon naturelle[14,74].

Dans les trois chapitres suivants, nous allons étudier les conséquences phénoménologiques des extensions supersymétriques de la chromodynamique quantique. Notre conclusion est :

(i) Les violations de l'invariance d'échelle observées dans la diffusion profondément inélastique des leptons par les nucléons sont compatibles également avec la présence de gluinos légers. On peut faire une analyse similaire à celle du cas standard et trouver les distributions des gluinos dans le nucléon.

(ii) Les modifications qu'apportent les gluinos aux sections efficaces de production des "jets" dans $p\bar{p}$ et à l'asymétrie de production d'un photon directe à grand moment transverse sont négligeables.

(iii) Par contre les modifications dûes à des quarks scalaires sont importantes et en principe détectables expérimentalement.

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LIGHT GLUINOS IN DEEP INELASTIC SCATTERING

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We extend the standard QCD predictions, up to second order, to the scaling violation of the deep inelastic structure functions in the presence of light gluinos (supersymmetric partners of gluons). We show that these new particles give rise to scaling violations which are very close to the standard ones. Therefore present experimental data are also consistent with the existence of light gluinos of mass $m_\lambda < 10$ GeV. Only more precise data would be able to conclude on their existence. All scaling violation effects observed in non-singlet and singlet structure functions are obtained with a single mass scale parameter Λ_{eff}. We also find the gluon and gluino distributions inside the nucleon, as a function of the gluino mass m_λ.

1. Introduction

The existence of a symmetry between bosons and fermions [1] is a very attractive idea in particle physics. This kind of symmetry, called supersymmetry, classifies particles with different statistics in supermultiplets. The spin one-half supersymmetry generator transforms fermions into bosons of the same supermultiplet, and conversely. This implies that every particle must have a superpartner with a spin shifted by $\frac{1}{2}$ a unit. The gluons and photon have spin-$\frac{1}{2}$ partners named the gluinos and photino, while quarks and leptons are associated with spin-0 partners [2].

In realistic models, supersymmetry must be broken [3] in such a way that all successful phenomenological implications of QCD and electroweak gauge interactions are reproduced. One solution is to break the supersymmetry explicitly [4]. In that case all the supersymmetric partners of the known particles become heavy and decouple in the low energy region. More attractively, the breaking can be realized spontaneously [2, 3] or maybe dynamically [5]. In these cases the scalar partners of quarks and leptons can be quite heavy, and satisfy the experimental lower limit for their masses [6] ($\gtrsim 16$ GeV if one assumes for scalar quarks the same limit as for scalar leptons). On the other hand the gluinos, which are strongly interacting neutral fermions, may well have a mass of the order of a few GeV [5, 7]. There are, however, lower limits on the gluino hadronic bound-state masses. They are found to be bigger than 2 or 4 GeV from the non-production of such states in calorimeter and beam

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dump experiments [7–9], or from the analysis of the gluino pair production in e^+e^- scattering and in heavy quarkonium decays [10]. Assuming a binding energy of the order of 1 GeV for gluino-gluon or gluino-quark-antiquark states, one can deduce that the gluino mass m_λ should be larger than about 1 GeV.

The existence of light gluinos implies, in general, some modification of the strong interaction experiment analysis, in particular for the scaling violation of the structure functions. One can expect these modifications to be quite large since the gluinos (which are Majorana fermions) belong to the adjoint representation, so that large group factors naturally appear. In some models, massive Dirac gluinos [7, 11] may appear by combination of the gauge Majorana fermions with matter Majorana ones. In this case the modification must be even larger.

The aim of this work is to estimate quantitatively the effect of light gluinos (1 GeV $< m_\lambda <$ 10 GeV) in the evolution of the structure functions, and to study the compatibility of this effect with the deep inelastic data. For this purpose we review in sect. 2 the situation of the observed scaling violations in deep inelastic scattering and we present the modifications implied by the existence of light gluinos, on calculable quantities, up to the two-loop approximation.

In sect. 3 we calculate the modified evolution kernel of singlet and non-singlet structure functions. The comparison with the standard case is made in sect. 4. This analysis involves only a scale parameter Λ^S and the gluino mass m_λ. It turns out that for every m_λ there exists a value $\Lambda^S(m_\lambda)$ such that one can reproduce the observed scaling violations with the same accuracy as the standard analysis.

We also give gluon and gluino distributions for different values of the gluino mass.

Sect. 5, finally, is devoted to general conclusions.

2. Scaling violation predictions

The factorization theorem and the renormalization group equations can be summarized into the following equations which describe scaling violations of the structure function $F_i(x, Q)$ in standard QCD:

$$F_i(x,Q) = e_{i,\mathrm{NS}}(x,\alpha) \otimes q_{\mathrm{NS}}(x,Q) + e_{i,\mathrm{S}}(x,\alpha) \otimes q_{\mathrm{S}}(x,Q) + e_{i,\mathrm{G}}(x,\alpha) \otimes G(x,Q), \tag{2.1}$$

$$Q\frac{\partial}{\partial Q} q_{\mathrm{NS}}(x,Q) = P_{\mathrm{NS}}(x,\alpha) \otimes q_{\mathrm{NS}}(x,Q), \tag{2.2}$$

$$Q\frac{\partial}{\partial Q}\begin{pmatrix} q_{\mathrm{S}}(x,Q) \\ G(x,Q) \end{pmatrix} = \begin{pmatrix} P_{\mathrm{qq}}(x,\alpha) & P_{\mathrm{qG}}(x,\alpha) \\ P_{\mathrm{Gq}}(x,\alpha) & P_{\mathrm{GG}}(x,\alpha) \end{pmatrix} \otimes \begin{pmatrix} q_{\mathrm{S}}(x,Q) \\ G(x,Q) \end{pmatrix}. \tag{2.3}$$

The coefficient functions $e_{i,\tau}$ and the anomalous dimensions P_{ij} in x space are calculable in perturbative series of the running coupling constant $\alpha(Q)$. $\otimes$ means convolution in x space

$$A(x)\otimes B(x)=\int_x^1 A(y)B\left(\frac{x}{y}\right)\frac{\mathrm{d}y}{y}. \tag{2.4}$$

The quark $q_\tau(x,Q)$ (τ = non-singlet or singlet) and gluon $G(x,Q)$ distributions have to be determined from experiment. Let us note that only the generalized effective charges are process dependent.

If we consider non-singlet and singlet combination of structure functions, eqs. (2.1)–(2.3) imply the integro-differential equations

$$Q\frac{\partial}{\partial Q}F_{\mathrm{NS}}(x,Q)=K_{\mathrm{NS}}(x,\alpha)\otimes F_{\mathrm{NS}}(x,Q), \tag{2.5}$$

$$Q\frac{\partial}{\partial Q}\begin{pmatrix}F_{\mathrm{S}}(x,q)\\ G(x,Q)\end{pmatrix}=\begin{pmatrix}K_{\mathrm{FF}}(x,\alpha) & K_{\mathrm{FG}}(x,\alpha)\\ K_{\mathrm{GF}}(x,\alpha) & K_{\mathrm{GG}}(x,\alpha)\end{pmatrix}\otimes\begin{pmatrix}F_{\mathrm{S}}(x,Q)\\ G(x,Q)\end{pmatrix}. \tag{2.6}$$

The evolution kernels K_{NS} and K_{ij} are known up to order α^2 and are combinations [12] of the coefficient functions e_i and of the anomalous dimensions P_{ij} [12, 13]. The consistent two-loop expression for $\alpha(Q)$ is

$$\frac{\alpha(Q)}{4\pi}=\left[\beta_0\ln\frac{Q^2}{\Lambda^2}+\frac{\beta_1}{\beta_0}\ln\ln\frac{Q^2}{\Lambda^2}\right]^{-1}, \tag{2.7}$$

with

$$\beta_0=\tfrac{1}{3}(11C_{\mathrm{A}}-2N_{\mathrm{F}}), \tag{2.8}$$

$$\beta_1=\tfrac{1}{3}(34C_{\mathrm{A}}^2-10C_{\mathrm{A}}N_{\mathrm{F}}-6C_{\mathrm{F}}N_{\mathrm{F}}), \tag{2.9}$$

where C_{A} and C_{F} are the second-order Casimirs of the gluon and quark representations, and N_{F} is the number of quark flavors.

Eqs. (2.5), (2.6) allow one to predict the structure function for any Q and $x>x_0$, when the initial distributions $F_{\mathrm{NS}}(x,Q_0)$, $F_{\mathrm{S}}(x,Q_0)$ and $G(x,Q_0)$ for $x>x_0$ are given. The initial structure functions $F_{\mathrm{NS}}(x,Q_0)$ and $F_{\mathrm{S}}(x,Q_0)$ can be directly taken from experiment and the initial gluon distribution $G(x,Q_0)$ can be computed, in the leading log approximation, by using the antiquark distribution [14] which can be corrected in the next to leading approximation [15]. A good agreement with the deep inelastic data is obtained [16] with $\Lambda_{\overline{\mathrm{MS}}}\sim 0.200$ GeV.

Let us now examine the modification of eqs. (2.5)–(2.9) in a supersymmetric theory. In fact we want to compare the corresponding predictions to the experimen-

tal data which are restricted to the domain $2 < Q < 15$ GeV, which is below the expected scalar-quark threshold. Thus it is evident, in the calculation of the anomalous dimensions of the cut vertices [17], that the scalar quarks decouple in the evolution kernel. From the decoupling theorem [18] we are allowed to consider a low-energy effective theory in which only the particles which can be produced contribute. This means that we consider only the contribution of the quarks, gluons and gluinos to the β function and to the evolution kernel, and only quarks and gluons below the 2-gluino threshold.

When including the spin-$\frac{1}{2}$ gluinos, the coefficients β_0 and β_1 of the β function read

$$\beta_0^S = \tfrac{1}{3}(11C_A - 2N_F - 2N_\lambda), \tag{2.10}$$

$$\beta_1^S = \tfrac{1}{3}(34C_A^2 - 10C_A N_F - 10C_A N_\lambda - 6C_F N_F - 6C_\lambda N_\lambda), \tag{2.11}$$

where group factors of the gluino representation are $C_\lambda = C_A$, $N_\lambda = 2T_\lambda = 2C_A$ for Dirac gluinos and $N_\lambda = 2T_\lambda = C_A$ for Majorana ones. For SU(3) with 4 flavors, β_0 and β_1 change from the values $\frac{25}{3}$ and $\frac{154}{3}$ to the values $\frac{19}{3}$ and $\frac{10}{3}$ for Majorana gluinos and to the values $\frac{13}{3}$ and $-\frac{134}{3}$ for Dirac gluinos.

According to the definition of the effective low-energy theory, the running coupling constant is defined as follows:

$$\frac{\alpha^S(Q)}{4\pi} = \left[\beta_0 \ln\frac{Q^2}{\Lambda_{\text{eff}}} + \frac{\beta_1}{\beta_0}\ln\ln\frac{Q^2}{\Lambda_{\text{eff}}^2}\right]^{-1} \qquad \text{for } Q < 2m_\lambda$$

$$= \left[\beta_0^S \ln\frac{Q^2}{\Lambda_S^2} + \frac{\beta_1^S}{\beta_0^S}\ln\ln\frac{Q^2}{\Lambda_S^2}\right]^{-1} \qquad \text{for } Q > 2m_\lambda, \tag{2.12}$$

with the relation between Λ_{eff} and Λ_S given by the equality of the two expressions at $Q = 2m_\lambda$ (m_λ the gluino mass).

As far as the evolution kernel is concerned, there is an α^2 contribution of the gluinos to the non-singlet piece K_{NS} and a mixing of the singlet structure function with gluon and gluino distributions. Indeed from the mixing under renormalization of the quark, gluon and gluino operators (or cut vertices), eq. (2.3) now becomes

$$Q\frac{\partial}{\partial Q}\begin{pmatrix} q_S(x,Q) \\ G(x,Q) \\ L(x,Q) \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qG} & P_{qL} \\ P_{Gq} & P_{GG} & P_{GL} \\ P_{Lq} & P_{LG} & P_{LL} \end{pmatrix} \otimes \begin{pmatrix} q_S(x,Q) \\ G(x,Q) \\ L(x,Q) \end{pmatrix}. \tag{2.13}$$

Thus eqs. (2.5) and (2.6) are written

$$Q\frac{\partial}{\partial Q}F_{\mathrm{NS}}(x,Q)=K^{\mathrm{S}}_{\mathrm{NS}}(x,\alpha^{\mathrm{S}})\otimes F_{\mathrm{NS}}(x,Q), \tag{2.14}$$

$$Q\frac{\partial}{\partial Q}\begin{pmatrix}F_{\mathrm{S}}(x,Q)\\ G(x,Q)\\ L(x,Q)\end{pmatrix}=\begin{pmatrix}K^{\mathrm{S}}_{\mathrm{FF}} & K^{\mathrm{S}}_{\mathrm{FG}} & K^{\mathrm{S}}_{\mathrm{FL}}\\ K^{\mathrm{S}}_{\mathrm{GF}} & K^{\mathrm{S}}_{\mathrm{GG}} & K^{\mathrm{S}}_{\mathrm{GL}}\\ K^{\mathrm{S}}_{\mathrm{LF}} & K^{\mathrm{S}}_{\mathrm{LG}} & K^{\mathrm{S}}_{\mathrm{LL}}\end{pmatrix}\otimes\begin{pmatrix}F_{\mathrm{S}}(x,Q)\\ G(x,Q)\\ L(x,Q)\end{pmatrix}. \tag{2.15}$$

As in the standard case, given the initial conditions $F_{\mathrm{NS}}(x,Q_0)$, $F_{\mathrm{S}}(x,Q_0)$, $G(x,Q_0)$ and $L(x,Q_0)$ we can predict the structure functions at any Q. In fact, according to the effective theory definition, we can take $L(x,Q_0)=0$ if $Q_0<2m_\lambda$ and we obtain

$$F_{\mathrm{NS}}(x,Q)=F_{\mathrm{NS}}(x,Q_0)+\int_{Q_0}^{2m_\lambda}\mathrm{d}\ln Q\,K_{\mathrm{NS}}(x,\alpha^{\mathrm{S}}(Q))\otimes F_{\mathrm{NS}}(x,Q)$$
$$+\int_{2m_\lambda}^{Q}\mathrm{d}\ln Q\,K^{\mathrm{S}}_{\mathrm{NS}}(x,\alpha^{\mathrm{S}}(Q))\otimes F_{\mathrm{NS}}(x,Q), \tag{2.16}$$

$$\begin{pmatrix}F_{\mathrm{S}}(x,Q)\\ G(x,Q)\\ L(x,Q)\end{pmatrix}=\begin{pmatrix}F_{\mathrm{S}}(x,Q_0)\\ G(x,Q_0)\\ 0\end{pmatrix}+\int_{Q_0}^{2m_\lambda}\mathrm{d}\ln Q\,K(x,\alpha^{\mathrm{S}}(Q))\otimes\begin{pmatrix}F_{\mathrm{S}}(x,Q)\\ G(x,Q)\\ 0\end{pmatrix}$$
$$+\int_{2m_\lambda}^{Q}\mathrm{d}\ln Q\,K^{\mathrm{S}}(x,\alpha^{\mathrm{S}}(Q))\otimes\begin{pmatrix}F_{\mathrm{S}}(x,Q)\\ G(x,Q)\\ L(x,Q)\end{pmatrix}, \tag{2.17}$$

where K_{NS} and K are the $K^{\mathrm{S}}_{\mathrm{NS}}$ and K^{S} kernels with $C_\lambda=N_\lambda=0$.

3. Supersymmetric evolution kernel

Since gluinos are spin-$\frac{1}{2}$ particles, their contribution can be obtained from the corresponding diagrams with quarks by an appropriate change of group factors. However, we want to keep the parton interpretation of the distributions, i.e. to define $q(x,Q)$, $G(x,Q)$ and $L(x,Q)$ as quark, gluon and gluino distributions. Since these densities depend on the chosen renormalization scheme, we have to use supersymmetric renormalization for the cut vertices, as is the case for the dimensional reduction renormalization scheme [19]. Let us emphasize that such a supersymmetric scheme has to be used not only at higher energies where supersymmetry breaking can be neglected, but also at lower energies if we want to define $\int x\,\mathrm{d}x\,G(x,Q)$ and $\int x\,\mathrm{d}x\,L(x,Q)$ as the energy fractions carried by gluons and gluinos.

We start by computing the standard QCD quantities [eqs. (2.1)–(2.3)] in the dimensional reduction scheme ($\overline{\mathrm{DR}}$) which can be obtained [20] from the coefficient functions and anomalous dimensions known up to next to leading order in the $\overline{\mathrm{MS}}$ scheme. For this purpose we consider the corresponding quantities in moment space (in order to have multiplication instead of convolution):

$$C_\tau^n(\alpha) = \int x^{n-1}\,\mathrm{d}x\, e_\tau(x,\alpha) = C_\tau^{(0)n} + \frac{\alpha}{4\pi} C^{(1)n}, \qquad \tau = \mathrm{q}, \mathrm{G}, \tag{3.1}$$

$$\gamma_{ij}^n(\alpha) = -\int x^{n-1}\,\mathrm{d}x\, P_{ij}(x,\alpha) = \frac{\alpha}{4\pi}\gamma_{ij}^{(0)n} + \left(\frac{\alpha}{4\pi}\right)^2 \gamma_{ij}^{(1)n}. \tag{3.2}$$

Then the $\overline{\mathrm{DR}}$ expressions are obtained from the known $\overline{\mathrm{MS}}$ ones (omitting n dependence and matrix indices):

$$C^{(0)}_{\tau,\overline{\mathrm{DR}}} = C^{(0)}_{\tau,\overline{\mathrm{MS}}}, \tag{3.3}$$

$$\gamma^{(0)}_{\overline{\mathrm{DR}}} = \gamma^{(0)}_{\overline{\mathrm{MS}}}, \tag{3.4}$$

$$C^{(1)}_{\mathrm{q},\overline{\mathrm{DR}}} = C^{(1)}_{\mathrm{q},\overline{\mathrm{MS}}} - \Delta O_{\mathrm{qq}}, \tag{3.5}$$

$$C^{(1)}_{\mathrm{G},\overline{\mathrm{DR}}} = C^{(1)}_{\mathrm{G},\overline{\mathrm{MS}}}, \tag{3.6}$$

$$\gamma^{(1)}_{\overline{\mathrm{DR}}} = \gamma^{(1)}_{\overline{\mathrm{MS}}} + 2\beta_0 \Delta O - \left[\gamma^{(0)}, \Delta O\right], \tag{3.7}$$

where the matrix ΔO is

$$\Delta O = \begin{pmatrix} \Delta O_{\mathrm{qq}} & \Delta O_{\mathrm{qG}} \\ \Delta O_{\mathrm{qG}} & \Delta O_{\mathrm{GG}} \end{pmatrix} = \begin{pmatrix} C_{\mathrm{F}}\left(\dfrac{2}{n} - \dfrac{2}{n+1} - 1\right) & 0 \\ -\dfrac{2C_{\mathrm{F}}}{n+1} & C_{\mathrm{A}}\left(\dfrac{4}{n+1} - \dfrac{4}{n+2} - \dfrac{1}{3}\right) \end{pmatrix}. \tag{3.8}$$

From these quantities of standard QCD in $\overline{\mathrm{DR}}$ scheme, we can obtain the anomalous dimension of (2.13) with an appropriate change of group factors. The non-zero $\gamma^{(0)}_{\lambda\lambda}$, $\gamma^{(0)}_{\lambda\mathrm{G}}$ and $\gamma^{(0)}_{\mathrm{G}\lambda}$ are obtained respectively from $\gamma^{(0)}_{\mathrm{qq}}$, $\gamma^{(0)}_{\mathrm{qG}}$ and $\gamma^{(0)}_{\mathrm{Gq}}$ with the changes $C_{\mathrm{F}} \to C_\lambda$, $N_{\mathrm{F}} \to N_\lambda$ and the $\gamma^{(1)}$ from table 1. The coefficient functions are unchanged (the gluino effective charge remains zero owing to the decoupling of scalar

TABLE 1
The second-order anomalous dimensions in the presence of gluinos (column 1) are obtained from those without gluino (column 2) by group factor changes (other column)

	From	C_F^2	$C_A C_F$	$C_F N_F$	$C_A N_F$
NS_q	NS	C_F^2	$C_A C_F$	$C_F(N_F + N_\lambda)$	
Δ_{qq}	Δ			$C_F N_F$	
qG	qG			$C_F N_F$	$C_A N_F$
qL	Δ			$C_\lambda N_F$	
Gq	Gq	C_F^2	$C_A C_F$	$C_F(N_F + N_\lambda)$	
GG	GG			$C_F N_F + C_\lambda N_\lambda$	$C_A(N_F + N_\lambda)$
GL	Gq	C_λ^2	$C_A C_\lambda$	$C_F(N_F + N_\lambda)$	
Lq	Δ			$C_F N_\lambda$	
LG	qG			$C_\lambda N_\lambda$	$C_A N_\lambda$
NS_L	NS	C_λ^2	$C_A C_\lambda$	$C_F(N_F + N_\lambda)$	
Δ_{LL}	Δ			$C_\lambda N_\lambda$	

The matrix elements γ_{ij} are denoted ij and $\Delta_{qq(LL)} = \gamma_{qq(LL)} - \gamma_{NSq(L)}$.

quarks), and we obtain the kernels (2.14) and (2.15):

$$K^S_{3,NS} = \frac{\alpha^S}{4\pi} 4C_F \left(\frac{1+x^2}{1-x}\right)_+ + \left(\frac{\alpha^S}{4\pi}\right)^2$$

$$\times \Big\{ 4C_F^2 \Big[\left(\tfrac{3}{4} + 12\ \mathrm{Li}_3(1) - \pi^2\right) \delta(1-x) - 10(1-x) - (1+x)\ln^2(x)$$

$$- 4\frac{(1+x^2)}{1-x} \ln x \ln(1-x) - 2\left(2x + \frac{3}{1-x}\right) \ln x \Big]$$

$$+ 16\eta C_F \left(C_F - \tfrac{1}{2} C_A\right) \Big[2(1-x) + (1+x)\ln x + \frac{1+x^2}{1+x} FF(x) \Big]$$

$$+ \tfrac{4}{3} C_A C_F \Big[\left(\tfrac{215}{4} - 18\,\mathrm{Li}_3(1) + \tfrac{22}{3}\pi^2\right) \delta(1-x) + \left(\frac{\frac{367}{2} - 2\pi^2 - 22\ln(1-x)}{1-x} \right)_+$$

$$- \tfrac{1}{3}(13 + 220x) - 16\left(1 + x - \frac{11}{4}\frac{1}{1-x}\right) \ln x + 3\frac{1+x^2}{1-x} \ln^2 x$$

$$+ (1+x)\left(11\ln(1-x) + \pi^2\right) \Big]$$

$$+\tfrac{8}{3}C_F(N_F+N_\lambda)\Big[-\left(\tfrac{19}{4}+\tfrac{2}{3}\pi^2\right)\delta(1-x)-\left(\frac{\frac{29}{6}-2\ln(1-x)}{1-x}\right)_+$$

$$+\tfrac{1}{3}(5+14x)-2\frac{1+x^2}{1-x}\ln x-(1+x)\ln(1-x)\Big]\Big\}, \tag{3.9}$$

$$K^S_{2,\mathrm{NS}}=K^S_{3,\mathrm{NS}}+\left(\frac{\alpha^S}{4\pi}\right)^2\tfrac{8}{3}C_F(N_F+N_\lambda)(1+x), \tag{3.10}$$

$$K^S_{FF}=\frac{\alpha^S}{4\pi}4C_F\left(\frac{1+x^2}{1-x}\right)_+ +\left(\frac{\alpha^S}{4\pi}\right)^2$$

$$\times\Big\{4C_F^2\Big[\left(\tfrac{3}{4}+12\,\mathrm{Li}_3(1)-\pi^2\right)\delta(1-x)$$

$$-2(1-x)-(1+x)\ln^2(x)+2\left(2-\frac{3}{1-x}\right)\ln(x)$$

$$-4\frac{1+x^2}{1-x}\ln(x)\ln(1-x)+4\frac{1+x^2}{1+x}FF(x)\Big]$$

$$+\tfrac{4}{3}C_AC_F\Big[\left(\tfrac{215}{4}-18\,\mathrm{Li}_3(1)+\tfrac{22}{3}\pi^2\right)\delta(1-x)+\left(\frac{\frac{367}{2}-2\pi^2-22\ln(1-x)}{1-x}\right)_+$$

$$-\tfrac{1}{3}(82+217x)+\frac{(1+x^2)}{1-x}(3\ln^2x+22\ln x)$$

$$+(1+x)\left(11\ln(1-x)+\pi^2\right)-6\frac{1+x^2}{1+x}FF(x)\Big]$$

$$+C_F(N_F+N_\lambda)\Big[-\left(\tfrac{38}{3}+\tfrac{16}{9}\pi^2\right)\delta(1-x)-\left(\frac{\frac{116}{9}-\frac{16}{3}\ln(1-x)}{1-x}\right)_+\Big]$$

$$+\tfrac{8}{3}C_FN_F\Big[\frac{1}{3}\left(\frac{26}{x}-79+68x+10x^2\right)+\left(2-4x+12x^2-\frac{4}{1-x}\right)\ln x$$

$$-6(1+x)\left(\ln^2x+\mathrm{Li}_2(x)-\tfrac{1}{6}\pi^2\right)+2\left(\frac{2}{x}+1-2x-2x^2\right)\ln(1-x)\Big]$$

$$+\tfrac{8}{3}C_FN_\lambda\Big[\tfrac{1}{3}(8+17x)-2\frac{1+x^2}{1-x}\ln x-(1+x)\ln(1-x)\Big]\Big\}, \tag{3.11}$$

$$K_{\mathrm{FL}}^{\mathrm{S}}=\left(\frac{\alpha^{\mathrm{S}}}{4\pi}\right)^2 8C_\lambda N_{\mathrm{F}}\left[\tfrac{1}{9}\left(\frac{26}{x}-87+51x+10x^2\right)-2x(1-2x)\ln x-2(1+x)\right.$$

$$\left.\times\left(\ln^2 x+\mathrm{Li}_2(x)-\tfrac{1}{6}\pi^2\right)+\frac{1}{3}\left(\frac{4}{x}+3-3x-4x^2\right)\ln(1-x)\right], \tag{3.12}$$

$$K_{\mathrm{FG}}^{\mathrm{S}}=\frac{\alpha^{\mathrm{S}}}{4\pi}4N_{\mathrm{F}}(1-2x+2x^2)$$

$$+\left(\frac{\alpha^{\mathrm{S}}}{4\pi}\right)^2\left\{4C_{\mathrm{A}}N_{\mathrm{F}}\left[\frac{1}{9}\left(\frac{52}{x}-180-390x+539x^2\right)-14x(4-3x)\ln x\right.\right.$$

$$-(5+14x+2x^2)\ln^2 x+(1-2x+2x^2)$$

$$\times\left(2\ln^2(1-x)-4\ln x\ln(1-x)-\tfrac{1}{3}\pi^2\right)$$

$$+\frac{2}{3}\left(\frac{4}{x}-3+60x-67x^2\right)\ln(1-x)-4(1+4x)$$

$$\left.\times\left(\mathrm{Li}_2(x)-\tfrac{1}{6}\pi^2\right)+2(1+2x+2x^2)FF(x)\right]$$

$$+4C_{\mathrm{F}}N_{\mathrm{F}}\left[2-7x+4x^2+(1+4x-12x^2)\ln x\right.$$

$$+(1-2x+4x^2)\ln^2 x-4(1-2x+2x^2)(\ln x\ln(1-x)$$

$$\left.\left.+\mathrm{Li}_2(x))-2(1+6x-6x^2)\ln(1-x)\right]\right\}, \tag{3.13}$$

$$K_{\mathrm{LF}}^{\mathrm{S}}=\left(\frac{\alpha^{\mathrm{S}}}{4\pi}\right)^2 8C_{\mathrm{F}}N_\lambda\left[\frac{1}{9}\left(\frac{20}{x}-27+45x-38x^2\right)\right.$$

$$\left.+\tfrac{1}{3}(3+9x+8x^2)\ln x-(1+x)\ln^2 x\right], \tag{3.14}$$

$$K^{S}_{LG} = \frac{\alpha^S}{4\pi} 4N_\lambda (1 - 2x + 2x^2)$$

$$+ \left(\frac{\alpha^S}{4\pi}\right)^2 \Big\{ 4C_A N_\lambda \Big[\frac{1}{9}\Big(\frac{40}{x} - 51 + 336x - 304x^2 \Big) + \tfrac{2}{3}(3 + 12x + 32x^2) \ln x$$

$$- (3 + 6x + 2x^2) \ln^2 x - 8x(1-x)\ln(1-x)$$

$$- (1 - 2x + 2x^2)\big(2\ln^2(1-x) - \tfrac{1}{3}\pi^2\big) + 2(1 + 2x + 2x^2) FF(x) \Big]$$

$$+ 4C_\lambda N_\lambda \big[9 - 25x + 20x^2 + (1 - 8x + 8x^2)\ln x + 8x(1-x)\ln(1-x)$$

$$+ (1 - 2x + 4x^2)\ln^2 x + 2(1 - 2x + 2x^2)$$

$$\times \big(\ln^2(1-x) - 2\ln x \ln(1-x) - \tfrac{1}{3}\pi^2\big)\big] \Big\}, \qquad (3.15)$$

$$K^{S}_{LL} = \frac{\alpha^S}{4\pi} 4C_\lambda \left(\frac{1+x^2}{1-x}\right)_+ + \left(\frac{\alpha^S}{4\pi}\right)^2$$

$$\times \Big\{ 4C_\lambda^2 \Big[\big(\tfrac{3}{4} + 12\,\mathrm{Li}_3(1) - \pi^2\big)\,\delta(1-x) - 2(1-x) + 2\Big(2 - \frac{3}{1-x}\Big)\ln x$$

$$- (1+x)\ln^2 x - 4\frac{1+x^2}{1-x}\ln x \ln(1-x) + 4\frac{1+x^2}{1+x} FF(x) \Big]$$

$$+ 4C_A C_\lambda \Big[\big(\tfrac{13}{4} - 6\,\mathrm{Li}_3(1) + \tfrac{11}{9}\pi^2\big)\,\delta(1-x) + \left(\frac{\frac{134}{9} - \frac{2}{3}\pi^2}{1-x}\right)_+ - \tfrac{2}{9}(8 + 59x)$$

$$+ \frac{1+x^2}{1-x}\big(\tfrac{11}{3}\ln x + \ln^2 x\big) + (1+x)\tfrac{1}{6}\pi^2 - \frac{1+x^2}{1+x} FF(x) \Big]$$

$$- 2C_\lambda (N_F + N_\lambda) \Big[\big(1 + \tfrac{4}{9}\pi^2\big)\,\delta(1-x) - \frac{40}{9}\frac{1}{(1-x)_+} \Big]$$

$$+ \tfrac{8}{3} C_\lambda N_F \Big[\tfrac{2}{3}(1 + 4x) - \frac{1+x^2}{1-x}\ln x \Big] + \tfrac{8}{3} C_\lambda N_\lambda \Big[\frac{1}{3}\Big(\frac{20}{x} - 25 + 53x - 38x^2 \Big)$$

$$+ \Big(4 + 10x + 8x^2 - \frac{2}{1-x}\Big)\ln x - 3(1+x)\ln^2 x \Big] \Big\}, \qquad (3.16)$$

$$K_{GF}^{S} = \frac{\alpha^S}{4\pi} 4C_F\left(\frac{2}{x} - 2 + 1\right) + \left(\frac{\alpha^S}{4\pi}\right)^2$$

$$\times \left\{ 8C_A C_F \left[\frac{1}{9}\left(\frac{3}{x} + 22 + \tfrac{23}{2}x + 38x^2\right) - \tfrac{1}{3}(36 + 9x + 8x^2)\ln x \right.\right.$$

$$+ \left(\frac{1}{x} + 3 + \tfrac{3}{2}x\right)\ln^2 x + \left(\frac{2}{x} - 2 + x\right)\left(\tfrac{11}{3}\ln(1-x)\right.$$

$$\left.\left. + \ln^2(1-x) - 2\ln x \ln(1-x) - \tfrac{1}{6}\pi^2\right) - \left(\frac{2}{x} + 2 + x\right) FF(x)\right]$$

$$- \tfrac{16}{9} C_F (N_F + N_\lambda)\left(\frac{2}{x} - 2 + x\right)(5 + 3\ln(1-x))$$

$$+ 4C_F^2 \left[2(2+x)\ln x - (4-x)\ln(1-x) + 4\left(\frac{2}{x} - 2 + x\right)\right.$$

$$\left.\left. \times \left(\ln x \ln(1-x) - \ln^2(1-x) + \mathrm{Li}_2(x)\right) + 2(2-x)\left(\mathrm{Li}_2(x) - \tfrac{1}{6}\pi^2\right)\right]\right\}, \tag{3.17}$$

$$K_{GG}^{S} = \frac{\alpha^S}{4\pi}\left[\tfrac{4}{3}\left(\tfrac{11}{2}C_A - N_F - N_\lambda\right)\delta(1-x) + 8C_A\left(\frac{1}{x} - 2 + x - x^2 + \frac{1}{(1-x)_+}\right)\right]$$

$$+ \left(\frac{\alpha^S}{4\pi}\right)^2 \left\{ 8C_A^2 \left[\left(\tfrac{107}{36} + 3\,\mathrm{Li}_3(1)\right)\delta(1-x) + \left(\frac{\frac{67}{9} - \frac{1}{3}\pi^2}{1-x}\right)_+ \right.\right.$$

$$- \tfrac{1}{18}(25 + 175x - 66x^2) - \tfrac{1}{3}(25 - 11x + 44x^2)\ln x$$

$$+ \left(\frac{1}{x} + 2 + 5x - x^2 + \frac{1}{1-x}\right)\ln^2 x - 4\frac{\ln x \ln(1-x)}{1-x}$$

$$- \left(\frac{1}{x} - 2 + x - x^2\right)\left(4\ln x \ln(1-x) - \tfrac{1}{3}\pi^2\right)$$

$$\left. - 2\left(\frac{1}{x} + 2 + x + x^2 - \frac{1}{1+x}\right) FF(x)\right]$$

$$+ \tfrac{8}{3} C_A (N_F + N_\lambda)\left[- \tfrac{13}{6}\delta(1-x) - \frac{10}{3}\frac{1}{(1-x)_+}\right.$$

$$\left. - \frac{1}{3}\left(\frac{23}{x} - 29 + 13x - 17x^2\right) - 2(1+x)\ln x\right]$$

$$+\tfrac{8}{3}C_F N_F\Big[-\tfrac{3}{2}\delta(1-x)-1+13x-12x^2-2(3-3x+2x^2)\ln x$$

$$-\Big(\frac{4}{x}+3-3x-4x^2\Big)\ln(1-x)+6(1+x)\big(\mathrm{Li}_2(x)-\tfrac{1}{6}\pi^2\big)\Big]$$

$$+\tfrac{8}{3}C_\lambda N_\lambda\Big[-\tfrac{3}{2}\delta(1-x)+\frac{1}{3}\Big(\frac{2}{x}-21+15x+4x^2\Big)-(1+x)(3+\ln x)\ln x\Big]\Big\}, \tag{3.18}$$

$$K^{\mathrm{S}}_{\mathrm{GL}}=\frac{\alpha^{\mathrm{S}}}{4\pi}4C_\lambda\Big(\frac{2}{x}-2+x\Big)+\Big(\frac{\alpha^{\mathrm{S}}}{4\pi}\Big)^2$$

$$\times\Big\{4C_\lambda^2\Big[-7-x+3x\ln x-(2-x)\ln^2 x-\Big(\frac{2}{x}-2+x\Big)\big(6\ln(1-x)$$

$$+2\ln^2(1-x)\big)\Big]-\tfrac{16}{9}C_\lambda(N_F+N_\lambda)\Big(\frac{2}{x}-2+x\Big)\big(5+3\ln(1-x)\big)$$

$$+8C_A C_\lambda\Big[\frac{1}{9}\Big(\frac{3}{x}+22+\tfrac{23}{2}x+38x^2\Big)$$

$$-\tfrac{1}{3}(36+9x+8x^2)\ln x+\Big(\frac{1}{x}+3+\tfrac{3}{2}x\Big)\ln^2 x$$

$$+\Big(\frac{2}{x}-2+x\Big)\big(\tfrac{11}{3}\ln(1-x)+\ln^2(1-x)$$

$$-2\ln x\ln(1-x)-\tfrac{1}{6}\pi^2\big)-\Big(\frac{2}{x}+2+x\Big)FF(x)\Big]\Big\}. \tag{3.19}$$

In these formulae

$$FF(x)=\tfrac{1}{2}\ln^2 x-2\ln x\ln(1+x)-2\,\mathrm{Li}_2(-x)-\tfrac{1}{6}\pi^2,$$

$$\mathrm{Li}_h(x)=\int_0^x\frac{\mathrm{Li}_{h-1}(z)}{z}\,\mathrm{d}z,\qquad \mathrm{Li}_1(x)=-\ln(1-x). \tag{3.20}$$

A consistent use of the kernel K^{S} implies a scale parameter $\Lambda_{\overline{\mathrm{DR}}}$. The relation to the $\Lambda_{\overline{\mathrm{MS}}}$ one is given by the relation

$$\frac{4\pi}{\alpha_{\overline{\mathrm{DR}}}}=\frac{4\pi}{\alpha_{\overline{\mathrm{MS}}}}-\tfrac{1}{3}C_A. \tag{3.21}$$

Thus if we want to compare the evolution with K^S with the standard one, we can use the K^S kernel with $\alpha_{\overline{MS}}$ along with a scale parameter $\Lambda_{\overline{DR}} = \Lambda_{\overline{MS}} e^{C_A/6\beta_0}$; that is, for SU(3) – 4 flavors, $\Lambda_{\overline{DR}} = 1.06\ \Lambda_{\overline{MS}}$, a change which is negligible.

4. Structure function evolution

We know that standard QCD is in good agreement with the deep inelastic data [16]. This agreement is achieved with a scale parameter $\Lambda_{\overline{MS}} \sim 200$ MeV with an uncertainty larger than 50 MeV. Thus instead of directly comparing the predicted evolution of the structure function, as given by the supersymmetric kernel, to the experimental data, we compare this evolution to the standard QCD one, where the experimental error is translated into an error in the $\Lambda_{\overline{MS}}$ determination. The two parameters, in the supersymmetric case, are the mass of the Majorana or Dirac gluinos and the scale parameter Λ_{eff}. As experimental data are limited to the range $2 < Q < 15$ GeV/c, a sizable mean effect of the gluinos in this range can only exist if the gluino mass is not too high. We have therefore restricted our analysis to the case $1 \leqslant m_\lambda \leqslant 4$ GeV. We have found that, with $\Lambda_{\text{eff}} \simeq \Lambda_{\overline{MS}}$, the effect of gluinos on the predicted structure function is not appreciable inside the "experimental errors". To be more precise, let us consider the following quantities

$$I(Q) = \frac{F(\Lambda_{\overline{MS}} = 0.2) - F^S(m_\lambda, \Lambda_{\text{eff}} = 0.2)}{F(\Lambda_{\overline{MS}} = 0.2)}, \tag{4.1}$$

$$I_\pm(Q) = \frac{F(\Lambda_{\overline{MS}} = 0.2) - F(\Lambda_{\overline{MS}} = 0.2 \pm 0.05)}{F(\Lambda_{\overline{MS}} = 0.2)}, \tag{4.2}$$

where $F(\Lambda)$ and $F^S(\Lambda_{\text{eff}})$ are the structure functions obtained at the Q value using the standard QCD evolution for F or the supersymmetric one for F^S, and starting from the same input distribution at $Q_0 = 2$ GeV. Thus the comparison of $I(Q)$ with $I_\pm(Q)$ is nothing but the comparison of the gluino effect with the "experimental errors" as translated in Λ uncertainties.

In the non-singlet case, we have used as input function at $Q_0 = 2$ GeV

$$xF_3(x, Q_0) = 2.24\, x^{0.42}(1-x)^{2.26}, \tag{4.3}$$

and the corresponding quantity $I(17\text{ GeV})$ is compared to $I_\pm(17)$ as function of x in fig. 1 [these quantities are unaffected by slight changes of the parametrization (4.3)]. It is clear that the effect of the gluinos is small as compared to a change of Λ in standard QCD evolution, except at small x for Dirac gluinos with 1 GeV mass. However, this comparison is misleading for $x \sim 0.15$–0.2 because we know that the quantities $I_\pm$ have a zero in this range. This is not the case for the quantity I which

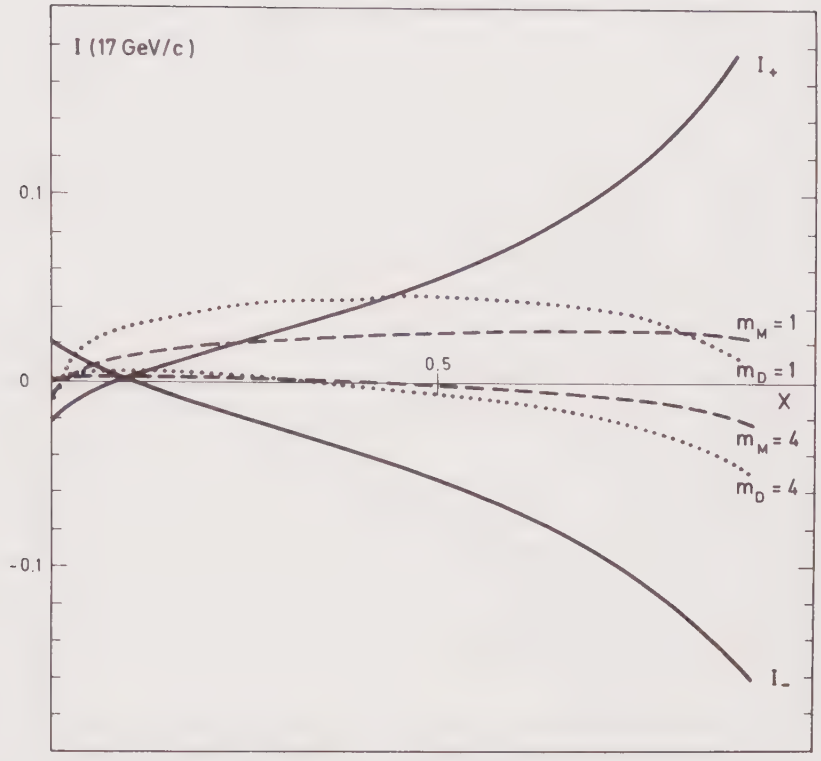

Fig. 1. Comparison of gluino effects and change of scale parameter in non-singlet structure functions as defined in eqs. (4.1) and 4.2. Solid lines represent $I_{\pm}$(17 GeV), dashed lines I(17 GeV) for 1 and 4 GeV Majorana gluino masses ($m_M = 1$ and 4) and dotted lines I(17 GeV) for 1 and 4 GeV Dirac gluino masses ($m_D = 1$ and 4).

compares structure functions obtained with different kernel and very different $\alpha(Q)$ behaviour. Nevertheless, in all cases, I(17 GeV) remains less than 4% in all the x range. The fact that we found this result for $\Lambda_{\text{eff}} = \Lambda_{\overline{\text{MS}}}$, whatever the (Majorana or Dirac) gluino mass, is not a trivial result. The reason is that the modifications of the coupling constant and of the kernel act in opposite direction to the change of Λ_{eff}. The β function modifications push down the Λ_{eff} parameter whereas the kernel modifications push this parameter up. In fig. 2 we plot $\alpha(Q)$ and $\alpha^{\text{S}}(Q)$ as functions of Q^2 in the case $m_\lambda = 1$ GeV for Majorana and Dirac gluinos. To visualize the difference between the kernel K_{NS} and K^{S}_{NS}, we plot their effect on the same structure function $xF_3(x, Q_0 = 2)$ in fig. 3:

$$\frac{\partial \ln F(x,Q)}{\partial \ln Q^2} = \frac{K_{\text{NS}}(x,\alpha) \otimes F(x,Q)}{2F(x,Q)} . \tag{4.4}$$

The total effect with the Q variation is shown in fig. 4 where for the highest Q value

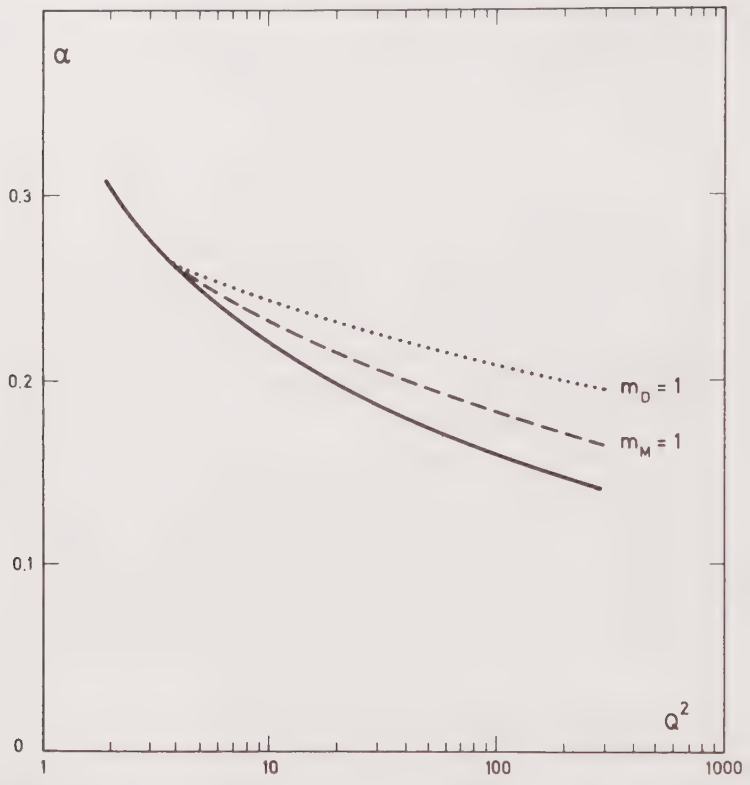

Fig. 2. Variation with Q of the coupling constant in the standard case (solid line) compared to $m_\lambda = 1$ GeV variation for Majorana (dashed line) and Dirac (dotted line) gluinos.

the change of the coupling constant dominates. Let us note that change of Λ_{eff} gives an effect very similar to the corresponding change of $\Lambda_{\overline{\text{MS}}}$.

The first and second order Λ^S corresponding to $\Lambda_{\text{eff}} = 0.2$ are given in table 2 for different gluino mass values, below and above the bottom threshold. The fact that Λ^S becomes smaller (mainly in the Dirac case) expresses the smaller Q^2 dependence of the $\alpha^S(Q)$ coupling constant after the gluino threshold (see fig. 2).

In the singlet case, our analysis starts with the following input at $Q_0 = 2$ GeV:

$$F_2(x, Q_0) = 1.1\,(1 + 3.7x)(1 - x)^{3.19}, \tag{4.5a}$$

$$xG(x, Q_0) = 2.7\,(1 + 8.5x)(1 - x)^9 + 0.26\,(1 - x)^{3.19}, \tag{4.5b}$$

$$xL(x, Q_0) = 0, \tag{4.5c}$$

where F_2 is taken from CDHS data [14] and G from the analysis [15] taking into

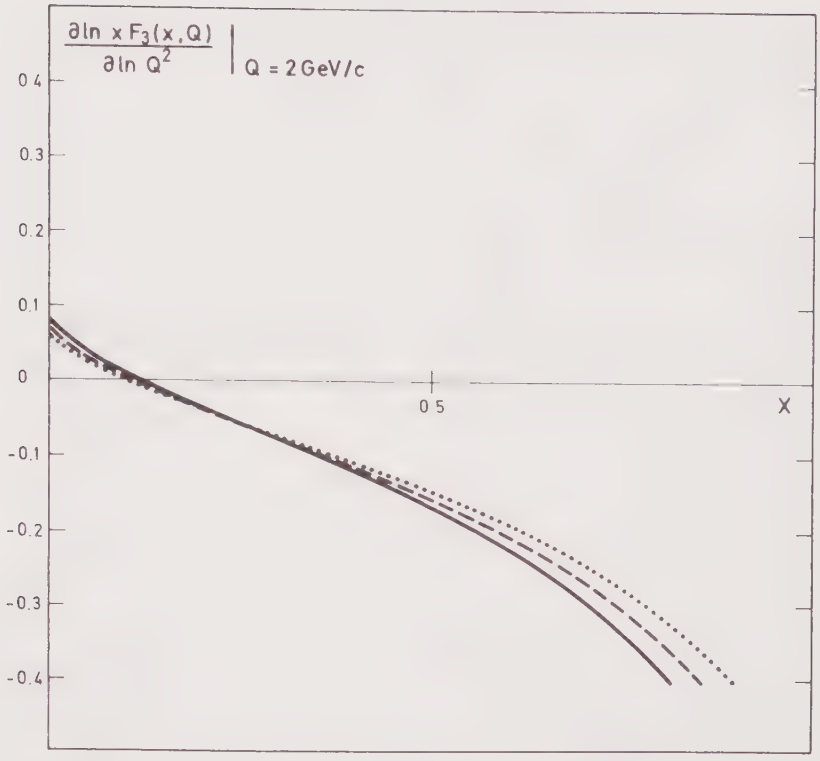

Fig. 3. Logarithmic derivative of xF_3 with respect to $\ln Q^2$ taken at $Q = 2$ GeV/c. Different curves as in fig. 2.

account second-order corrections. This gluon distribution is obtained by solving [see (2.6)]

$$Q\frac{\partial}{\partial Q}F_2^{\exp}(x,Q) = K_{\mathrm{FF}}(x,\alpha)\otimes F_2^{\exp}(x,Q) + K_{\mathrm{FG}}(x,\alpha)\otimes G(x,Q) \quad (4.6)$$

at $Q = Q_0$.

The distribution (4.5b) corresponds to this resolution with the kernels K_{FF} and K_{FG} taken in the $\overline{\mathrm{MS}}$ scheme. As explained in the last section, the gluon distribution to be used as input of the evolution equation in the supersymmetric case has to be computed in the $\overline{\mathrm{DR}}$ scheme. However, we have checked that the distribution (4.5b) with kernels taken in $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes gives for the left-hand side of eq. (4.6) the same result with less than 2% change, a difference which is negligible as compared to the corresponding experimental errors.

The results for the singlet structure function is very similar to the non-singlet case, as shown in fig. 5 for the corresponding $I(Q)$ quantities defined in (4.1) and (4.2).

TABLE 2
Λ^{S} and Λ^{St} values corresponding to $\Lambda_{eff}^{LO} = \Lambda_{eff} = 0.2$, for 4 flavors (first two lines) or five flavors with bottom threshold at 10 GeV (last line)

	m_M		m_D		
	1	4	1	4	Standard
$\Lambda_{eff}^{LL} = 0.2$	0.097	0.062	0.024	0.0066	0.2
$\Lambda_{eff} = 0.2$	0.05	0.026	0.00026	0.000035	0.2
$\Lambda_{eff} = 0.2$ $Q > 10$ GeV/c	0.0165	0.0079	1.82×10^{-6}	0.16×10^{-5}	0.127

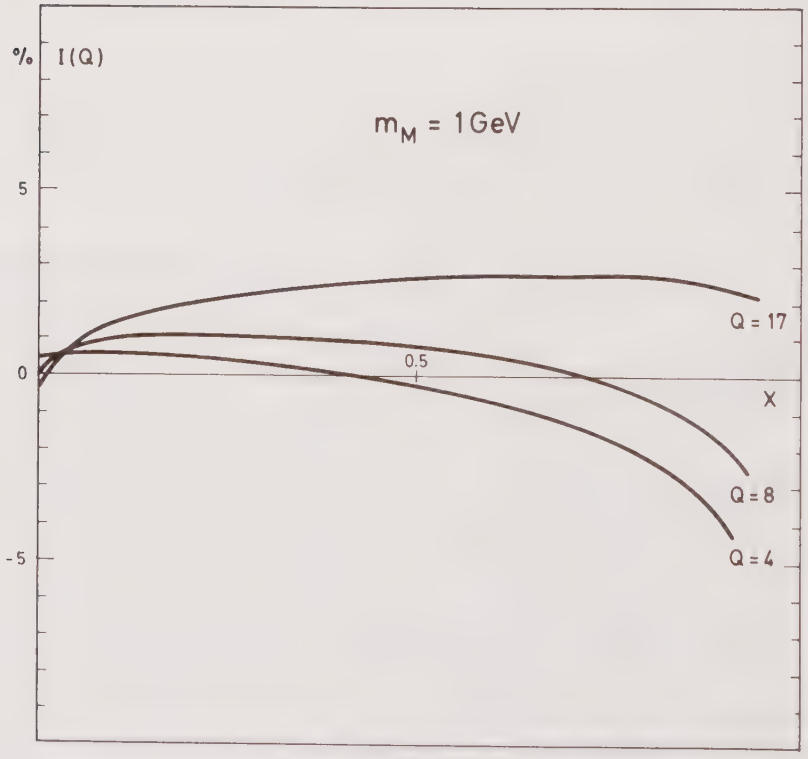

Fig. 4. Change with Q of $I(Q)$ for 1 GeV Majorana gluino mass.

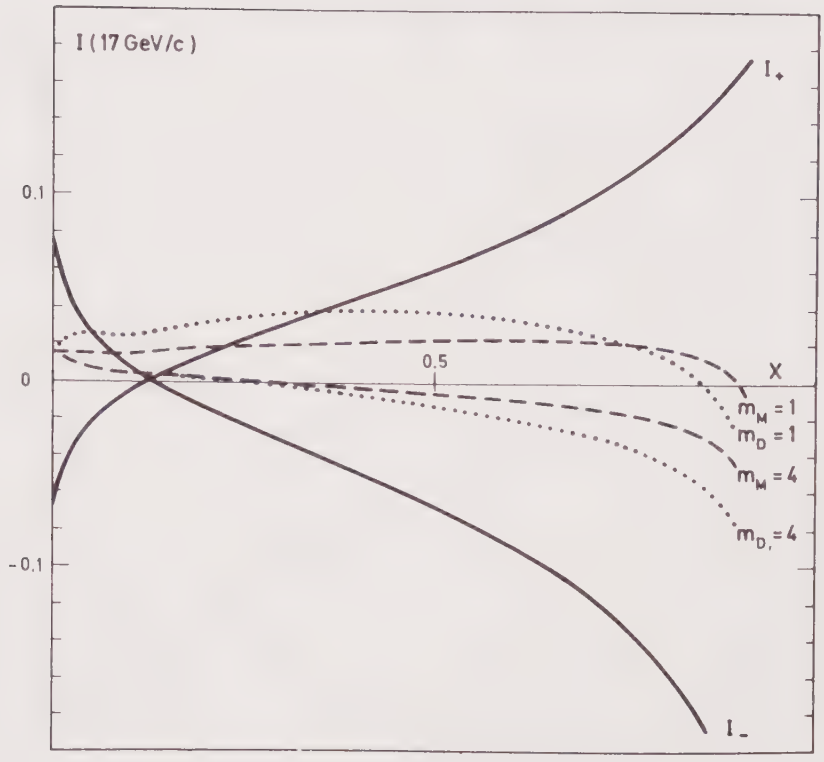

Fig. 5. Same as fig. 1 for singlet structure functions.

Note that these $I(17\ \text{GeV})$ curves are obtained with the same Λ_{eff} value as in the non-singlet case. The same compensation phenomenon between coupling constant and kernel variations is observed. The effect of these modifications of the kernel is exhibited in fig. 6 at $Q_0 = 2$ GeV for 1 GeV gluino mass.

The above results show that all the present deep inelastic data can be explained with the same accuracy with or without light gluinos. One can expect that increasing the maximum available value of Q would enhance the difference between the two possibilities. However, this description has to be modified if one goes above the scalar quark threshold. Thus in the absence of very accurate data in the present Q range, one cannot exclude the existence of gluinos. In this case one can predict their distribution inside the nucleon. If fig. 7 are plotted the gluon and gluino distributions as functions of x at $Q = 17$ GeV for $m_\lambda = 1$ GeV. For higher values of gluino mass, we have

$$G(x, m_\lambda = 1) < G(x, m_\lambda) < G_{\text{st}}(x), \tag{4.7a}$$

$$0 < L(x, m_\lambda) < L(x, m_\lambda = 1). \tag{4.7b}$$

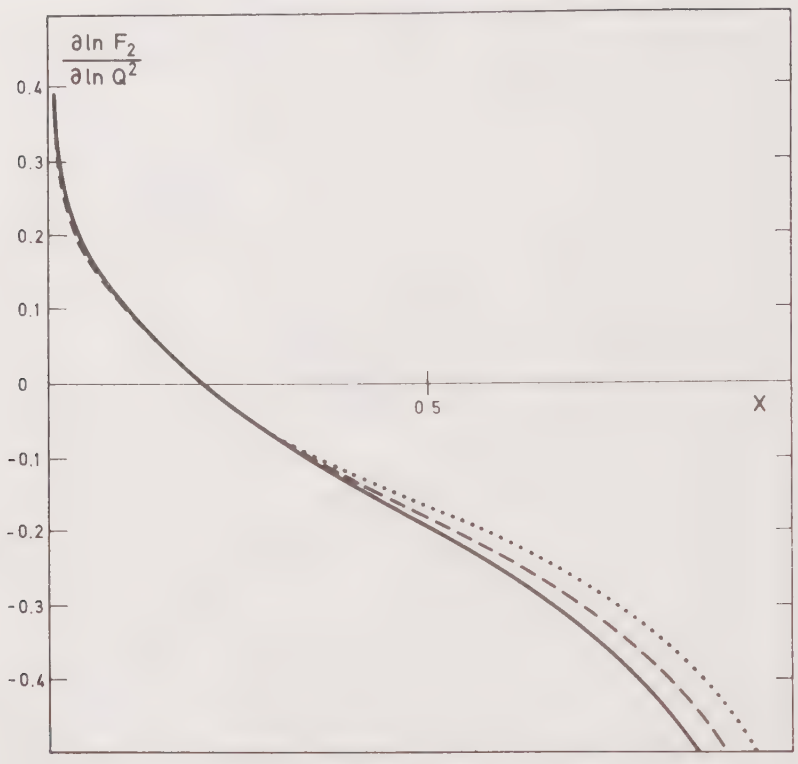

Fig. 6. Same as fig. 3 for F_2.

We finally calculate the energy fraction carried by quarks, gluons and gluinos, which is defined from the second moment of $q(x, Q)$, $G(x, Q)$ and $L(x, Q)$. In table 3 we give the numerical fractions at some Q values for different gluino masses (and in the standard case) when starting at $Q_0 = 2$ GeV from an initial energy fraction of 47% as carried by the quarks and of 53% by the gluons. The whole energy evolution in the most significant case ($m_\lambda = 1$ GeV) is shown in fig. 8. In this figure as well in table 3 one observes that almost all the energy fraction of the gluinos is transferred from the gluons, while the energy fraction of the quarks remains essentially unchanged. On the other hand the energy fraction of the Dirac gluinos is two times bigger than that of Majorana ones.

This energy distribution we have obtained is quite different from that of ref. [10]. There are at least two reasons for that. First the different initial energy fraction of quarks and gluons (we took respectively 47% and 53% as given by experiment), also the second-order effects we have taken into account. Let us also note that there is no meaning in doing extrapolations at infinite Q if one does not take into account the other particles of the supersymmetric theory.

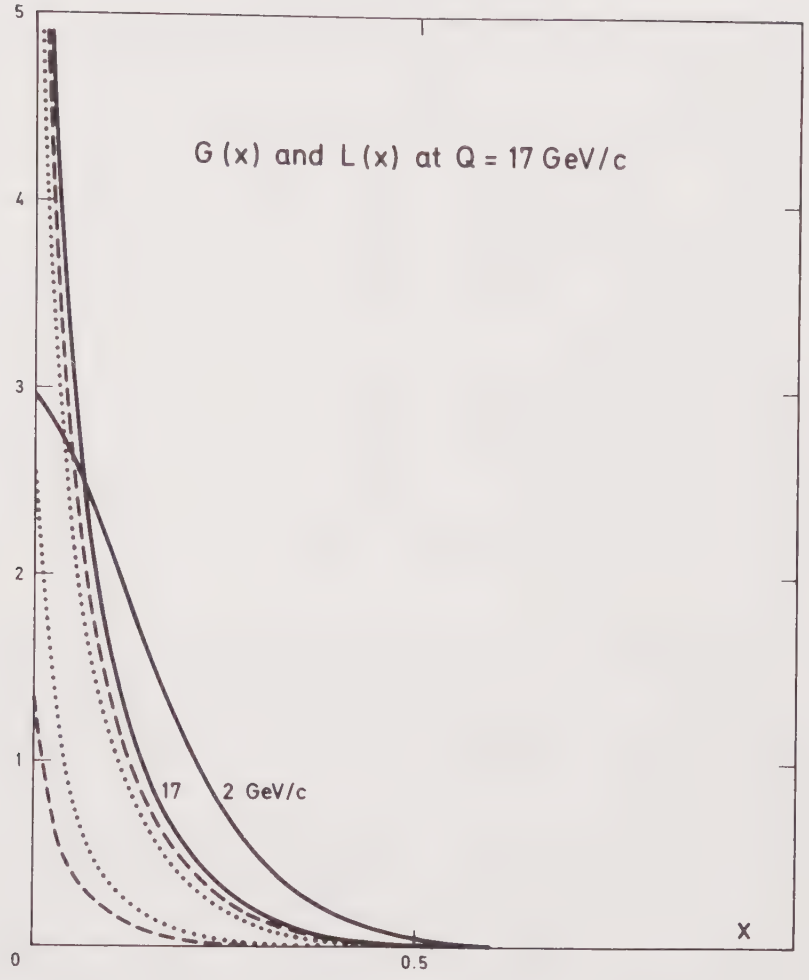

Fig. 7. Gluon (upper lines) and 1 GeV mass gluino (2 lowest lines) distributions at $Q = 17$ GeV. Different curves as in fig. 2.

As the net effect of gluinos on the structure function evolution is small, it is not necessary to introduce accurate threshold effect. So it seems reasonable to start without any intrinsic gluino distribution below their threshold. Moreover this has the advantage that we can directly take the "experimental" gluon distribution. However, it is clear that a small initial gluino distribution does not change our final results for scaling violation.

As a limiting case we want to ask if the deep inelastic data can exclude the possibility of massless gluinos. Our analysis of non-singlet structure functions with $m_\lambda = 1$ corresponds in fact to the situation $m_\lambda \leqslant 1$ with $\Lambda^S = 0.05$ or $\Lambda^S = 0.00026$ (Majorana or Dirac gluinos). Thus the observed scaling violation of xF_3 is compati-

TABLE 3
Numerical fraction (%) of the nucleon energy carried by quark (q), gluon (G) and gluino (L)

		m_{Majorana}					m_{Dirac}			
		St	1	2	3	4	1	2	3	4
	q	46.6	46.6	46.6	46.6	46.6	46.6	46.6	46.6	46.6
$Q = 4$	G	53.4	50.0	53.4	53.4	53.4	46.5	53.4	53.4	53.4
	L	0	3.4	0	0	0	6.9	0	0	0
	q	46.2	46.0	46.2	46.3	46.2	45.9	46.3	46.3	46.2
$Q = 8$	G	53.8	48.3	51.1	52.8	53.8	42.8	48.4	51.8	53.8
	L	0	5.7	2.7	0.9	0	11.3	5.3	1.9	0
	q	46.5	46.1	46.4	46.5	46.5	45.6	46.3	46.5	46.5
$Q = 16$	G	53.5	46.9	49.1	50.4	51.3	40.3	44.7	47.0	49.1
	L	0	7.0	4.5	3.1	2.2	14.1	9.0	6.5	4.4
	q	46.8	46.2	46.6	46.7	46.8	45.4	46.3	46.6	46.7
$Q = 32$	G	53.2	45.8	47.6	48.7	49.4	38.6	41.9	44.1	45.7
	L	0	8.0	5.8	4.6	3.8	16.0	11.8	9.3	7.6

ble with massless gluinos. As concerns the singlet case, one needs to extract initial gluon and gluino distributions from experimental data. According to the observation we have made before, about the energy transfer from gluons to gluinos, it seems that one could transfer part of the gluon distribution to gluinos. As an example we have arbitrarily assigned $\frac{1}{3}$ of distribution (4.5b) to gluinos, and $\frac{2}{3}$ to gluons. The corresponding quantities $I(17)$ [eq. (4.1)] then take values around 10% for $x = 0.05$ and 3% for $x = 0.5$. The large change of 10% near $x = 0$ is only reached for large Q values where there are no corresponding data. So this massless gluino situation seems to require a direct confrontation with data. However, let us note that in a first-order analysis [in which gluinos do not contribute directly to F_2 evolution – see eqs. (2.15) and (3.12)] the obtained energy fractions of gluons and quarks do not leave very much room for gluinos. As it would be unlikely that second-order effects drastically change this description, massless gluinos are not favoured by data.

5. Conclusion

In this work we have extended the standard QCD predictions for the scaling violation of the deep inelastic structure functions assuming the presence of light gluinos with a mass smaller than 10 GeV. We give all calculable quantities (evolution kernels, coefficient functions and anomalous dimensions) relative to the second-order analysis, in a renormalization scheme which preserves supersymmetry ($\overline{\text{DR}}$).

We show that the existence of light gluinos (Majorana or Dirac) can explain the present experimental data with the same accuracy as the standard second-order QCD analysis performed with or without heavy gluinos. This result is obtained with

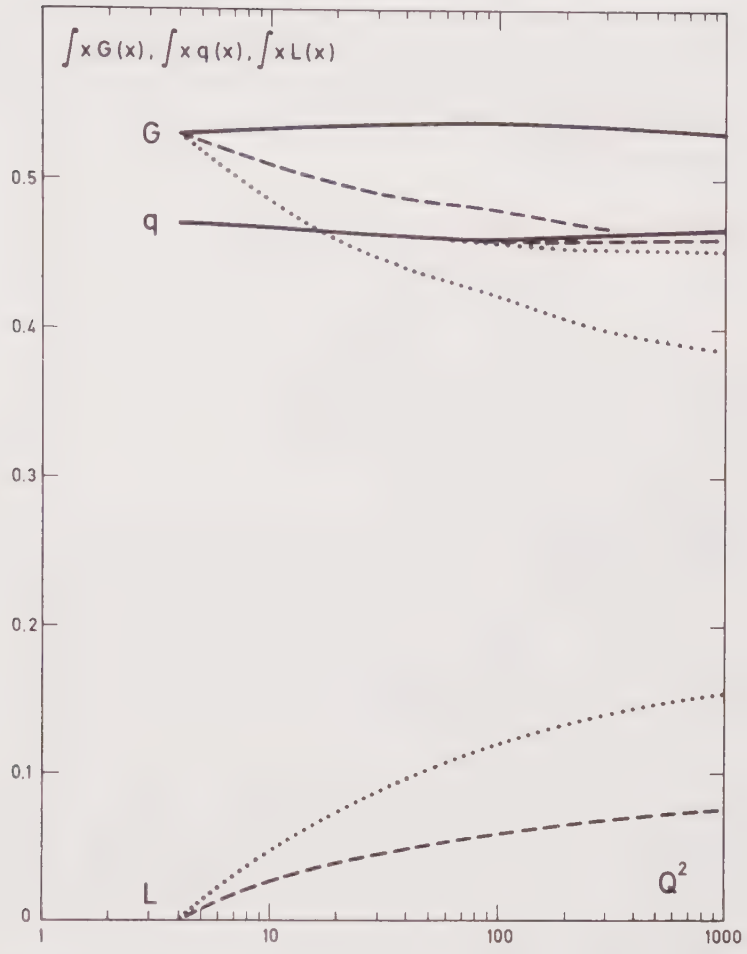

Fig. 8. Q evolution of the energy fraction carried by quarks (q), gluons (G) and gluinos (L). Different curves as in fig. 2.

a single mass scale parameter Λ_{eff} or $\Lambda_{\overline{\text{MS}}}$ which controls the Q^2 behavior of the strong coupling constant for Q below or above the gluino threshold.

We also derive the gluon $G(x, Q)$ and gluino $L(x, Q)$ distributions inside the nucleon for different values of the gluino mass (Majorana or Dirac). These distributions can be used for further analysis on the gluino hadronic bound-state production or to find predictions in some other processes (Drell-Yan or high $P_\perp$, etc), where these quantities may play a significant role. In that case a well defined lower bound or even a precise determination of the gluino mass can be obtained.

Note that from the above analysis in the finite region of $Q = 2$–17 GeV we predict the behavior and the evolution of the structure functions, gluon and gluino distribution for Q larger than 17 GeV which may be verified in future experiments.

Such gluinos have to be taken into account in supersymmetric GUT predictions like the proton lifetime or $\sin^2\theta_w$. Our results imply that the strong coupling constant has to be defined with a supersymmetric scale Λ^S which takes a very different value of the low-energy effective scale $\Lambda_{\text{eff}} \sim \Lambda_{\overline{\text{MS}}}$. For light gluinos our Λ^S determination can give significant effects.

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SUPERSYMMETRIC ENHANCEMENT FACTOR FOR THE 1-JET CROSS-SECTION IN p-$\bar{p}$ REACTIONS

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ABSTRACT :

We compare the 1-jet inclusive cross-section at high $p_\perp$ in proton-anti-proton reaction at SPS collider predicted by standard QCD and by its simplest supersymmetric extension (SQCD). We have analyzed two cases :

(i) $p_\perp$ is small compared to the masses of squarks, and only light gluinos (2 GeV) are considered. The enhancement factor between QCD and SQCD predictions is found to be small (of order 1.3).

(ii) $p_\perp$ is large compared to the mass of squarks (17 GeV in our analysis) and both gluinos and squarks are taken into account. Then, the enhancement factor gets much larger. It is roughly equal to 2 for $p_\perp$ = 90 GeV, and it is an increasing function of the transverse momentum $p_\perp$.

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1. INTRODUCTION

In recent years supersymmetric models have emerged as possible realistic extensions of the standard SU(3)xSU(2)x U(1) model of fundamental interactions[1]. The supersymmetric(SUSY) models have a richer particle content than the standard model, and they have been built such that they reproduce the low energy predictions of the standard model. With the increase in energy of present experiments, one may hope to be able to probe the existence of the new SUSY particles.

In this paper we analyse the 1-jet production cross section in p-$\bar{p}$ collisions at CERN, where the center of mass energy is large ($\sqrt{s}$ = 540 GeV), and allows for jets with a large transverse momentum as compared to the existing mass limits on SUSY particles. Indeed, if the supersymmetric partners of gluons (gluinos) and quarks (squarks) exist, one expect a jet cross section larger than if they are absent. This is true for 2 reasons :

i) new production channels open,

ii) the running coupling constant $\overline{\alpha}_s$ increases.

Using the parton model hypothesis and parton densities measured from deep inelastic lepton-hadron scattering data, we have computed the enhancement factor in two cases :

i) The squark mass is large enough for the proton content to be modified only by the presence of gluinos.

ii) Gluinos and squarks are taken into account. We found a significant effect in the second case only (a factor 2 to 3 when $p_\perp$ is 80 to 150 GeV).

One should note that our computation is by nature sensitive to the parton model uncertainties, which are not fully controlled in p-$\bar{p}$ reactions analysis. However our purpose is mainly to compare the QCD and super QCD predictions, and the parton model uncertainties should play a minor role in the enhancement factor which is a ratio of cross sections. Finally, let us point out that the smallness of the enhancement factor when only gluinos are considered can be traced back to supersymmetry relations between parton scattering cross sections in the dominant channels.

The paper is organized as follows. In section (2) we expose our strategy for computing the 1 jet cross-section in p-$\bar{p}$ collisions. Detailed analytical

formula are displayed in section (3). In particular we show the existence of supersymmetric relations making the comparison between the standard and SUSY predictions easier. In the last section our numerical results are studied in details.

2. THEORETICAL FRAMEWORK AND QUALITATIVE DISCUSSION

If strong interactions are described by an arbitrary asymptotically free (A.F.) theory, then the proton should appear as made of quasi independent partons in short distance experiments. This picture has been rigorously proven in deep inelastic lepton-hadron inclusive reactions and in e^+e^- semi inclusive annihilation[2] , and is widely believed to hold in hadron-hadron collisions such as $p\text{-}\bar{p}$ reactions in some specific kinematic regions for the latter reactions, the theoritical status is less clear than for lepton-hadron and e^+e^- reactions, but the present experimental data are quite in favor of a parton description of the proton in hadron-hadron high energy collision[3,4].

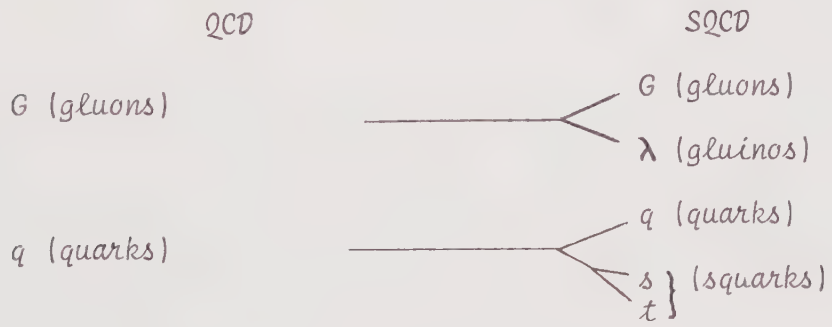
Deep inelastic lepton-hadron collisions can be clearly described despite our ignorance as to the way partons bind into hadrons. One can rigorously define parton distributions which express the probability of finding partons inside a hadron at a given Q^2, and the lepton-hadron cross-section is simply obtained by doing the convolution of these parton distributions with the parton-lepton cross-sections[5]. The latter quantities are computable at any given order in perturbation theory, the expansion parameter being the running coupling constant of the A.F. theory $\bar{\alpha}_s(Q^2)$. The energy dependance of the parton distributions is governed by evolution equations, so that once they have been determined at some energy, they can be computed at any other energy. Asymptotically, one recovers all results of the original Feynman parton model. One should note however that the above description is only valid far from thresholds.

For the hadron-hadron reactions, the same picture is expected to hold true, although a rigorous proof of the relevant factorization theorem is still lacking. One assumes therefore that the cross-sections are convolutions of the parton distributions extracted from lepton-hadron deep inelastic experiments with computable parton-parton cross-sections. This approach has been followed in ref (4) to study the large $p_\perp$ 1-jet inclusive cross-section in $p\text{-}\bar{p}$ collisions, assuming that QCD describes the parton interactions.

Chromodynamics (Q.C.D.) is the simplest candidate for an asymptotically free theory of strong interactions. It is based on the gauge group SU(3) and the partons consist of a vector gluon octet mediating the strong interaction force and of quarks belonging to the fundamental representation of SU(3) color. This theory accomodates quite well the present experimental data. In this framework the gluon and quark densities have been consistently extracted from deep inelastic lepton-hadron structure functions. The quark density measurement is direct , whereas the gluon density requires an analysis of scaling violations[6]. In turn, these distributions have allowed a reasonable parametrization of other deep inelastic experiments.

What about asymptotically free supersymmetric theories of strong interactions ? In these theories, new particles are associated with the quarks and gluons according to the boson-fermion symmetry, but the parton picture works exactly as in QCD.

In the N = 1 supersymmetric realistic models (called SQCD), the color gauge group is still SU(3). There is an octet of gluons mediating the strong interactions, and its supersymmetric partner is an octet of Majorana spinors. A pair of complex scalar quarks belonging to the fundamental representation of SU(3) is associated with each ordinary quark. The passage from QCD to SQCD is symbolized by the picture



and the classical SQCD Lagrangian reads :

$$\mathcal{L}_{SQCD} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \bar{\lambda}^a \not{D} \lambda^a + i \bar{q}_i \not{D} q_i + (D_\mu s_i)^+ (D_\mu s_i) +$$

$$+ (D_\mu t_i)^+ (D_\mu t_i) + ig\sqrt{2} \left(\bar{\lambda}^a \frac{1+\gamma_5}{2} s_i^+ T^a q_i + \bar{\lambda}^a \frac{1-\gamma_5}{2} t_i^+ T^a q_i - \right. \quad (2.1)$$

$$\left. - h.c. \right) - \frac{1}{2} g^2 \left(s_i^+ T^a s_i - t_i^+ T^a t_i \right)^2 +$$

$$+ \text{mass terms} \quad ,$$

where D_μ is the covariant derivative and $F^a_{\mu\nu}$ is the field strength of the gluon field. The index i runs over all flavors, a is the color index and T_a are the SU(3) generators in the fundamental representation.

Existing data have already provided mass limits for the squarks[8] and gluinos[9,10]. e^+e^- reactions offer the cleanest way to detect charged new particles. The absence of new thresholds at PETRA provides a lower bound for the squark mass of about 17 GeV. Calorimeter and beam dump experiments on one hand and, on the other hand, the absence of production of bound states pairs involving gluinos in e^+e^- collisions and in heavy quarkonium decays, show that the gluino mass must be higher than about 2 GeV[11].

Lepton-hadron deep inelastic reactions may appear as a way to provide a hint for gluino existence (when $Q \gtrsim 4$ GeV). Indeed, the particle content of SQCD changes the coefficient of the one loop β - function, so that, in principle, by analyzing flavour non singlet structure functions, a precise measurement of the change with Q^2 of the running coupling constant could distinguish between QCD and SQCD. In addition, the two loop anomalous dimension kernel which governs scaling violations changes from QCD to SQCD[10]. However, the effects are only logarithmic, and the Q^2 range is too small to allow for a measurement of the slope of the running coupling constant. Further, there is a compensation between the β-function variation and the change of kernel, and, in conclusion, the deep inelastic data can be fitted as well in the QCD as in the SQCD framework. The QCD analysis provides densities for the up and down valence quarks, sea quarks and gluon which characterize the proton in this theory. In a SQCD analysis, the squarks must be

neglected, because their mass limit from PETRA is above the present $\sqrt{Q^2}$ range of experiments. If one assumes the existence of light gluinos, in addition to the usual partons, one finds that quarks densities are nearly equal to those of QCD, and that the sum of gluon and gluino densities is nearly equal to the gluon density of the QCD analysis. These results follow from a next to leading logarithmic analysis[10]. In order to predict the shape of squarks densities at larger values of Q^2, an approximate treatment of threshold effects has been proposed in ref. (12), where values of squarks densities at high Q^2 compatible with low Q^2 data have been given.

Let us come to the study of high $p_\perp$ 1-jet production in p-$\bar{p}$ reactions, and show that its cross-section is quite sensitive to the parton content of the proton, and should be a meaningfull observable to probe the possible existence of supersymmetric partners of gluons and quarks. In a parton model approach, the cross section for the production of a jet with transverse momentum $p_\perp$ can be symbolically written as[3] :

$$\frac{d\sigma}{dp_\perp d\cos\theta} = \sum_{a,b} \sum_{i,j} q_i^p \otimes \frac{d\hat{\sigma}_{ij}^{ab}}{dt} \otimes q_j^{\bar{p}} \quad , \qquad (2.2)$$

where the indices i,j,a,b stand for types of partons in the proton. q_i^p and $q_i^{\bar{p}}$ are the densities of parton i in the proton and in the antiproton. $\frac{d\hat{\sigma}_{ij}^{ab}}{dt}$ is the differential cross section for the reaction :

$$\text{parton i + parton j} \longrightarrow \text{parton a + parton b} \quad ,$$

where the parton a carries a transverse momentum $p_\perp$ and is emitted at an angle θ with respect to the beam axis (fig(1)). $\frac{d\hat{\sigma}_{ij}^{ab}}{dt}$ is proportionnal to the squared running coupling constant $\bar{\alpha}_s$ of the A.F. theory describing the partons strong interactions. The details of formula (2.2), and the definition of the convolution $\otimes$ will be given in section 3.

Now, supposing that SQCD is the basic strong interaction theory, one expects gluino jets to be produced (with $p_\perp \gtrsim 4$ GeV) from processes involving ordinary ingoing partons, for instance through the reactions : $G+G \longrightarrow \lambda+\lambda$ or $q+\bar{q} \longrightarrow \lambda+\lambda$. These processes exist even if the gluino density in the proton is very small and, moreover, are of the same order of magnitude as $G+G \longrightarrow G+G$, or $q+\bar{q} \longrightarrow G+G$. Thus, one expects the jet production to be substantially larger for

SQCD than for QCD. Furthermore, the numerical value of the squared running coupling constant of SQCD has been found to be larger than that of QCD by about 25 % in the next to leading order analysis of ref.(10). Since the jet cross-section is proportionnal to the squared coupling constant, one may therefore conclude that all factors contribute to enlarge the jet cross-section in p-$\bar{p}$ reactions when passing from QCD to SQCD. Obviously, in order to implement quantitatively the enhancement factor, the change in parton densities when lepton-hadron scattering is analyzed with SQCD instead of QCD must be taken into account.

Consequently we have computed all parton-parton cross-sections involved in eq.(2.2) in SQCD, extending the QCD results of refs.(13,14). We have assumed the gluinos to be light (2 GeV), and have considered both cases when the squarks are taken into account or not. Then, using previous estimations of partons densities from lepton hadron deep inelastic scattering[10], we have computed the 1-jet cross section.

3. *THE 1-JET FORMULA IN THE p-$\bar{p}$ REACTION*

Let us consider the reference frame in which the proton and the antiproton are collinear with opposite momenta $p = \frac{\sqrt{s}}{2}$. We neglect the proton mass with respect to the center of mass reaction energy $\sqrt{s}$. We want to compute the semi-inclusive p-$\bar{p}$ cross-section when one hadronic jet is observed with the transverse momentum $p_\perp$ at an angle θ with respect to the beam axis (fig.1). In the parton model leading approximation, this cross-section reads :

$$\frac{d\sigma}{dp_\perp\, d\cos\theta} = \frac{2\pi p_\perp}{\sin^2\theta} \sum_{i,j} \sum_{a,b} \bar{\alpha}_s^2(p_\perp^2)\, \frac{1}{s^2} \int_0^1 dx_1 \int_0^1 dx_2\, \delta\left[x_1 x_2 - \frac{p_\perp}{\sqrt{s}}\left(x_1 \operatorname{tg}\frac{\theta}{2} + x_2 \operatorname{cotg}\frac{\theta}{2}\right)\right]$$

$$\cdot\, q_i^p\left(x_1, \bar{\alpha}_s(p_\perp^2)\right) \left| A_{ij}^{ab}(x_1, x_2)\right|^2 q_j^{\bar{p}}\left(x_2, \bar{\alpha}_s(p_\perp^2)\right) , \qquad (3.1)$$

where $\bar{\alpha}_s(p_\perp^2)$ is the running coupling constant of the A.F. theory describing the constituent partons strong interaction. The choice of $p_\perp$ as the dimensioned parameter in $\bar{\alpha}_s$, though widely used (14,4), cannot be rigorously justified. Using

another parameter would not alter significantly our conclusions. The Feynman amplitude A_{ij}^{ab} refers to the process :

$$\text{parton i + parton j} \longrightarrow \text{parton a + parton b} \quad , \tag{3.2}$$

where the partons i and j are collinear to the p-$\bar{p}$ beam and carry respectively the longitudinal momenta $x_1 \frac{\sqrt{s}}{2}$ and $-x_2 \frac{\sqrt{s}}{2}$. Parton a builds the observed hadronic jet of momentum $(p_\perp, \theta)$ and we sum over all partons species. Densities $q_i^p\left(x_1, \bar{\alpha}_s(p_\perp^2)\right)$ and $q_j^{\bar{p}}\left(x_2, \bar{\alpha}_s(p_\perp^2)\right)$ are the probabilities that the proton and the antiproton respectively emit partons i and j with momenta $x_1 \frac{\sqrt{s}}{2}$ and $-x_2 \frac{\sqrt{s}}{2}$. We have assumed no intrinsic transverse momentum dependence in the parton densities.

3.1. *Kinematical analysis*

For the reaction (3.2), the phase space constraints are such that :

$$x_2 = \frac{x_1 P_\perp/\sqrt{s} \; tg\frac{\theta}{2}}{x_1 - P_\perp/\sqrt{s} \; \cot g \frac{\theta}{2}} \quad , \tag{3.3a}$$

$$x_{min} = \frac{P_\perp/\sqrt{s} \; \cot g \frac{\theta}{2}}{1 - P_\perp/\sqrt{s} \; tg\frac{\theta}{2}} \leqslant x_1 \leqslant 1 \quad . \tag{3.3b}$$

Then formula (3.1) can be written under the following form, well suited for practical computations :

$$\frac{d\sigma}{dP_\perp \, d\cos\theta} = \frac{2\pi}{s^{3/2}} \frac{1}{\sin^2\theta \; tg\frac{\theta}{2}} \bar{\alpha}_s^2(P_\perp^2) \int_{x_{min}}^{1} \frac{dx_1}{x_1^2} \cdot$$

$$\cdot \sum_{i,j} \sum_{a,b} q_i^p\left(x_1, \bar{\alpha}_s(P_\perp^2)\right) \left| A_{ij}^{ab}(x_1, x_2) \right|^2 q_j^{\bar{p}}\left(x_2, \bar{\alpha}_s(P_\perp^2)\right) \quad . \tag{3.4}$$

The amplitudes A_{ij}^{ab} must be computed from all Feynman diagrams contributing to the process (2.2), and the squared amplitudes $\left| A_{ij}^{ab} \right|^2$ are functions of the Mandelstam variables $\hat{u}, \hat{s}, \hat{t}$:

$$\hat{s} = (P_i + P_j)^2 = x_1 x_2 s \quad ,$$

$$\hat{t} = (P_i - P_a)^2 = - x_1 P_\perp \sqrt{s}\ \mathrm{tg}\frac{\theta}{2} \quad ,$$

$$\hat{u} = (P_i - P_b)^2 = - x_2 P_\perp \sqrt{s}\ \mathrm{cotg}\frac{\theta}{2} \quad . \tag{3.5}$$

3.2. *SQCD without squarks*

From PETRA results, the squark mass is larger than 17 GeV. For consistency, one must neglect their contribution for values of $p_\perp$ smaller than twice this limit, i.e. $p_\perp \lesssim 34$ GeV. We have computed all amplitudes $|A_{ij}^{ab}|^2$ when the partons are quarks, antiquarks, gluons and gluinos, in the zero mass limit. The relevant Feynman rules can be found from Lagrangian (2.1). These Born squared amplitudes are displayed in Table 1. They are dimensionless and depend on $\hat{s}$, $\hat{t}$ and $\hat{u}$ through the combinations :

$$\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2} \ , \ \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \ , \ \frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} \ , \ \frac{\hat{s}^2}{\hat{u}\hat{t}} \ , \ \frac{\hat{t}^2}{\hat{u}\hat{s}} \ , \ \frac{\hat{u}^2}{\hat{s}\hat{t}} \quad . \tag{3.6}$$

In the kinematical domain characterized by eq.(3.3), and when $p_\perp \ll \sqrt{s}$, there are strong inequalities among the quantities in eq. (3.6), provided that $\frac{\sqrt{s}}{p_\perp} \gg \mathrm{tg}\frac{\theta}{2} \gg \frac{p_\perp}{\sqrt{s}}$, and that x_1 is not too close to x_2. These inequalities express that either

$$\hat{s} \sim |\hat{t}| \gg |\hat{u}| \sim p_\perp^2 \tag{3.7a}$$

or

$$\hat{s} \sim |\hat{u}| \gg |\hat{t}| \sim p_\perp^2 \tag{3.7b}$$

and therefore :

$$\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \ \text{or} \ \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2} \gg \frac{\hat{s}^2}{\hat{u}\hat{t}} \ , \ \frac{\hat{t}^2}{\hat{s}\hat{u}} \ , \ \frac{\hat{u}^2}{\hat{s}\hat{t}} \ , \ \frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2} \quad . \tag{3.8}$$

The inequalities (3.8) are illustrated in fig (2) for $\sqrt{s} = 540$ GeV, $\theta = 90°$,

$p_\perp = 20$ GeV.

When only the dominant terms on the left side of (3.8) are retained in the Born squared amplitudes, their form gets much simpler. This approximation can be diagrammatically formulated as follows :

i) retain only the $\hat{u}$ and $\hat{t}$ channels diagrams contributing to the Born amplitudes A_{ij}^{ab} , and, among them, neglect those with the exchange of a fermion (quark or gluino).

ii) When computing the squared amplitudes, neglect all interference terms.

Let us note that the above approximation is coherent with the choice of $p_\perp^2$ as the scale of the hard scattering process among partons. Indeed, according to eq. (3.7), $p_\perp^2$ is precisely the order of magnitude of the squared virtual mass of the particule exchanged in the hard process.

Since we neglect squarks, the relevant parton densities q_i in the proton are the up and down quark densities u and d, the sea quarks density Σ and the gluon and gluino densities G and λ . The sea quarks density is uniformly distributed among all flavors, so that :

$$\Sigma = c = s = \bar{c} = \bar{s} = \bar{u} = \bar{d} \quad ,$$

where c and s stand for the charmed and strange components of the sea. The symbol $\bar{}$ stands for the antiparticle components ; u and d are the complete up and down quark densities, equal to the sum of the valence and sea quark densities. The anti-proton densities can be deduced from those of the proton by charge conjugation ($u \leftrightarrow \bar{u}$, $d \leftrightarrow \bar{d}$, $\Sigma \leftrightarrow \Sigma$, $G \leftrightarrow G$, $\lambda \leftrightarrow \lambda$).

It is convenient to build a 5-vector $Q = (u, d, \Sigma, G, \lambda)$ with the parton densities as components. Then formula (3-4) can be written in a matricial form :

$$\frac{d\sigma}{dp_\perp \, d\cos\theta} = \frac{2\pi}{s^{3/2}} \frac{1}{\sin^2\theta \; tg\frac{\theta}{2}} \bar{\alpha}_s(p_\perp^2) \int_{x_{min}}^{1} \frac{dx_1}{x_1^2} \, Q^t(x_1) \, A(\hat{s}, \hat{t}, \hat{u}) \, Q(x_2) \quad , \tag{3.9}$$

where x_2 and $\hat{s},\hat{t},\hat{u}$ are given in eq. (3.3) and (3.5), and A is a 5x5 matrix with components :

$$A(\hat{s},\hat{t},\hat{u})_{ij} = \sum_{a,b} \left| A^{ab}_{ij}(x_1,x_2) \right|^2 \quad . \tag{3.10}$$

This matrix is symmetric and invariant when $\hat{u} \leftrightarrow \hat{t}$. We have computed its elements, and the result is displayed in table 3-a. If the gluino sector is left out, the QCD result of ref. (13) is recovered and displayed in table 2-a.

In the region $p_\perp \ll \sqrt{s}$ we are interested in, one may take advantage of the inequalities (3-8). Then the matrix elements of A get much simpler. Tables 2-b and 3-b display the matrix A', approximation of the matrix A when eqs. (3.8) are used.

The most interesting result in this approximation is the following set of relations between the matrix elements of A' (table 3-b) :

$$\begin{aligned} A'_{Gu} &= A'_{\lambda u} = A'_{uG} = A'_{u\lambda} \quad , \\ A'_{Gd} &= A'_{\lambda d} = A'_{dG} = A'_{d\lambda} \quad , \\ A'_{GG} &= A'_{\lambda\lambda} = A'_{G\lambda} = A'_{\lambda G} \quad . \end{aligned} \tag{3.11}$$

These equalities allow one to reduce the parton density 5-vector (u,d,Σ,G,λ) to a 4-vector $(u,d,\Sigma,G+\lambda)$ and the 5x5 matrix A' to a 4x4 matrix. Moreover, this 4x4 reduced matrix is precisely equal to the A' matrix one would compute in QCD in the same approximation. Consequently, provided this approximation is correct, the QCD and SQCD predictions are easy to compare : one can use the same formula up to the changes $\bar{\alpha}_{QCD} \leftrightarrow \bar{\alpha}_{SQCD}$ and $G_{QCD} \leftrightarrow G_{SQCD} + \lambda_{SQCD}$. On the other hand, the numerical analysis of ref.(10) shows that the gluon density in QCD is nearly equal to the sum of the gluon and gluino densities in SQCD on the whole $\sqrt{Q^2}$ range studied ($4 \leqslant \sqrt{Q^2} \leqslant 120$ GeV). It follows that the enhancement factor from QCD to SQCD is governed only by the change of running coupling constant. In section 4 we will verify quantitatively the relevance of this approximation. However, we have calculated the ratio of the 1-jet inclusive cross-sections in QCD and SQCD using

the complete matrix A, without any approximation, and we have found that it is given by the ratio of the squared running coupling constants. The approximation (3.8) only helps us to interpret this result.

3.3. *SQCD with squarks*

Supposing the existence of squarks, and a large enough value of $P_\perp$, one must introduce an 8-vector to describe the parton densities in the proton $Q = (u, u_s, d, d_s, \Sigma, \Sigma_s, G, \lambda)$ where the symbol s indexes the supersymmetric partners of a given quark or antiquark. Besides the A matrix in eq. (3.9) is now 8x8. When they are computed in the zero mass limit, there are relations between the matrix elements of A, because supersymmetry is now exact. Thus A can be reduced to an effective 4x4 matrix A_S whereas the vector of densities is reduced to a 4-vector $Q_S = (u + u_s, d + d_s, \Sigma + \Sigma_s, G + \lambda)$. The A_S matrix is displayed in table 3.

The approximation (3.8) turns out to be very bad in this case, contrarily to the case of section 3.2 where squarks were neglected. It is however interesting to note that the A_S matrix when approximation (3.8) is used is identical to the corresponding approximated matrix computed in pure QCD.

Once the A_S matrix has been computed, the main problem is to estimate the squarks densities from low energy results. Obviously, squarks are not massless and the difficulty to overcome is our ignorance about the threshold effects they induce. To keep simple, we have used quark + squark and gluon + gluino distributions respectively equal to the quark and gluon distributionsof standard QCD multiplied by a Q^2-dependent proportionality factor :

$$\left[G(x,Q^2) + \lambda(x,Q^2)\right]_{SQCD} = G_{QCD}(x,Q^2) \cdot f_G(Q^2) \quad , \tag{3.12a}$$

$$\left[q(x,Q^2) + s(x,Q^2)\right]_{SQCD} = q_{QCD}(x,Q^2) \cdot f_q(Q^2) \quad , \tag{3.12b}$$

with :

$$f_G(Q^2) = \frac{\left[G^{(2)} + \lambda^{(2)}\right]_{SQCD}}{G^{(2)}_{QCD}} \quad , \quad f_q(Q^2) = \frac{\left[q^{(2)} + s^{(2)}\right]_{SQCD}}{q^{(2)}_{QCD}} \quad , \tag{3.13}$$

where the index (2) refers to the second moment of the corresponding density. $G^{(2)}_{QCD}$ and $q^{(2)}_{QCD}$ are very slowly varying functions of Q^2 and their approximate values in the Q^2 range we explore are : $G^{(2)}_{QCD} = .53$, $q^{(2)}_{QCD} = .47$. For $(G+\lambda)^{(2)}_{SQCD}$, $(q+s)^{(2)}_{SQCD}$, we used the leading order analysis of ref. (12), and followed their treatment of threshold effects. We have assumed the squark mass to be equal to the present lower bound $m_{s,t} = 17$ GeV.

We also checked that the precise form $G + \lambda$ and $q + s$ distributions does not influence significantly our results. For example taking $f_G(Q) = f_q(Q) = 1$, the jet cross sections are modified by less than 15 % (for $34 \lesssim p_\perp \lesssim 150$ GeV).

4. *NUMERICAL RESULTS*

We have computed the 1-jet cross section in eq. (3.9) as a function of $p_\perp$ and θ, using the QCD matrix (table 2) and the SQCD matrix without (table 3) and with (table 4) squarks. In all our computations, we have done the hypothesis of 4 flavors. Using 6 flavours should not change drastically our numerical results. How we obtained the partons densities has been described in the previous section.

We first discuss the case without squarks. In figs. 3,4 are plotted the production cross sections for the various partons as functions of the rapidity $y = \text{Arg th } (\cos\theta)$ for $p_\perp = 20$ GeV and $p_\perp = 100$ GeV. Fig. 3 describes the QCD case, and fig. 4 the SQCD case without squarks. In both cases the 1-jet cross section is dominated by the gluons at low $p_\perp$ and by the quarks at high $p_\perp$, while the gluino production is always small (of the order of 10 %).

In fig. 5 we plot the total 1-jet inclusive cross section as a function of the rapidity y in the QCD and SQCD cases for $p_\perp = 60$ GeV and $p_\perp = 120$ GeV. We find that the shape in rapidity of the QCD and SQCD (without squarks) cross sections can hardly be distinguished. Therefore one can define an enhancement factor K between these cross sections which depends only on $p_\perp$. K is given in fig. 7 as a function of $p_\perp$. It increases slowly with $p_\perp$ and is roughly 1.3. As can be seen on this figure, up to $p_\perp \simeq 100$ GeV, $K(p_\perp)$ is approximatively

identical to the ratio : $\bar{\alpha}^2_{SQCD}(p_\perp^2) \,/\, \bar{\alpha}^2_{QCD}(p_\perp^2)$ (for the running coupling constants we have used the second order formula with $\Lambda_{\overline{MS}} = .2$ GeV in standard QCD with 4 flavors).

This indicates that the approximation (3.8), discussed in section 3.2, is good. We have also checked this approximation by replacing the matrix A in table 1 by A' in table 2. At $\theta = 90°$ and $p_\perp = 20$ GeV, the approximation reproduces 85 % of the total cross section for QCD and 82 % for SQCD. This percentage is almost independent from θ and it decreases very slowly with $p_\perp$. It is about 82 % and 79 % respectively at $p_\perp = 60$ GeV.

Finally fig. 6 shows the evolution of the total 1-jet cross sections as functions of $p_\perp$, at fixed angles $\theta = 90°$ and $\theta = 40°$.

For taking into account squarks, we have used the A-matrix in table 4. The partons densities ($u + u_s$, $d + d_s$, $\Sigma + \Sigma_s$, $G + \lambda$) which we use have been discussed in section 3-3. The running coupling constant is given by the one loop formula :

$$\bar{\alpha}_{SQCD}(p_\perp) = 4\pi \left[\frac{4\pi}{\bar{\alpha}_s(2m_s)} + (9 - N_F) \ln\left(\frac{p_\perp}{2m_s}\right)^2 \right]^{-1} ,$$

where $2m_s = 34$ GeV is the threshold for production of squarks, and $\bar{\alpha}_{SQCD}(2m_s)$ is calculated without squarks ; the second term incorporates the contribution of squarks to the β-function.

The corresponding total 1-jet inclusive cross-section is plotted (together with the two other cases) as a function of the rapidity y in fig. 5 and as a function of $p_\perp$ in fig. 6. The enhancement factor between QCD and SQCD with squarks, turns out to be quite large (of the order of 2 to 3). It is plotted in fig. 7 as a function of at = 90° and in fig. 8 as a function of the rapidity y at = 60 GeV.

The curves displayed in figs. (7,8) in the case of SQCD without squarks are for $N_F = 4$. With $N_F = 6$, cross sections are slightly different, but the ratios of SQCD versus QCD quantities are nearly identical. When squarks are included, the difference between $N_F = 4$ and $N_F = 6$ is more significant, and we have treated both cases in figs. (7,8).

CONCLUSION

In this paper, we have studied the enhancement factor that supersymmetric particles, gluinos and squarks would induce for the 1-jet inclusive cross section in $p\text{-}\bar{p}$ collisions. If the contribution of squarks is neglected, as it must be when $p_\perp$ is not too large ($p_\perp \lesssim 40$ GeV), this enhancement factor is not significant, even with light gluinos. For large values of $p_\perp$, supposing a squark mass equal to 17 GeV, we have found that the enhancement factor is quite large, of the order of 2 for $p_\perp = 80$ GeV and $\theta = 90°$. Although the calculated jet cross-sections are sensitive to the uncertainties related to the parton model, the enhancement factor should not depend drastically on them.

However the comparison with experiment cannot yet be performed because of large experimental uncertainties in the SPS collider at CERN[15], which are related to the definition of the jet cross section. Therefore the comparison would require quantities which do not depend on the absolute normalization of the cross-sections. For instance one could consider the shape of the jet cross-sections as functions of $p_\perp$ (see figure 6) provided a more precise measurement is performed.

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NOTE ADDED

We have compared the results for the squared Born amplitudes of partons cross sections with the expressions given by P.R. Harrison and C.H. Llewellyn Smith in ref.(15) . We agree with their results except for the processes $G + G \longrightarrow s_i + \bar{s}_i$ and $G + q_i \longrightarrow s_i + \lambda$.

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TABLE CAPTIONS

Table 1 : The Born squared amplitude for the process parton a + parton b $\longrightarrow$ parton c + parton d. The indices i,j are flavour indices. Color and spin are averaged. We have defined $T^{ab}_{cd} = \left| A^{ab}_{cd} \right|^2$. The symbols $q, \bar{q}$, s, $\bar{s}$, G, λ stand for quark, antiquark, squark, antisquark, gluon and gluino. The vertical bar connects the parton emitted with momentum $x_1 p$ from the parton with the parton a which builds the observed jet (this distinction is not necessary if the final or initial particles are identical). The Born squared amplitude for the process $q_i + q_i \longrightarrow \lambda + \lambda$ is different when squarks are considered (δ=1) or not (δ=0).

Table 2 : (2-a) The A matrix for eq. (3.9) in the QCD case, with the quantities from Table 1. We have used the notation $T^{ab}_{\nwarrow cd} = T^{ab}_{|cd} + T^{ab}_{\searrow cd}$. N_f is the flavour number.

(2-b) The same matrix in the approximation (3.8). We have defined

$$T = \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2} + \frac{\hat{t}^2 + \hat{s}^2}{\hat{u}^2} .$$

Table 3 : (3-a) The A matrix in the SQCD case without squarks.
(3-b) The same matrix in the approximation (3-8).

Table 4 : The A matrix in the SQCD case with gluinos and squarks.

FIGURE CAPTIONS

Figure 1

The jet cross section according to the parton model picture. The parton a is emitted with transverse momentum $p_\perp$ and angle θ in the p-$\bar{p}$ center of mass frame.

Figure 2

The dominant term $\frac{\hat{t}^2 + \hat{s}^2}{\hat{u}^2}$ as compared to other functions : $\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2}$, $\frac{\hat{t}^2}{\hat{s}\hat{u}}$, $\frac{\hat{s}^2}{\hat{t}\hat{u}}$ occuring in the Born squared amplitudes.

$p_\perp = 20$ GeV, $\theta = 90°$, $\sqrt{s} = 540$ GeV.

Figure 3

Production cross sections of the various partons as functions of the rapidity y in the standard QCD case for $p_\perp = 20$ GeV (3-a) and $p_\perp = 100$ GeV (3-b).

Figure 4

The same quantities as in fig.3 are plotted in the SQCD case without squarks.

Figure 5

Total 1-jet inclusive cross-section as a function of the rapidity y for $p_\perp = 60$ GeV (5-a) and $p_\perp = 120$ GeV (5-b) in the three cases : standard QCD (dotted line), SQCD without squarks (dashed line) and SQCD with squarks (solid line).

Figure 6

Total 1-jet inclusive cross section as a function of the transverse momentum $p_\perp$ for $\theta = 90°$ (6-a) and $\theta = 40°$ (6-b) in the three cases (conventions analogous to fig.5).

Figure 7

The enhancement factor between SQCD and QCD 1-jet cross sections is plotted versus $p_\perp$ for $\theta = 90°$ in the two cases : SQCD without squarks (dashed line) and SQCD with squarks (full line).
The ratio $\bar{\alpha}^2_{SQCD} / \bar{\alpha}^2_{QCD}$ is also plotted in the case without squarks (dotted line) and in the case with squarks (dashed-dotted line).

Figure 8

Rapidity distribution of the enhancement factor between QCD and SQCD for $p_\perp = 60$ GeV. The ratio $\bar{\alpha}^2_{SQCD} / \bar{\alpha}^2_{QCD}$ is also plotted (conventions analogous to fig.7).

$$T^{q_i q_j}_{q_i q_j} = T^{q_i \bar{q}_j}_{q_i \bar{q}_j} = \frac{4}{9}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \qquad (i \neq j)$$

$$T^{q_i q_i}_{q_i q_i} = \frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right) - \frac{8}{27}\,\frac{\hat{s}^2}{\hat{u}\hat{t}}$$

$$T^{q_i \bar{q}_i}_{q_j \bar{q}_j} = \frac{4}{9}\,\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} \qquad (i \neq j)$$

$$T^{q_i \bar{q}_i}_{q_i \bar{q}_i} = \frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right) - \frac{8}{27}\,\frac{\hat{u}^2}{\hat{s}\hat{t}}$$

$$T^{gg}_{q_i \bar{q}_i} = \frac{32}{27}\,\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}} - \frac{8}{3}\,\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2} = \frac{64}{9}\,T^{q_i \bar{q}_i}_{gg}$$

$$T^{q_i g}_{q_i g} = \frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2} - \frac{4}{9}\,\frac{\hat{u}^2+\hat{s}^2}{\hat{u}\hat{s}}$$

$$T^{gg}_{gg} = T^{\lambda\lambda}_{\lambda\lambda} = \frac{9}{4}\left(\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2} + \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2} + \frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} + 3\right)$$

$$T^{\lambda\lambda}_{q_i \bar{q}_i} = \frac{8}{3}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}\right) - \frac{8}{27}\,\delta = \frac{64}{9}\,T^{q_i \bar{q}_i}_{\lambda\lambda}$$

$$T^{\lambda g}_{\lambda g} = \frac{9}{4}\left(\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2} - \frac{\hat{u}^2+\hat{s}^2}{\hat{u}\hat{s}}\right)$$

$$T^{gg}_{\lambda\lambda} = T^{\lambda\lambda}_{gg} = \frac{9}{4}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}} - \frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}\right)$$

$$T^{q_i \lambda}_{q_i \lambda} = \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{1}{9}\,\delta$$

$$T^{s_i \bar{s}_j}_{q_i \bar{q}_j} = T^{s_i s_j}_{q_i q_j} = \frac{4}{9}\,\frac{\hat{u}}{\hat{t}} \qquad (i \neq j)$$

$$T^{s_j \bar{s}_j}_{q_i \bar{q}_i} = \frac{8}{9}\,\frac{\hat{u}\hat{t}}{\hat{s}^2} \qquad (i \neq j)$$

$$T^{s_i \bar{s}_i}_{q_i \bar{q}_i} = \frac{4}{9}\,\frac{\hat{u}}{\hat{t}} + \frac{8}{9}\,\frac{\hat{u}\hat{t}}{\hat{s}^2} - \frac{8}{27}\,\frac{\hat{u}}{\hat{s}}$$

$$T^{s_i s_i}_{q_i q_i} = \frac{2}{9}\,\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}}$$

$$T^{\lambda s_i}_{q_i g} = -\frac{\hat{s}}{\hat{u}} - \frac{4}{9}\,\frac{\hat{u}}{\hat{s}} - 1$$

$$T^{s_i \bar{s}_i}_{gg} = -\frac{3}{4}\,\frac{\hat{u}\hat{t}}{\hat{s}^2} + \frac{1}{3}$$

TABLE 1

$T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{gg}_{q_i\bar{q}_i} + 2(N_f-1)T^{q_j\bar{q}_j}_{q_i\bar{q}_i}$	$T^{q_i\bar{q}_j}_{q_i\bar{q}_j}$	$T^{q_iq_i}_{q_iq_i} + (N_f-1)T^{q_iq_j}_{q_iq_j} + (N_f-2)T^{q_i\bar{q}_j}_{q_i\bar{q}_j}$	$T^{q_ig}_{q_ig}$
sym	$T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{gg}_{q_i\bar{q}_i} + 2(N_f-1)T^{q_j\bar{q}_j}_{q_i\bar{q}_i}$	$T^{q_iq_i}_{q_iq_i} + (N_f-1)T^{q_iq_j}_{q_iq_j} + (N_f-2)T^{q_i\bar{q}_j}_{q_i\bar{q}_j}$	$T^{q_ig}_{q_ig}$
sym	sym	$(4N_f^2-12N_f+10)T^{q_iq_j}_{q_iq_j} + 4(N_f-1)^2T^{q_i\bar{q}_i}_{q_j\bar{q}_j} + 2(N_f-2)T^{q_iq_i}_{q_iq_i} + 2(N_f-2)(T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{gg}_{q_i\bar{q}_i})$	T_{gg}
sym	sym	sym	$T^{gg}_{gg} + 2N_f T^{q_i\bar{q}_i}_{gg}$

TABLE 2-a

$\frac{4}{9}T$	$\frac{4}{9}T$	$2(N_f-1)\frac{4}{9}T$	T
sym	$\frac{4}{9}T$	$2(N_f-1)T$	T
sym	sym	$4(N_f-1)^2\frac{4}{9}T$	$2(N_f-1)T$
sym	sym	sym	$\frac{9}{4}T$

TABLE 2-b

$T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + 2(N_f-1)\,T^{q_i\bar{q}_i}_{q_j\bar{q}_j} + T^{gg}_{q_i\bar{q}_i} + T^{\lambda\lambda}_{q_i\bar{q}_i}$	$T^{q_i\bar{q}_j}_{q_i\bar{q}_j}$	$T^{q_iq_i}_{q_iq_i} + (N_f-1)\,T^{q_iq_j}_{q_iq_j} + (N_f-2)\,T^{q_i\bar{q}_j}_{q_i\bar{q}_j}$	$T^{q_ig}_{q_ig}$	$T^{q_i\lambda}_{q_i\lambda}$
sym.	$T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + 2(N_f-1)\,T^{q_i\bar{q}_i}_{q_j\bar{q}_j} + T^{gg}_{q_i\bar{q}_i} + T^{\lambda\lambda}_{q_i\bar{q}_i}$	$T^{q_iq_i}_{q_iq_i} + (N_f-1)\,T^{q_iq_j}_{q_iq_j} + (N_f-2)\,T^{q_i\bar{q}_j}_{q_i\bar{q}_j}$	$T^{q_ig}_{q_ig}$	$T^{q_i\lambda}_{q_i\lambda}$
sym.	sym.	$(4N_f^2-12N_f+10)\,T^{q_iq_j}_{q_iq_j} + 4(N_f-1)^2\,T^{q_i\bar{q}_j}_{q_j\bar{q}_j} + 2(N_f-2)\,T^{q_iq_i}_{q_iq_i} + 2(N_f-1)\,(T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{gg}_{q_i\bar{q}_i} + T^{\lambda\lambda}_{q_i\bar{q}_i})$	$2(N_f-1)\,T^{q_ig}_{q_ig}$	$2(N_f-1)\,T^{q_i\lambda}_{q_i\lambda}$
sym.	sym.	sym.	$T^{gg}_{gg} + T^{\lambda\lambda}_{gg} + 2N_f\,T^{q_i\bar{q}_i}_{gg}$	$T^{g\lambda}_{g\lambda}$
sym.	sym.	sym.	sym.	$T^{\lambda\lambda} + T^{gg}_{\lambda\lambda} + 2N_f\,T^{q_i\bar{q}_i}_{\lambda\lambda}$

TABLE 3-a

$\frac{4}{9}\,T$	$\frac{4}{9}\,T$	$2(N_f-1)\,\frac{4}{9}\,T$	T	T
sym.	$\frac{4}{9}\,T$	$2(N_f-1)\,\frac{4}{9}\,T$	T	T
sym.	sym.	$4(N_f-1)^2\,\frac{4}{9}\,T$	$2(N_f-1)\,T$	$2(N_f-1)\,T$
sym.	sym.	sym.	$\frac{9}{4}\,T$	$\frac{9}{4}\,T$
sym.	sym.	sym.	sym.	$\frac{9}{4}\,T$

TABLE 3-b

$T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{s_i\bar{s}_i}_{q_i\bar{q}_i} + T^{g\bar{g}}_{q_i\bar{q}_i} + T^{\lambda\bar{\lambda}}_{q_i\bar{q}_i} + 2(N_f-1)\left(T^{q_j\bar{q}_j}_{q_i\bar{q}_i} + T^{s_j\bar{s}_j}_{q_i\bar{q}_i}\right)$	$T^{q_i\bar{q}_j}_{q_i\bar{q}_j} + T^{s_i\bar{s}_j}_{q_i\bar{q}_j}$	$T^{q_iq_i}_{q_iq_i} + T^{s_is_i}_{q_iq_i} + (N_f-1)T^{q_iq_j}_{q_iq_j} + (N_f-2)T^{q_i\bar{q}_j}_{q_i\bar{q}_j} + (2N_f-3)T^{s_i\bar{s}_j}_{q_i\bar{q}_j}$	$T^{q_ig}_{q_ig} + T^{s_i\lambda}_{q_ig}$
sym.	$T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{s_i\bar{s}_i}_{q_i\bar{q}_i} + T^{g\bar{g}}_{q_i\bar{q}_i} + T^{\lambda\bar{\lambda}}_{q_i\bar{q}_i} + 2(N_f-1)\left(T^{q_j\bar{q}_j}_{q_i\bar{q}_i} + T^{s_j\bar{s}_j}_{q_i\bar{q}_i}\right)$	$T^{q_iq_i}_{q_iq_i} + T^{s_is_i}_{q_iq_i} + (N_f-1)T^{q_iq_j}_{q_iq_j} + (N_f-2)T^{q_i\bar{q}_j}_{q_i\bar{q}_j} + (2N_f-3)T^{s_i\bar{s}_j}_{q_i\bar{q}_j}$	$T^{q_ig}_{q_ig} + T^{s_i\lambda}_{q_ig}$
sym.	sym.	$(4N_f^2-12N_f+10)\left(T^{q_iq_j}_{q_iq_j} + T^{s_is_j}_{q_iq_j}\right) + 4(N_f-1)^2\left(T^{q_i\bar{q}_i}_{q_j\bar{q}_j} + T^{s_i\bar{s}_i}_{q_j\bar{q}_j}\right) + 2(N_f-2)\left(T^{q_iq_i}_{q_iq_i} + T^{s_is_i}_{q_iq_i}\right) + 2(N_f-1)\left(T^{q_i\bar{q}_i}_{q_i\bar{q}_i} + T^{s_i\bar{s}_i}_{q_i\bar{q}_i} + T^{g\bar{g}}_{q_i\bar{q}_i} + T^{\lambda\bar{\lambda}}_{q_i\bar{q}_i}\right)$	$2(N_f-1)T^{q_ig}_{q_ig} + 2(N_f-1)T^{s_i\lambda}_{q_ig}$
sym.	sym.	sym.	$T^{gg}_{gg} + T^{\lambda\lambda}_{gg} + 2N_f T^{q_i\bar{q}_i}_{gg} + 2N_f T^{s_i\bar{s}_i}_{gg}$

TABLE 4

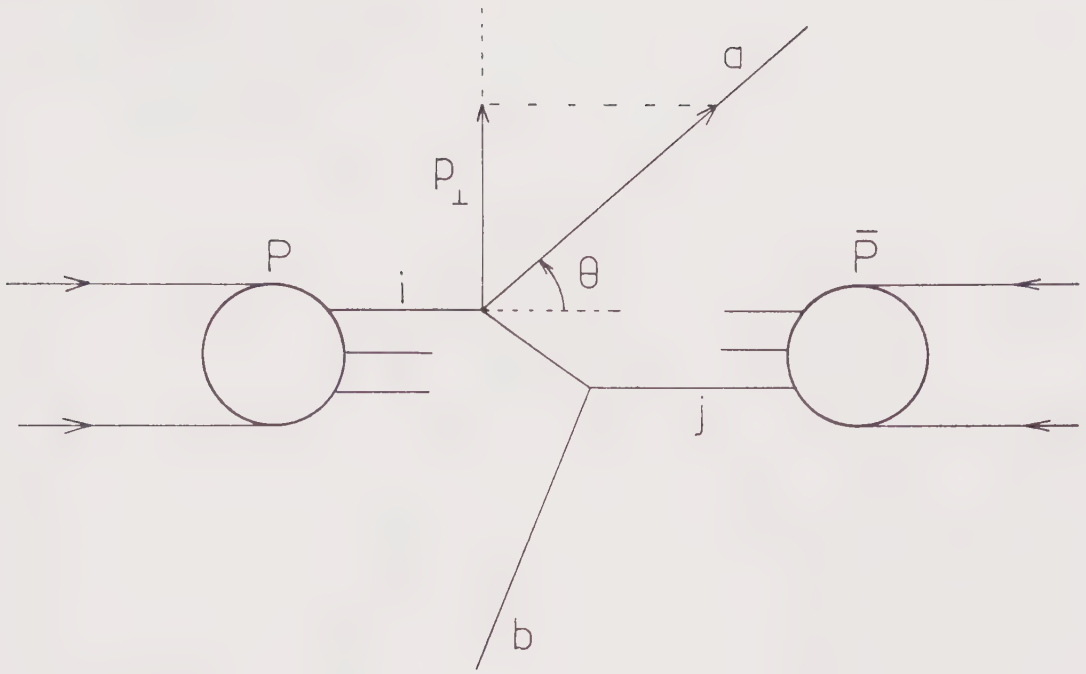

fig.1

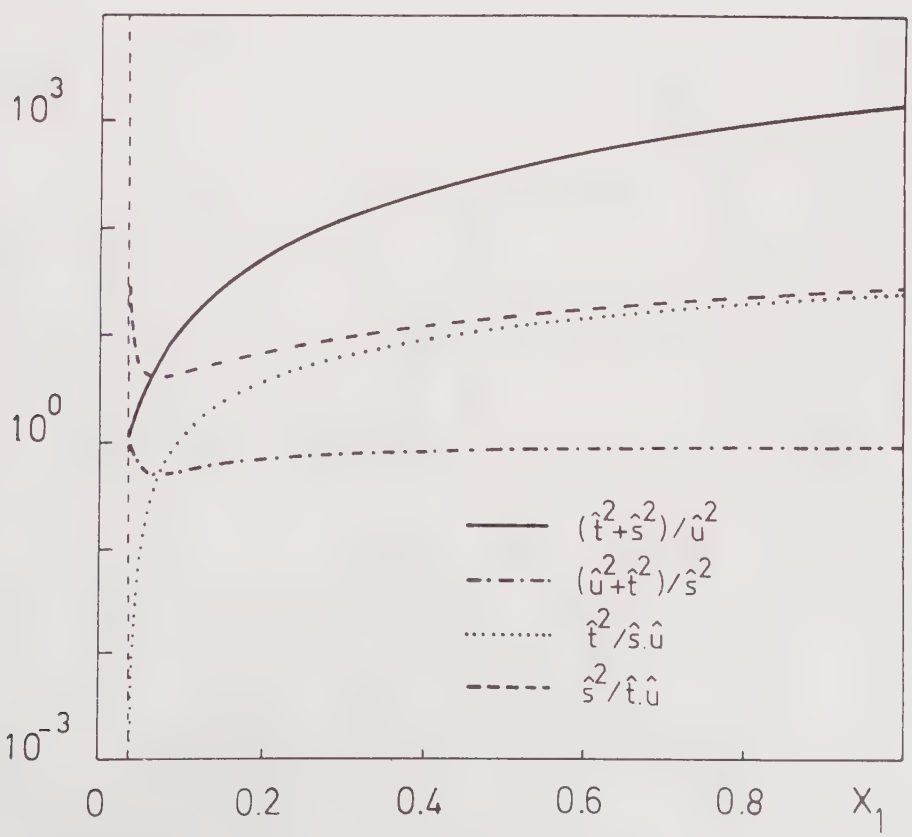

fig.2

$p_\perp$=20GeV , θ=90°

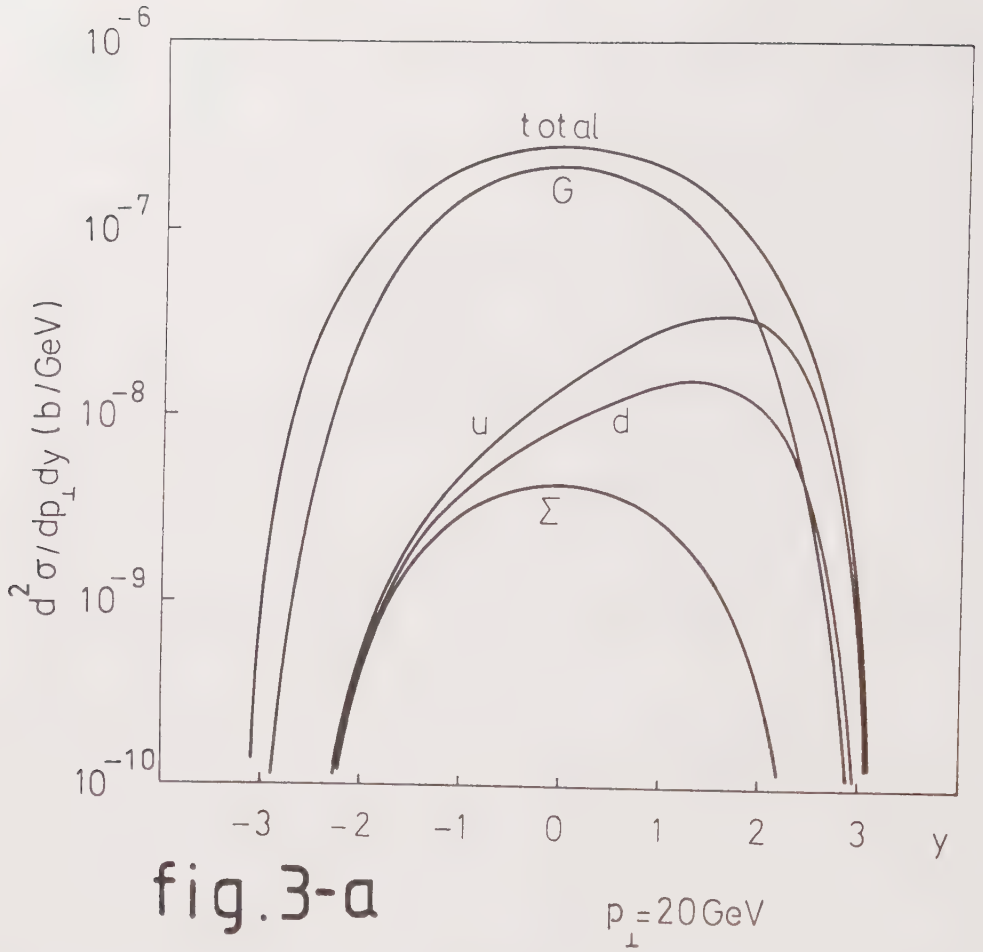

fig.3-a

$p_\perp = 20$ GeV

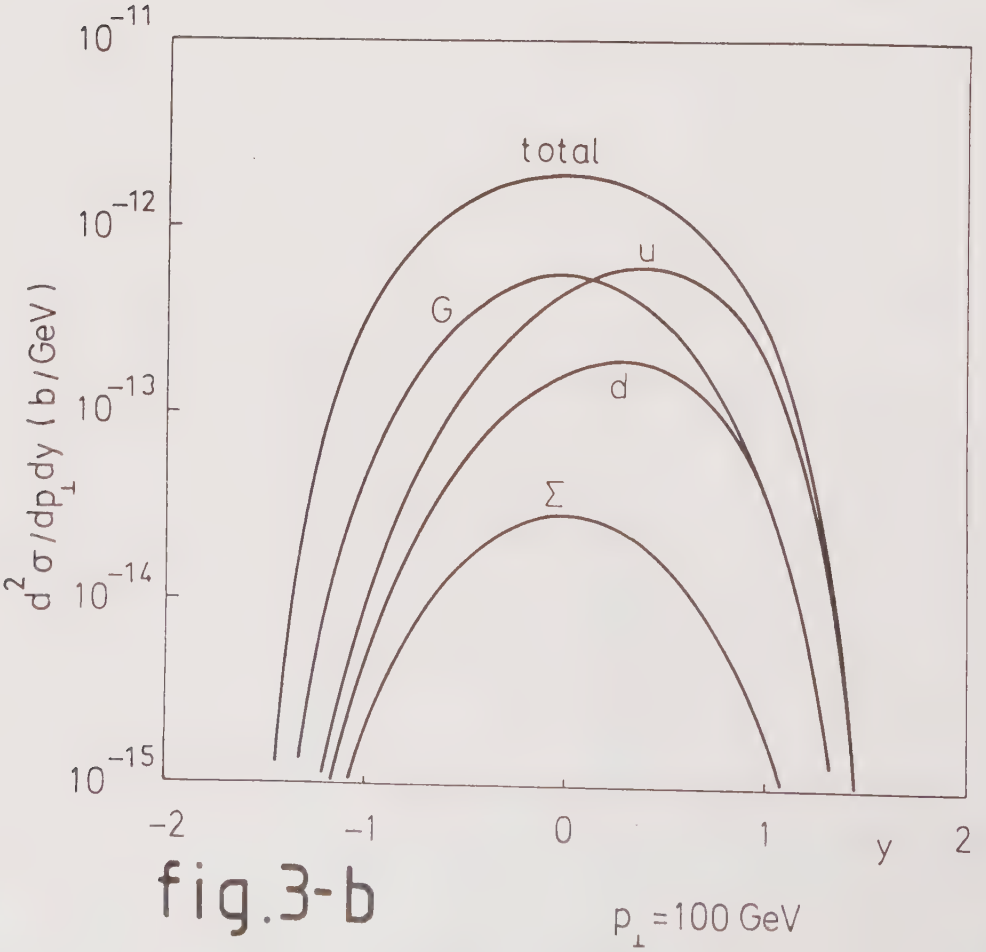

fig.3-b

$p_\perp = 100$ GeV

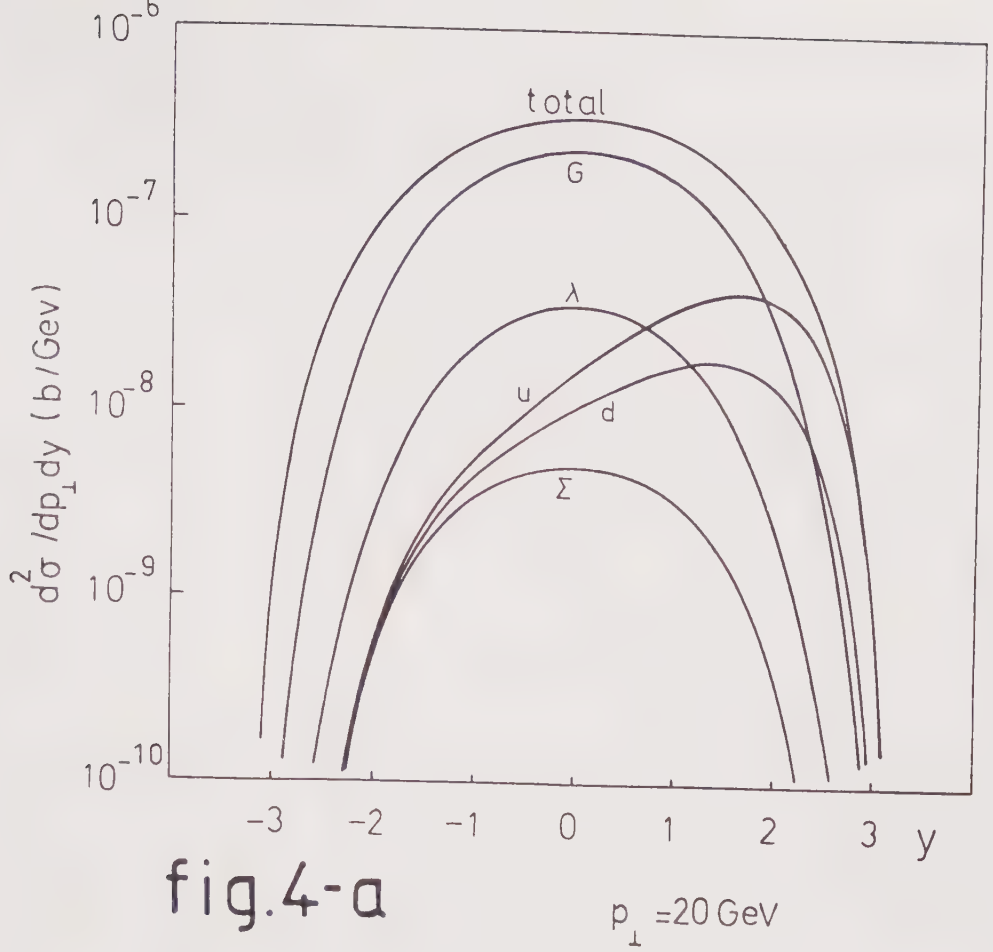

fig.4-a

$p_\perp$ =20 GeV

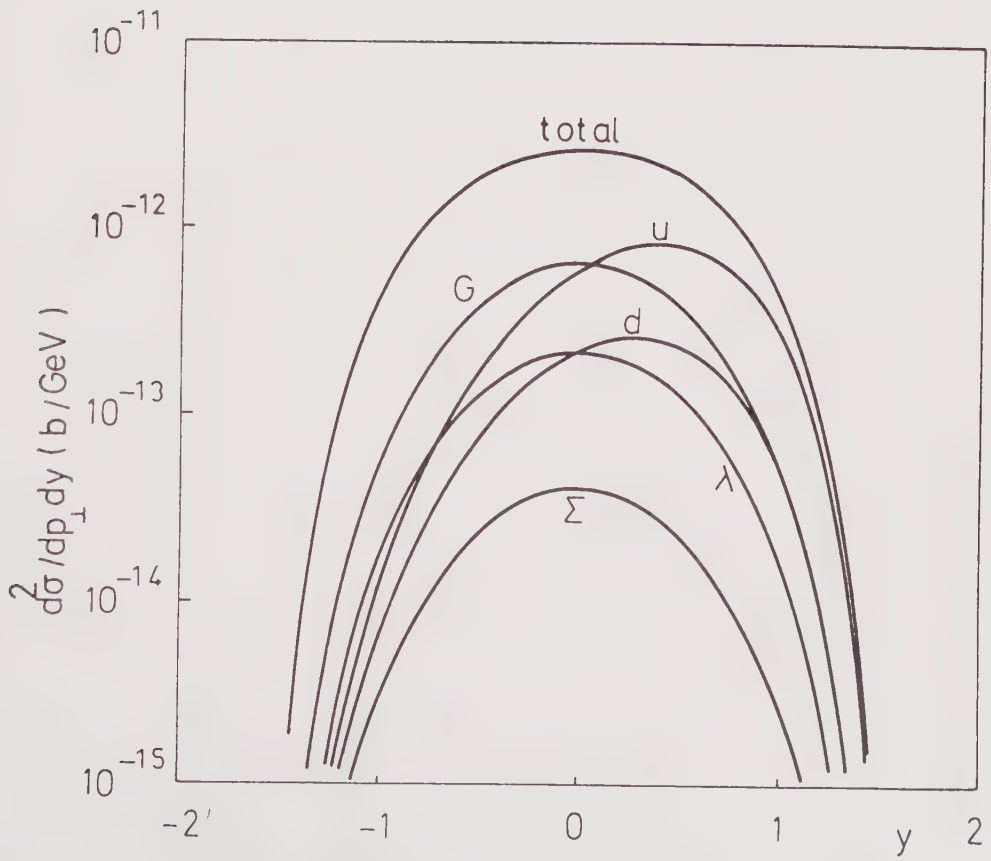

fig.4-b

$p_\perp$ = 100 GeV

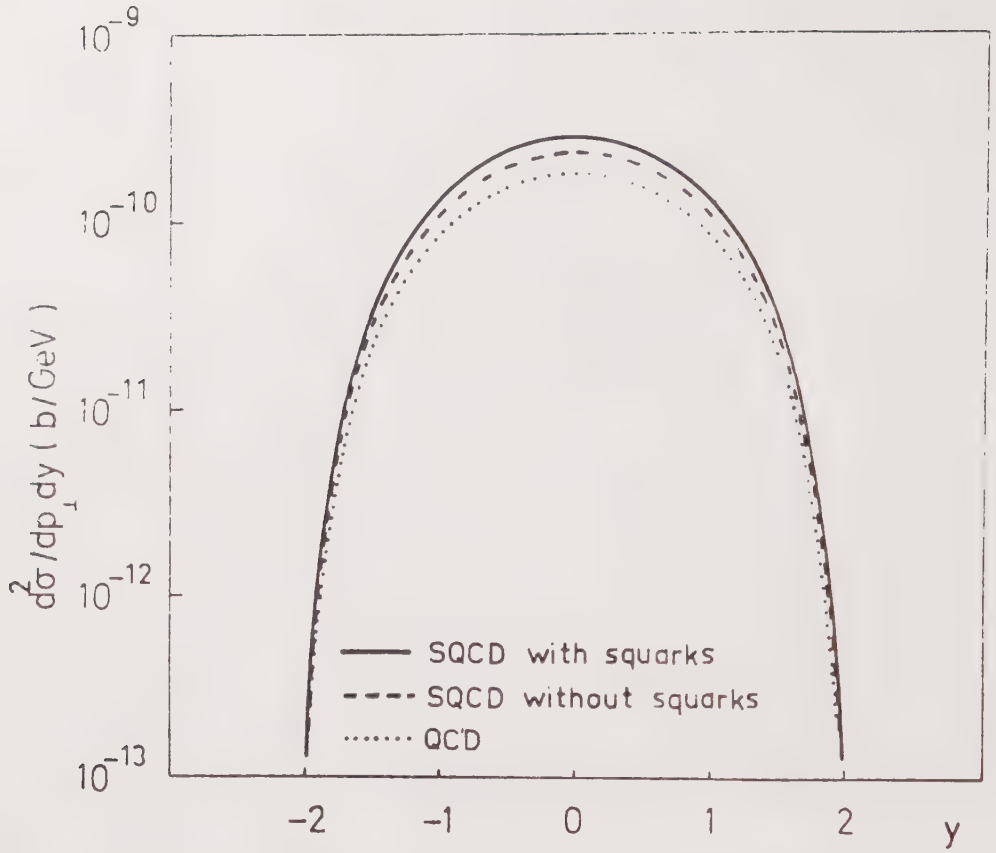

fig.5-a

$p_\perp = 60$ GeV

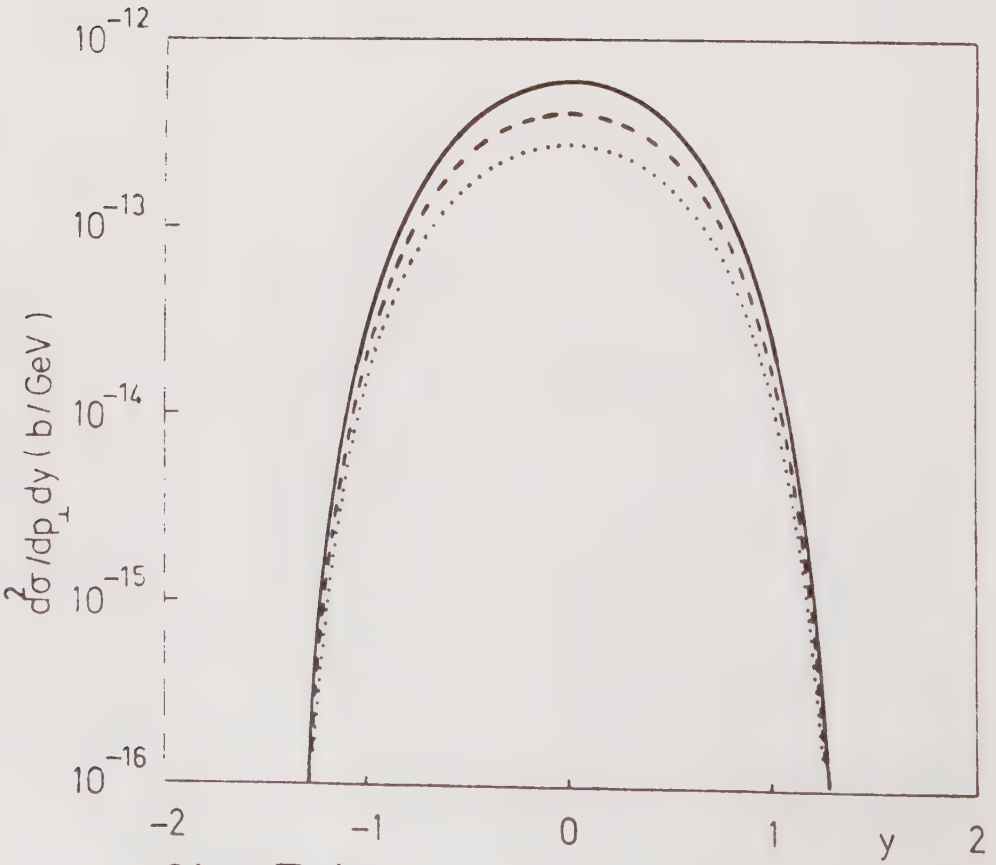

fig.5-b

$p_\perp = 120$ GeV

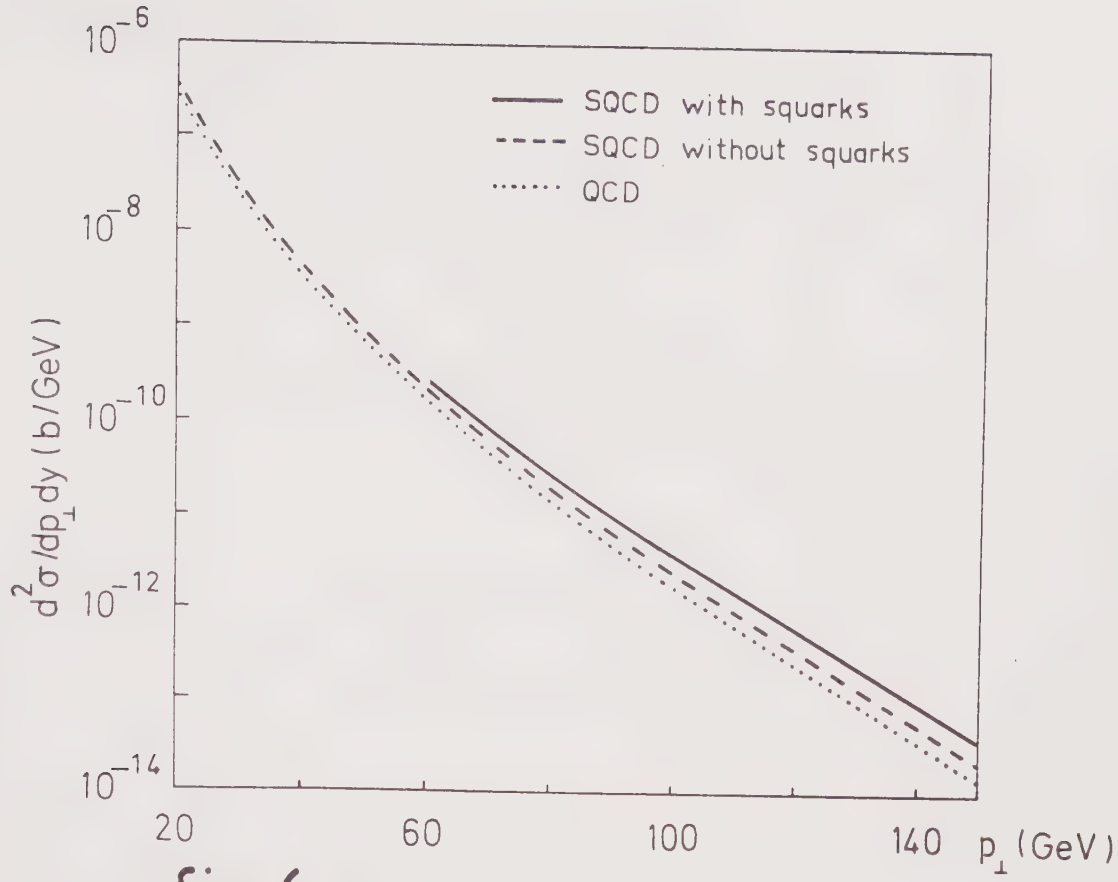

fig.6-a

$\theta = 90°$

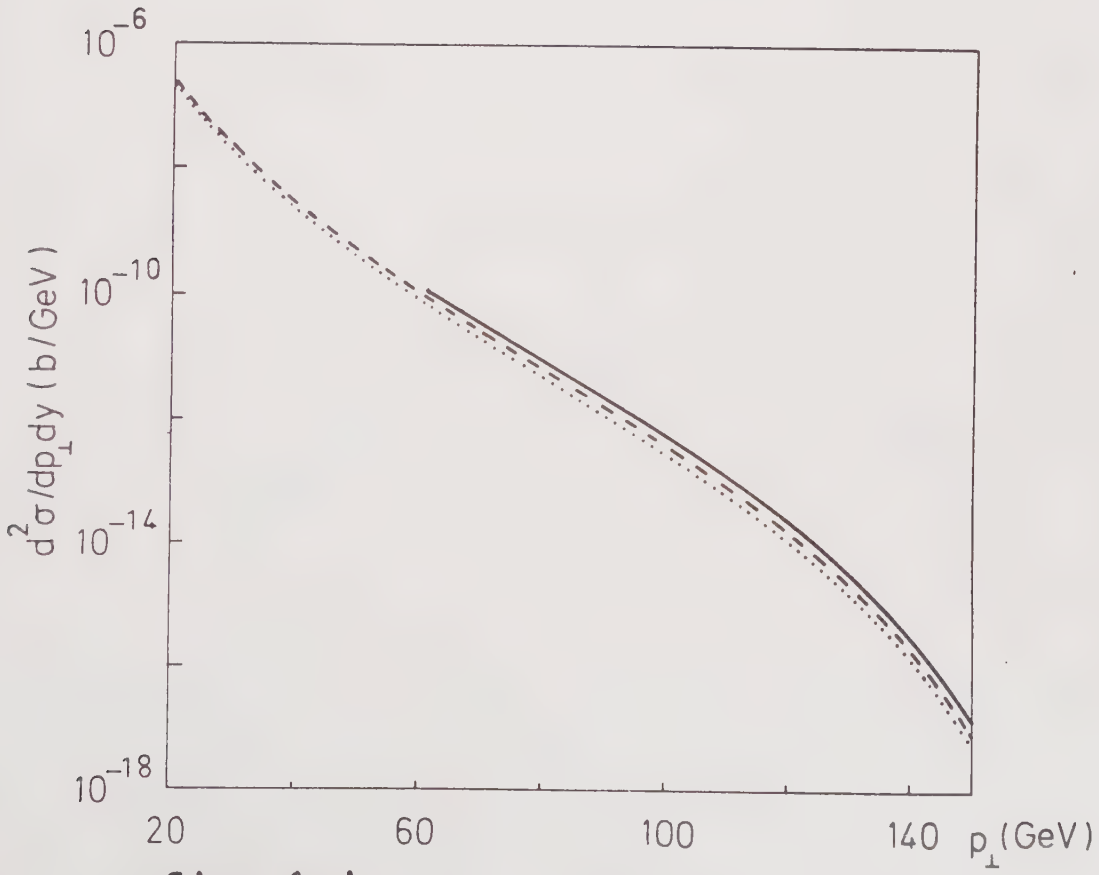

fig.6-b

$\theta = 40°$

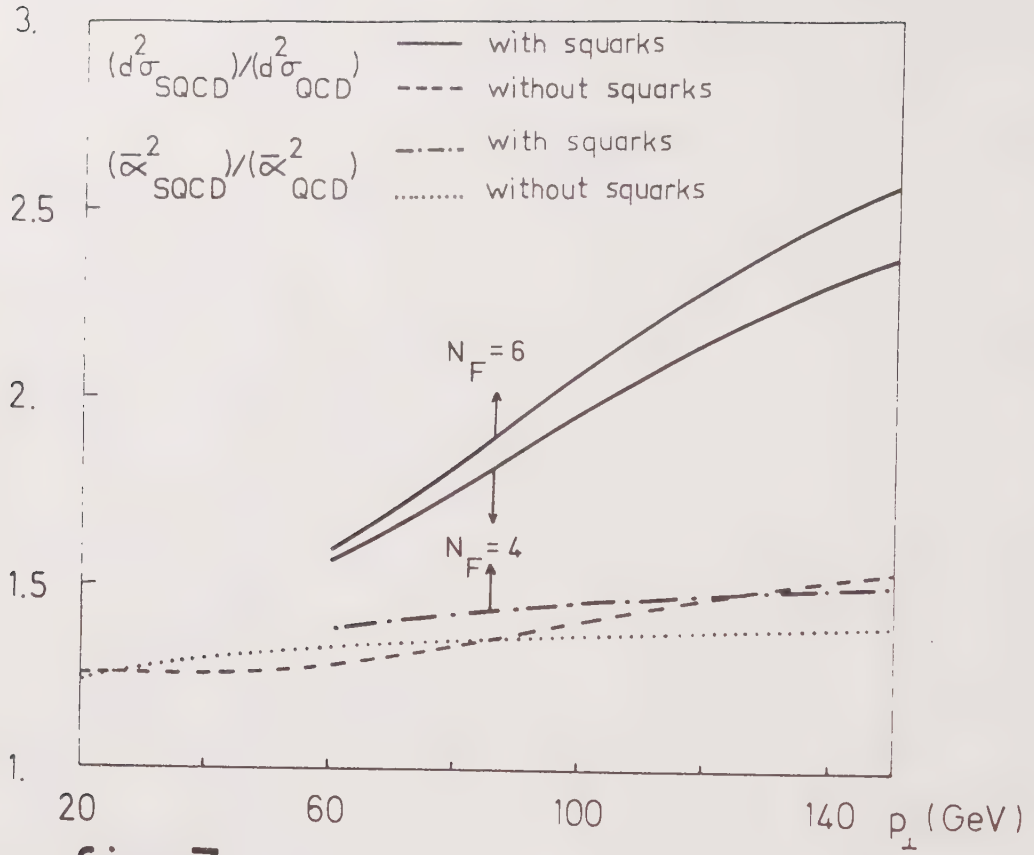

fig.7

$\theta = 90°$

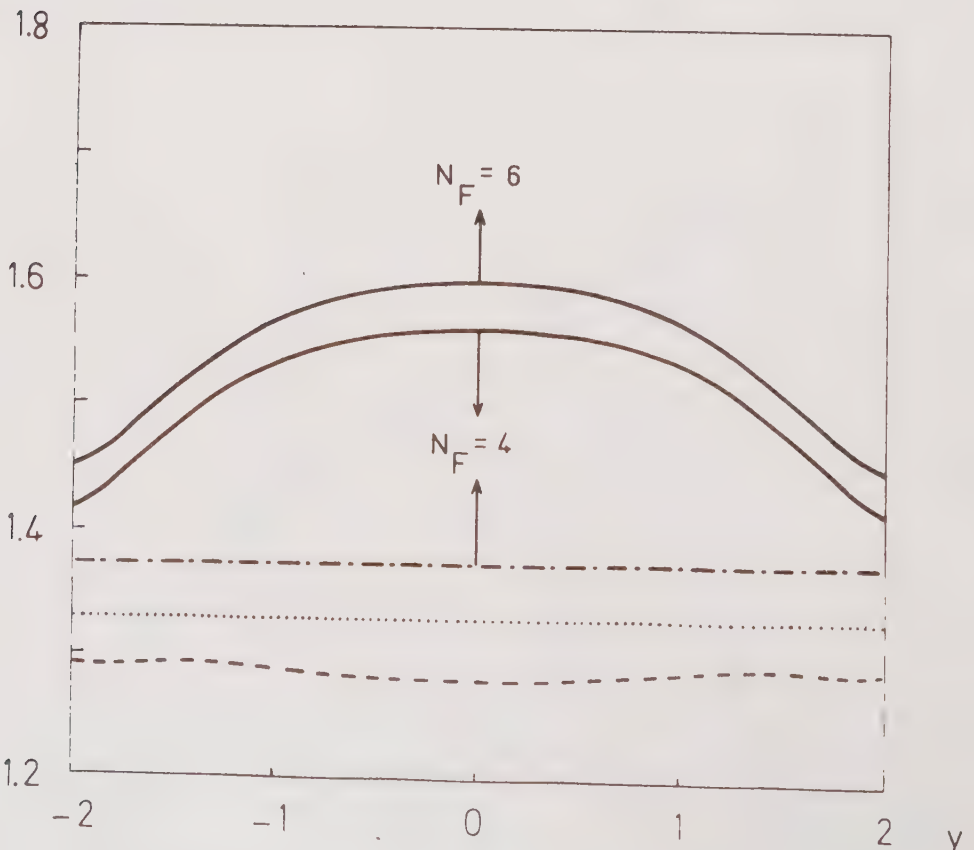

fig.8

$p_\perp = 60$ GeV

POLARIZATION EFFECTS OF SUPERSYMMETRIC QCD IN LARGE-P_T DIRECT PHOTON PRODUCTION

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ABSTRACT

The linear polarization P ($\sim$ perpendicular minus parallel to the scattering plane) of large-p_T direct photons from unpolarized hadrons is considered. Contrary to standard QCD, where to $0(1)$ P vanishes, and to $0(\alpha_s)$ P is very small and changes sign at angle $\theta_{cm} = 90°$, it is shown that supersymmetric theories (involving squarks and light gluinos), already to $0(1)$, imply a substantial and positive P through a wide range of angles including $\theta_{cm} = 90°$. For $\bar{p}p \to \gamma + X$ at collider energy and $p_T \gtrsim 30$ GeV, with squark mass 20 GeV we find P $\sim$ 10 % -5 % decreasing with p_T. We offer a qualitative understanding and discuss the significance of our results.

N° A550.0583 Mai 1983

In recent years supersymmetric (SUSY) theories have attracted much attention [1,3] ; the reasons, although mainly theoretical, are quite important [4]. It is therefore of interest to study the physical implications of, at least a certain class of, SUSY theories, and to advance detailed experimental tests.

Very recently, the standard QCD predictions, up to second order, regarding the scale violations of deep inelastic structure functions have been extended in the presence of light gluinos [5] and, for large values of Q^2, of squarks as well [6]. By an extension of the Altarelli-Parisi equations the gluino and squark distributions have been determined. Thus the standard QCD approach can be extended to include subprocesses involving such SUSY partons.

Here we consider hadroproduction of large transverse momentum (p_T) direct photons ($A+B \to \gamma+X$). It is well known that in standard QCD this is well described by [7,8]

$$q+g \to \gamma+q \qquad\qquad q+\bar{q} \to \gamma+g \tag{1}$$

Now consider the linear polarization of the outgoing photon

$$P = \frac{|M_\perp|^2 - |M_{/\!/}|^2}{|M_\perp|^2 + |M_{/\!/}|^2} \tag{2}$$

where $M_\perp (M_{/\!/})$ denotes the amplitude for γ to be polarized perpendicular (parallel) to the scattering plane. With unpolarized initial hadrons and massless quarks, P = 0 to 0(1) (leading order in α_s). To $0(\alpha_s)$, due to loop graph contributions, P is nonzero (but rather small, see below). However, even to this order, P = 0 at c.m. scattering angle $\theta = 90°$; in particular, P changes sign as θ varies [9].

Our purpose is to show that, *already to O(1)* (Born amplitudes), supersymmetric QCD (SQCD) implies a sizable polarization P for a wide range of angles, *including* $\theta = 90°$. In particular, $P > 0$ throughout this wide range.

We define the subprocess Mandelstam variables (Fig. 1(a)) :

$$s = (p_1 + p_2)^2 \qquad t = (q - p_1)^2 \qquad u = (q - p_2)^2 \tag{3}$$

To simplify notation we write the subprocess cross section :

$$\frac{d\sigma}{dt}(s,t) \equiv \pi \alpha \alpha_s \, \sigma(s,t) \tag{4}$$

($\alpha = 1/137$), and denote by $\sigma_\Delta(s,t)$ the difference of the corresponding quantities for photons polarized perpendicular and parallel to the scattering plane

$$\sigma_\Delta(s,t) \equiv \sigma_\perp(s,t) - \sigma_{/\!/}(s,t) \tag{5}$$

We use the Lagrangian [6,10]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + i\bar{q}_i \not{D} q_i + \frac{i}{2} \bar{\lambda}^a \not{D} \lambda^a + (D_\mu s_i)^+ (D_\mu s_i) \\ & + (D_\mu t_i)^+ (D_\mu t_i) + ig\sqrt{2}\,(\bar{\lambda}_a \frac{1+\gamma_5}{2} s_i^+ T_a q_i + \bar{\lambda}_a \frac{1-\gamma_5}{2} t_i^+ T^a q_i - \text{h.c.}) \\ & - \frac{1}{2} g^2 (s_i^+ T^a s_i - t_i^+ T^a t_i)^2 + \text{mass terms} \end{aligned} \tag{6}$$

where $G^a_{\mu\nu}$ is the gluon field strength, q_i and λ^a stand for quark and gluino fields, s_i and t_i for s- and t-squark fields, D_μ is the covariant derivative, i runs over flavors, a the color index and T_a the SU (3)

generators in the fundamental representation. We assume that the s- and t-squarks are degenerate with mass M, and subsequently we denote by s both types of them. We also take the gluinos as Majorana fermions.

For the subprocess $g+s \to \gamma+s$ (Fig. 1(a)) we find, including group factors :

$$\sigma_\Delta = \frac{10}{3} \frac{t(us - M^4)M^2}{(s-M^2)^4 (u-M^2)^2} \qquad \sigma = \frac{5}{3} \frac{1}{(s-M^2)^2} - \sigma_\Delta \tag{7}$$

and for $q + \bar{s} \to \gamma + \lambda$

$$\sigma_\Delta = \frac{16}{9} \frac{(us - M^2m^2)(M^2 - m^2)}{t(s - M^2)^2 (u - M^2)^2} \qquad \sigma = \frac{8}{9} \frac{m^2 - s}{t(s - M^2)^2} - \sigma_\Delta \tag{8}$$

where m is the gluino mass. We are subsequently restricted to $m \lesssim 4$ GeV. For $q+\lambda \to \gamma+s$ the contribution is obtained from (8) by $u \leftrightarrow s$ crossing the amplitudes (Fig. 1(a)) and replacing the factor 16/9 (8/9) by 2/3 (1/3).

Some care is needed in the running coupling constant $\alpha_s(Q^2)$ entering the contributions (4) - (8) [5]. We use

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \log(Q^2/\tilde{\Lambda}^2)} \tag{9}$$

Above the squark (and gluino) threshold the β-function :

$$\beta = 11 - \frac{2}{3} N_f - 2 - \frac{1}{3} N_f \tag{10}$$

where N_f is the number of flavors, and -2 and $-\frac{1}{3} N_f$ are the contributions of gluinos and squarks. $\tilde{\Lambda}$ is obtained from the standard QCD scale Λ by matching $\alpha_s(Q^2)$ at each threshold [5]. We use $\Lambda = 0.4$ GeV, and above the squark threshold $N_f = 6$.

For the contribution $Ed\sigma_\Delta/d^3_p$ to the physical process $A+B \to \gamma+X$ from σ_Δ of the subprocess $ab \to cd$, we first replace s, t, u by the usual $\hat{s}$, $\hat{t}$, $\hat{u}$; e.g. for (7), (8) :

$$\hat{s} = X_a X_b \frac{s}{2} + X_a (X_b^2 + \frac{4M^2}{s})^{1/2} \frac{s}{2} + M^2$$

$$\hat{t} = -X_a X_T \frac{s}{2} \tan \frac{\theta}{2} \qquad (11)$$

where now $s \equiv (p_A + p_B)^2$, $X_T = 2p_T/\sqrt{s}$ and X_a, X_b are momentum fractions ; then σ_Δ is multiplied by the parton distributions $F_{a/A}(X_a, Q^2)$, $F_{b/B}(X_b, Q^2)$ and integrated. For squarks and gluinos these are given in Refs[5, 6]. The polarization P is obtained by dividing $Ed\sigma_\Delta/d^3p$ by $Ed\sigma/d^3p$ of $A+B \to \gamma+X$.

The calculation is carried for $\bar{p}+p \to \gamma+X$ at $\sqrt{s} = 540$ and $M = 20$ GeV. To estimate the magnitude of P we determine $Ed\sigma/d^3p$ in terms of the standard subprocesses (1). For gluons and quarks we use the complete set of CDHS distributions including their scale violations [11]. We mention that the resulting $Ed\sigma/d^3p$ for $\bar{p}p \to \gamma+X$ is in agreement with preliminary data of the UA2 Collaboration in the range $15 \lesssim p_T \lesssim 30$ GeV (γ/jet $\sim 10^{-3}$) [12]. For $Ed\sigma_\Delta/d^3p$ we use the same gluon and quark distributions.

To lowest order in α_s there is a well-known ambiguity in the choice of the large variable Q^2. In Figs 1(b) and 2 we present results with $Q^2 = \hat{s}$ and comment below on other choices.

Fig. 1(b) shows the contributions $Ed\sigma_\Delta/d^3p$ of the SQCD subprocesses involved. It is remarkable that the contribution of $q\lambda \to \gamma s$ is way below the other two, and decreases very fast with p_T compared to $q\bar{s} \to \gamma\lambda$. The reason is that the gluino distribution decreases very fast with X [5,6]. This is in contrast with the squark distribution, which contains a valence part that decreases much slower [6] ; we discuss below the significance of this point. The contribution of $gs \to \gamma s$ decreases faster than of $q\bar{s} \to \gamma\lambda$; this is due to the steepness of the gluon distribution as compared to the quark valence [13].

Fig. 2 shows our results on the polarization P. Clearly SQCD implies a sizable P even at p_T as large as 80 GeV. In the same figure we plot the QCD prediction of Ref.[9](their Eq. (4) and Fig. 2) calculated at p_T = 45 GeV with $Q^2 = 2p_T^2$ and Λ = 0.4 GeV ; it is well below the corresponding SQCD. More important, consider the average polarization $\langle P \rangle$ in the range $60° \lesssim \theta \lesssim 120°$; QCD predicts $\langle P \rangle \sim 0$ almost independently of p_T ; SQCD predicts $\langle P \rangle \sim 10 - 3$ %, depending on p_T.

The order of magnitude and the dependence on p_T of P are understandable : First, our SQCD polarization is a mass effect, due to the (large) squark mass M (see Eqs. (7), (8)) ; this allows some helicity flip even to the lowest order in α_s. Thus, roughly, at the subprocess level, $\hat{P} \sim M^2/\hat{s}$ or $\hat{P} \sim M^2/(-\hat{t})$. With $X_a \sim X_b \sim X_T$ [14], this becomes $\hat{P} \sim M^2/p_T^2$. Now in the range of X of interest, the squark distribution (with which Eqs (7) and (8) are convoluted to obtain $Ed\sigma_\Delta/d^3p$) is, roughly, 10 -15 % of the quark and gluon distribution [6]. When p_T is not much larger than M, this finally implies $P \sim 10$ %. As p_T increases, P decreases.

Regarding the choice of the large variable Q^2, for $p_T \lesssim 30$ GeV (not far from M), the results are quite sensitive ; however, for $p_T \gtrsim 40$ GeV they are fairly stable [15].

One may ask whether other sources, hitherto neglected, may produce polarizations comparable to those of Fig. 2. First, higher twist effects [16] should be reasonably discarded at such high energies and p_T. A top quark (t), if it exists, should generally produce $P \neq 0$. Unfortunately, t-distributions are not available, so a detailed calculation cannot be carried out. However, we may speculate that the t-distribution in the nucleon would fastly decrease with X, since the t's may exist only in the sea ; this is in contrast to the total squark distribution. In addition, the t represents only one flavor, whereas the squark appears in several [5,6,10]. Thus we may anticipate that t polarizations will be significantly lower.

Finally, one may wonder why in SQCD polarizations are sizable (a few percent) even at p_T as large as 80 GeV (in fact, at such p_T our results are quite stable against various choices of Q^2). The reason, already stated, is that the squark distribution contains a valence part, which, although small, does not steeply fall with X [6]. But then, observation of the polarization effects of Fig. 2 will be an important verification of the short distance structure of SQCD. Of course, we are well aware of the practical difficulties of such an experiment.

It is a pleasure to thank P. Fayet, J.Iliopoulos and C. Kounnas for a number of discussions and remarks.

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[12] L. FAYARD. Presented in XVIII Rencontre de Moriond on Physics of $\bar{p}p$ Collisions, La Plagne, 19-25 March 1983.

[13] The CDHS gluon distribution [11] is $\sim (1+3.5\ X)(1-X)^{5.9}$ at $Q_o^2 = 5\ GeV^2$.

[14] This region is known to give a dominant part in the integral expressing $Ed\sigma/d^3p$ in terms of $d\sigma/d\hat{t}$ [8].

[15] We have also considered the choices $Q^2 = -\hat{t}$, $2p_T^2$ and $4p_T^2$.

[16] S. BRODSKY and P. LEPAGE in Proceed. of 1980 Intern. Symposium on Polarized Beams and Targets, Lausanne (Birkhäuser Verlag 1981), 169.

FIGURE CAPTIONS

Fig. 1(a) :

The SQCD Feynman graphs under consideration.

Fig. 1(b) :

The contributions of the SQCD subprocesses to the inclusive difference of cross sections $Ed\sigma_{\Delta}/d^3p$ at $\theta = 90°$.

Fig. 2 :

Linear photon polarizations. Solid lines : SQCD predictions. Dashed line : Standard QCD prediction involving subprocesses to $O(\alpha_s^2)$ for $p_T = 45$ GeV [9].

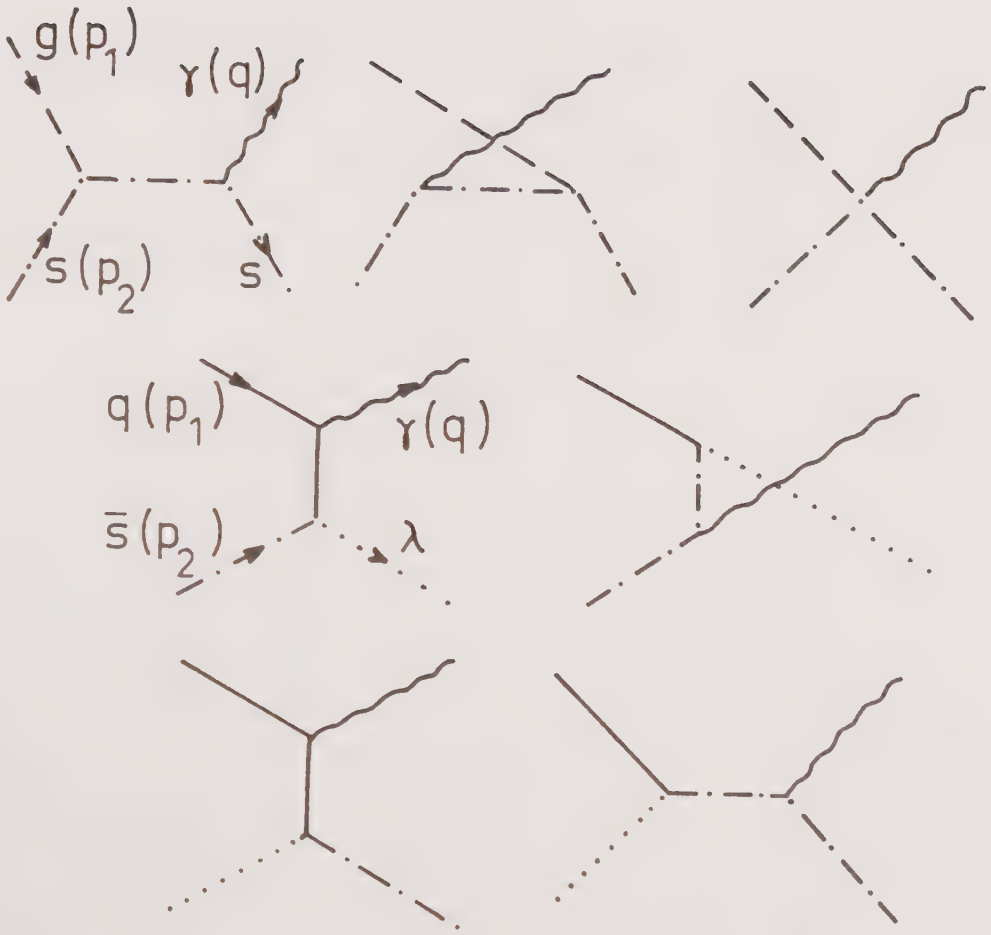

g(p1)
γ(q)
s(p2)
s
q(p1)
γ(q)
s̄(p2)
λ

Fig. 1(a)

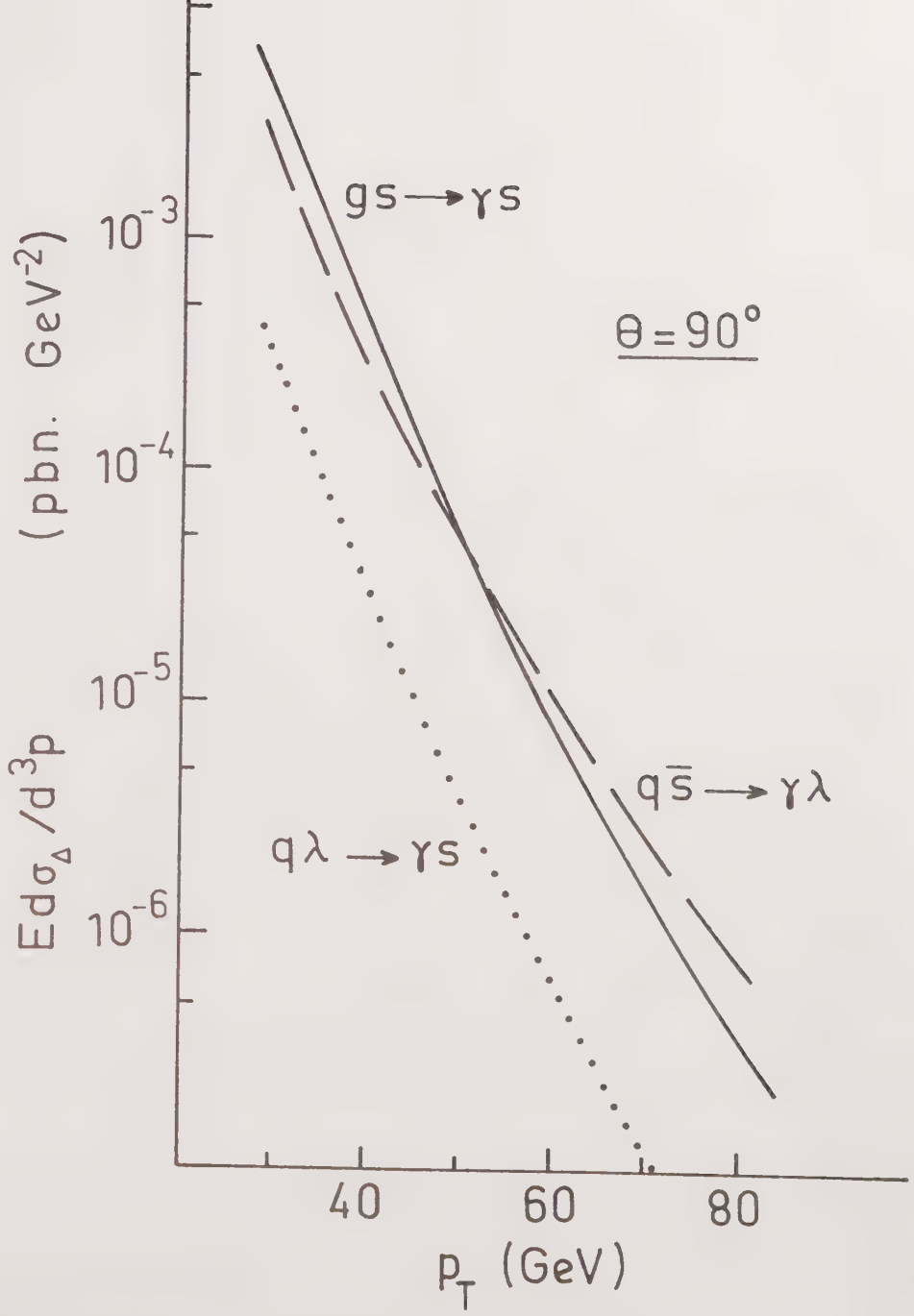

gs → γs
θ = 90°
qs̄ → γλ
qλ → γs
Edσ_Δ/d³p (pbn. GeV⁻²)
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
40
60
80
p_T (GeV)

Fig. 1(b)

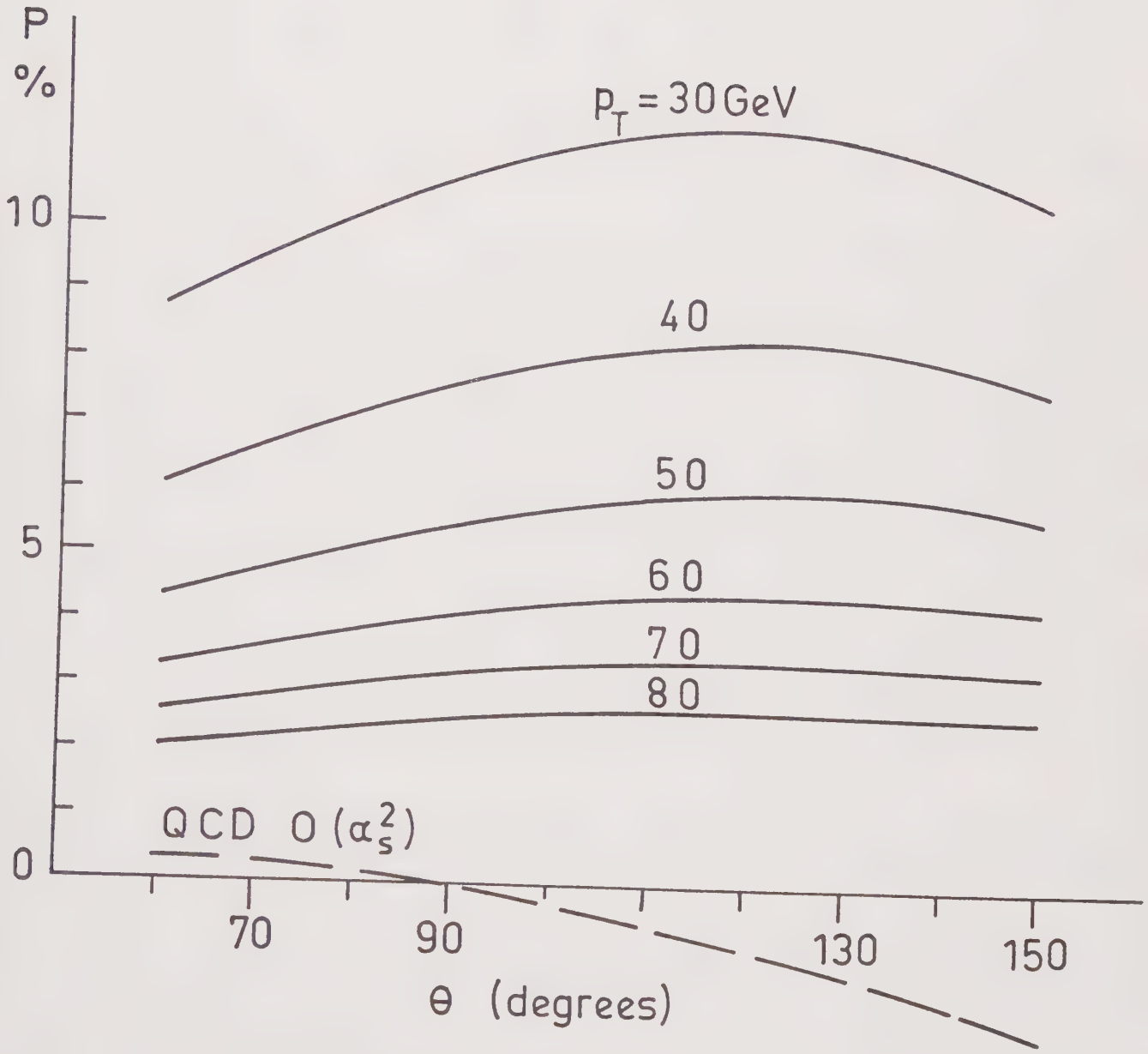

P
%
10
5
0
$P_T = 30$ GeV
40
50
60
70
80
QCD $O(\alpha_s^2)$
70
90
130
150
θ (degrees)

Fig. 2

VII.6 - Prédictions des théories grandement unifiées supersymétriques

Pour l'instant, il n'existe aucun modèle d'une théorie grandement unfiée supersymétrique qui n'ait pas de problèmes théoriques et qui soit en même temps acceptable phénoménologiquement. Une première difficulté est liée au problème de brisure spontanée de la supersymétrie ; on ne peut pas par exemple ajouter un terme de type D si le groupe unifié est simple (voir chapitre VII-1). Un autre problème est le fait que l'on peut être amené à élargir le groupe du modèle standrad par un facteur U(1) ce qui exclut de choisir SU(5) comme groupe unifié[13]. Ainsi les problèmes se compliquent encore. Certains de ces problèmes peuvent être évités en brisant la supersymétrie explicitement mais doucement (moyen qui n'apparait pas très esthétique)[72].

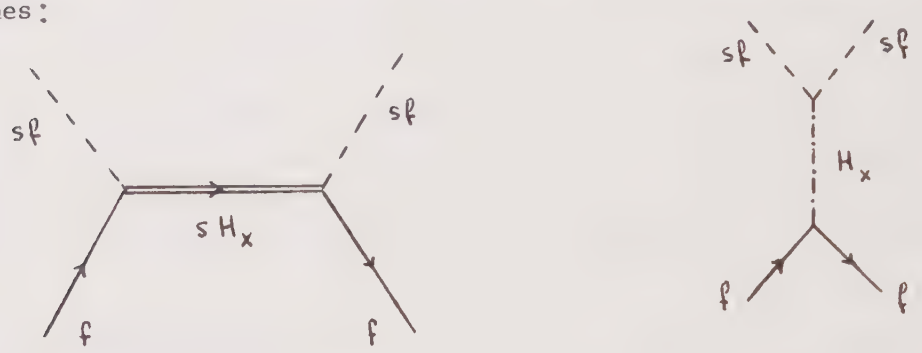
Dans ce chapitre, nous allons étudier les prédictions principales d'une théorie grandement unifiée supersymétrique (brisée doucement) basée sur le groupe minimal SU(5)[72], et les comparer avec celles du modèle unifié SU(5) standard. Pour obtenir ce modèle minimal, il faut associer à toute particule du modèle SU(5) ordinaire un supermultiplet correspondant (chiral ou vecteur de jauge), la seule différence étant l'existence de deux supermultiplets chiraux de Higgs, un dans la représentation 5 et l'autre dans le $\bar{5}$ de SU(5), à la place d'un Higgs complexe dans la représentation 5 au modèle ordinaire. Ses propriétés principales sont[75] :

(i) Il prédit un angle faible plus grand que la valeur expérimentale, contrairement à la prédiction remarquable des théories unifiées ordinaires (voir chapitre IV), $\sin^2\theta_w(M_w) = .233$.

(ii) Il prédit un rapport des masses $m_b/m_\tau = 2.85$ qui est essentiellement le même que dans la théorie SU(5) non supersymétrique.

(iii) Il prédit une masse des bosons de jauge X beaucoup plus lourde que celle des théories unifiées non supersymétriques, $M_X/M_W = 8.7 \times 10^{13}$. Ce résultat est dû au fait que la variation de l'évolution des constantes de couplage est beaucoup plus lente, à cause de l'existence des particules de spin - 0 et 1/2 qui contribuent avec un signe positif aux fonctions $-\beta$, ce qui oblige le point de rencontre M_X à se déplacer vers des valeurs plus grandes. Cette grande valeur de M_X impliquerait un temps de vie de proton très long (voir chapitre IV).

Néanmoins, dans ces théories supersymétriques grandement unifiées, le temps de vie du proton τ_p est proportionnel à M_x^2 au lieu de M_x^4[76]. La raison est l'existence d'opérateurs de dimensions 5 qui peuvent produire la désintégration du proton, et qui contiennent aussi des quarks et des leptons scalaires. Ces opérateurs viennent des diagrammes:



Une possibilité est d'imposer des symétries supplémentaires pour annuler ces opérateurs. Alors la désintégration du proton serait inobservable. Alternativement, on peut montrer[75] en utilisant les petites valeurs des couplages de Yukawa que, même en présence de ces nouveaux opérateurs, la prédiction pour le temps de vie du proton peut devenir un peu plus grande que la limite inférieure expérimentale. Finalement, dans ces théories, le proton se désintègre préférentiellement[75] en $\bar{\nu}K^+$ ou μ^+K plutôt que dans le canal $e^+\pi^0$ du cas standard. En effet, la désintégration est produite par un échange de Higgs virtuels qui sont couplés aux fermions proportionnellement à leurs masses. C'est pourquoi l'état final le plus probable contient les fermions des générations plus lourdes.

En conclusion, résumons les applications de la supersymétrie. Les théories supersymétriques (globales) ne semblent pas pour l'instant avoir trouvé leur place dans la physique des particules élémentaires. Elles nous ont fourni des nouvelles techniques intéressantes mais elles n'ont pas pu relier des particules de différents spins déjà existantes. Par contre, elles prédisent un spectre très riche de particules nouvelles, au-delà du modèle standard. Un avantage de ces théories est que les scalaires, dont l'existence pose des problèmes aux théories ordinaires, jouent un rôle fondamental et apparaissent naturellement comme les partenaires supersymétriques des fermions et même des bosons de jauge massifs. Elles peuvent

probablement être utilisées pour résoudre le problème de la "hiérarchie de jauge". Elles peuvent être, finalement, des théories très prédictives, avec beaucoup de symétrie et assez peu de paramètres. Elles réduisent aussi l'arbitraire dans le secteur des Higgs. Finalement, la supersymétrie est probablement réliée à la gravitation. En effet, la supersymétrie locale engendre la supergravité[15] qui réaliserait éventuellement une "ultime" unification de toutes les interactions fondamentales.

Dans le chapitre suivant, nous étudions de plus près les prédictions des théories supersymétriques grandement unfiées en utilisant une méthode qui simplifie beaucoup le traitement des effets des masses.

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SIMPLE TREATMENT OF THRESHOLD EFFECTS

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We demonstrate that contributions from threshold effects in coupling constant differences at low energies in GUTs are simply taken into account in a scheme that preserves supersymmetry (DR dimensional reduction). Therefore, automatically in supersymmetric GUTs the naive step approximation at the physical mass associated with the threshold gives the correct result.

From the present-day point of view, the observed strong, electromagnetic and weak interactions are described in terms of a grand unified gauge theory based on a simple Lie group. Perhaps the most striking prediction of grand unification is the mortality of the proton. While experimentalists are now moving ahead to observe proton decay in the near future, there are other important predictions of GUTs testable with existing data or with data obtainable by present-day machines. Needless to say that a precise evaluation of the various phenomenological predictions is necessary in order to confront experiment in an unambiguous way.

The existence of a unique gauge coupling constant in GUTs automatically implies relations between the low energy effective couplings. These relations can be cast in the form

$$1/\alpha_i(\mu) - 1/\alpha_j(\mu) = F_{ij}\,(\text{all independent parameters}; M_X/\mu), \qquad (1)$$

where M_X is the physical mass of the superheavy gauge bosons. If one considers as a prototype GUT the minimal SU(5) model, the standard approach based on the decoupling theorem [1] is to approximate the spontaneously broken SU(5) theory with the following effective theory

$$\begin{aligned} &\mathrm{SU}(5) && \mu \gg M_X, \\ &\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) && M_W \ll \mu \ll M_X, \\ &\mathrm{SU}(3)\times\mathrm{U}(1)_{em} && \mu \ll M_W. \end{aligned} \qquad (2)$$

Extrapolating the validity of (2) to the region $\mu \approx M_W$ and $\mu \approx M_X$ (step approximation) introduces an error. In the approach followed by two of the authors (I.A. and C.K.) and Roiesnel [2], the contribution of threshold effects has been calculated exactly to the order of two loops. A similar estimation of threshold effects has been done by other people as well [3–6]. It should be stressed that the incorporation of threshold effects is crucial since the error introduced by the step approximation leads to an appreciable uncertainty in important quantities as the proton lifetime.

The functions F_{ij} appearing in (1) are, in general [2],

$$F_{ij} = \int_{\mu^2}^{\infty} \frac{d\mu'^2}{\mu'^2}\left(\frac{\beta_i(\alpha_1,\ldots)}{\alpha_i^3} - \frac{\beta_j(\alpha_1,\ldots)}{\alpha_j^3}\right), \qquad (3)$$

where β_i are the mass dependent β functions. To the one-loop order, (3) becomes

$$F_{ij}^{(0)} = \int_{\mu^2}^{\infty} \frac{d\mu'^2}{\mu'^2}\,[b_0^i(\mu'^2) - b_0^j(\mu'^2)], \qquad (4)$$

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with b_0^i the mass dependent coefficients of the β functions. In the step approximation, we have [for i corresponding to SU(3), SU(2) or U(1)]

$$b_0^i(\mu^2) = b_0^i \cdot \Theta[M_X^2 - \mu^2],$$

which implies the equality of coupling constants at M_X

$$\alpha_1(M_X) = \alpha_2(M_X) = \alpha_3(M_X). \tag{5}$$

In reality, however, we can only have

$$\alpha_1(\mu) = \alpha_2(\mu) = \alpha_3(\mu) \quad \text{for} \quad \mu^2 \gg M_X^2 \tag{6}$$

which is equivalent to

$$4\pi/\alpha_1(M_X) + C_1 = 4\pi/\alpha_2(M_X) + C_2 = 4\pi/\alpha_3(M_X) + C_3 = 4\pi/\alpha_5(M_X). \tag{7}$$

The constant terms in the minimal SU(5) model, when the $\overline{\text{MS}}$ scheme is used are $C_1 = 0, C_2 = -2/3, C_3 = -1$. Note that these constants are gauge invariant quantities, as we shall show explicitly later on. The error introduced by the naive step approximation is to totally neglect these constant terms in the limit $M_X^2/\mu^2 \to \infty$.

In what follows we shall show that in the dimensional reduction scheme ($\overline{\text{DR}}$) [7] all constant contributions vanish and the naive condition (5) is effectively valid. The dimensional reduction scheme, designed to preserve supersymmetry, consists in keeping the algebra of fields in four dimensions while performing dimensional continuation in the momentum loop integrals. Integrations over mass dependent coefficient functions in expressions (4) generally lead to

$$\int \frac{d\mu'^2}{\mu'^2} b_0^i(\mu'^2) = b_0^i \ln(\mu^2/M_X^2) + 1/\alpha_5(M_X) + \phi^i(\mu^2/M_X^2),$$

where

$$\phi^i(\mu^2/M_X^2) = -C_i \quad \text{for } \mu^2/M_X^2 \to 0,$$

$$= (b_0^5 - b_0^i)\ln(\mu^2/M_X^2) \quad \text{for } \mu^2/M_X^2 \to \infty.$$

Thus, in general

$$F_{ij}(\mu^2) \xrightarrow[\mu^2/M_X^2 \to 0]{} (b_0^i - b_0^j)\ln(\mu^2/M_X^2) + (C_j - C_i).$$

As an example we have calculated the one-loop contributions of superheavy gauge particles together with the corresponding unphysical Goldstone and ghost particles, i.e., the total physically meaningful combination (see table 1). Although individual graphs in the limit $M_X^2/\mu^2 \to \infty$ have gauge dependent constant terms, their sum gives a gauge independent constant contributions [6]. Defining $g^{\mu\nu}g_{\mu\nu} = 4 - 2\epsilon'$ and performing integrations over $\int d^{4-2\epsilon}k$ we obtain in the low energy regime [in the minimal SU(5) model]

$$[-\tfrac{7}{2}\ln(M_X^2/\mu^2) + \tfrac{1}{3}\epsilon'/\epsilon][C(5) - C(i)], \tag{8}$$

where the Casimir coefficients are $C(1) = 0, C(2) = 2, C(3) = 3, C(5) = 5$. Heavy scalars give no contributions to the constant term in either the $\overline{\text{MS}}$ or $\overline{\text{DR}}$ scheme. No superheavy fermions are present in our minimal SU(5). In case we had superheavy fermions and followed Weinberg's prescription [3] of extrapolating the spinor algebra in $2^{2-\epsilon'}$ dimensions, we would find a $-2/3\,(\ln 2)\,\epsilon'$ constant piece. In the $\overline{\text{MS}}$ scheme $\epsilon' = \epsilon$ and a gauge independent constant term survives. On the contrary, in the $\overline{\text{DR}}$ scheme in which we maintain the algebra in four dimensions, $\epsilon' = 0$ and *no constant term appears*.

It is not difficult to see how this comes about in general in the $\overline{\text{DR}}$ scheme. Since the total constant term is gauge independent at low energies, it suffices to consider just the Feynman gauge. Then, two-point functions with external legs corresponding to light gauge bosons (gluons, $W_\pm, Z_0, \gamma$) will be transverse and only logarithmically divergent. They will be of the form

$$(g_{\mu\nu}p^2 - p_\mu p_\nu)\int d^{4-2\epsilon}k \frac{F(p^2/M^2)}{[(p+k)^2 - M^2]\,(k^2 - M^2)}, \tag{9}$$

where in the denominator only the same mass appears. In the limit $p^2/M^2 \to 0$, the integral will not give any constant term but just a $\ln M^2$. It should be stressed that this is due to keeping the algebra in four dimensions ($\epsilon' = 0$) on the one hand, and gauge invariance on the other. Similarly, one could argue about three-point functions.

It is not too hard to see that this property of the $\overline{\text{DR}}$ scheme will be preserved to the next order of perturbation theory. In the next order of the loop expansion the gauge couplings will be corrected by terms of the form

$$\ln[\alpha^{(i)}_{\text{one-loop}}(\mu^2)].$$

It is evident that constant terms cannot arise since they are not present at the one-loop level.

The alert reader has probably already noticed that

Table 1

$$= \tfrac{11}{6} + \tfrac{1}{2}\alpha + \tfrac{1}{3}\epsilon' + \epsilon\{[(1-\alpha^3)/3\alpha^3]\ln(1-\alpha) - [(3-4\alpha+\alpha^2)/2\alpha]\ln(1-\alpha) - \tfrac{25}{18} + \tfrac{1}{4}\alpha + 1/6\alpha + 1/3\alpha^2\}$$

$$= -\tfrac{1}{6} + \tfrac{1}{6}\epsilon\ln(1-\alpha)$$

$$= -\tfrac{1}{6} + \tfrac{1}{6}\epsilon\ln(1-\alpha)$$

$$= -\epsilon[(1/3\alpha^3)\ln(1-\alpha) + 1/3\alpha^2 + 1/6\alpha + 1/9]$$

$$= -1 + \alpha + \epsilon' + \epsilon[-\tfrac{1}{2} + \alpha + (1-\alpha)\ln(1-\alpha)]$$

$$= 3 - \tfrac{3}{2}\alpha - \epsilon' + \epsilon[2 - \tfrac{5}{4}\alpha + \tfrac{3}{2}\alpha^{-1}(1-\alpha)^2\ln(1-\alpha)]$$

gauge propagator = $[1/(k^2-M^2)]\{g_{\mu\nu} - \alpha K_\mu K_\nu/[k^2-(1-\alpha)M^2]\}$

$g^{\mu\nu}g_{\mu\nu} = 4 - 2\epsilon'$

integrations over $\int dp^{4-2\epsilon}$

this property of the $\overline{\text{DR}}$ scheme has important implications for supersymmetric GUTs. In a supersymmetric theory, in order to preserve supercurrent conservation, i.e., supersymmetry, the algebra must be kept in four dimensions [7]. Therefore, one is forced to use the $\overline{\text{DR}}$ scheme or perhaps some derivative subtraction scheme. But in the $\overline{\text{DR}}$ scheme, as we have shown, the supernaive step approximation at M_X (not $2M_X$ or elsewhere) gives the correct result. Thus, all naive renormalization group computations in SUSY GUTs, as long as they have been done in the $\overline{\text{DR}}$ scheme, do not have to be corrected with threshold effects. Of course, this property of the $\overline{\text{DR}}$ scheme is just a property of the scheme and is not connected with supersymmetry. Hence, although this scheme might break supersymmetry at higher loops and perhaps should be abandoned for SUSY GUTs, its legitimate use in ordinary GUTs is unquestionable.

As an application, let us consider the renormalization group equations for the three gauge couplings of a minimal supersymmetric SU(5) theory (with two Higgs doublets). They are (to the order of two loops)

$$1/\alpha_3(M_W) = 1/\alpha_5(M_X) - (b_3^{(0)}/2\pi)\ln(M_X/M_W) - \frac{1}{4\pi}\sum_{j=1}^{3}(b_{3j}^{(1)}/b_j^{(0)})\ln[\alpha_j(M_W)/\alpha_5(M_X)],$$

$$[1/\alpha(M_W)]\sin^2\theta_W(M_W) = 1/\alpha_5(M_X) - (b_2^{(0)}/2\pi)\ln(M_X/M_W) - \frac{1}{4\pi}\sum_{j=1}^{3}(b_{2j}^{(1)}/b_j^{(0)})\ln[\alpha_j(M_W)/\alpha_5(M_X)], \qquad (10)$$

$$\tfrac{3}{5}[1/\alpha(M_w)]\cos^2\theta_w(M_w)$$

$$= 1/\alpha_5(M_x) - (b_1^{(0)}/2\pi)\ln(M_x/M_w)$$

$$-\frac{1}{4\pi}\sum_{j=1}^{3}(b_{3j}^{(1)}/b_j^{(0)})\ln[\alpha_j(M_w)/\alpha_5(M_x)], \quad (10\ \text{con'd})$$

where

$$b_3^{(0)} = 9-2N_G, \quad b_2^{(0)} = 5-2N_G, \quad b_\Lambda^{(0)} = -2N_G - \tfrac{3}{5},$$

and

$$b_{ij}^{(1)} = \begin{bmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{bmatrix} \quad (\text{for } N_G = 3),$$

For $\Lambda_{\overline{MS}} = 0.1$ GeV, we obtain $M_x/M_w = 8.7 \times 10^{13}$ and $\sin^2\theta_w(M_w) = 0.233$.

In conclusion, we would like to restate our result. In the $\overline{DR}$ scheme, where we keep the algebra in four dimensions while performing momentum loop integrations in $4-2\epsilon$ dimensions, constant terms in coupling constant differences that correspond to the contribution of threshold effects are zero. Thus, the naive step approximation gives the correct result. It is evident that our result implies a great simplification in computations. By performing all calculations in the $\overline{DR}$ scheme, the naive step approximation at M_x contains all threshold effects.

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CHAPITRE VIII

STABILITE INFRAROUGE ET "ANTI-GRANDE UNIFICATION"

VIII. 2. On the infrared stability of gauge theories

VIII - STABILITE INFRAROUGE ET "ANTI-GRANDE UNIFICATION"

VIII.1 - Motivations et résultats précédents

Dans ce chapitre, nous présentons une méthode non conventionnelle pour comprendre l'origine des symétries de la nature. Nous avons vu que les interactions entre les particules élémentaires sont bien décrites, au moins aux basses énergies, à l'aide du modèle standard qui est une théorie de jauge basée sur le groupe de symétrie U(1)xSU(2)xSU(3). La philosophie que nous avons développée aux chapitres précédents, consiste à supposer que ce groupe de symétrie vient d'un groupe simple beaucoup plus large (par exemple SU(5), SO(10), E_6 etc.) qui est spontanément brisé à très haute énergie. C'est exactement l'idée de la "grande unification". Les théories grandement unifiées, ainsi que la supersymétrie sont des idées très attrayantes mais elles ont un désavantage important : elles ne peuvent pas répondre à la question de l'origine du groupe de symétrie de la théorie finale. La seule information expérimentale actuelle est que seulement les petits groupes unitaires semblent être préférés par la nature.

Dans ce chapitre, nous allons présenter une façon non orthodoxe pour approcher ce problème, suivant laquelle on postule qu'à basse énergie, on voit une symétrie maximale qui disparaît dans une théorie chaotique à très haute énergie. En d'autres mots, adoptons le point de vue suivant : en principe, il n'y a aucune raison pour choisir un groupe plutôt qu'un autre ; considérons alors qu'à très haute énergie, il n'y ait pas du tout de symétrie. Suivant l'idée de l'"anti-grande unification"[16], toutes nos équations actuelles avec leurs symétries et les valeurs spécifiques de leurs paramètres sont des points fixes attracteurs aux basses énergies de certaines transformations généralisées du groupe de renormalisation. L'espoir est que cette propriété d'être attracteur infrarouge sera vraie seulement pour les petits groupes unitaires qu'on observe. Ceci impliquerait que en commençant avec une théorie quelconque, prise au hasard, à très haute énergie, on se trouverait toujours aux basses énergies au même point fixe symétrique, le modèle standard U(1)xSU(2)xSU(3). Ainsi, nous avons l'ambition, non seulement de découvrir, grâce à cette procédure, les symétries actuelles de la nature (globales ou locales), mais aussi de montrer qu'elles sont les seules possibles.

Commençons alors avec une théorie arbitraire chaotique et essayons de découvrir les symétries qui sont stables dans l'infrarouge. La première observation est que tout terme non renormalisable (de dimension plus grande que quatre) ne contribue pas dans le domaine infrarouge par simple analyse dimensionnelle. D'autre part, tout terme de dimension plus petite que quatre (par ex. un terme de masse) est toujours dominant dans ce domaine (excepté dans quelques cas particuliers) ; cela donne un résultat trival auquel nous ne nous intéressons pas. En conclusion, il nous reste seulement tous les termes possibles de dimension quatre qui forment une théorie renormalisable.

Expliquons maintenant la méthode générale qu'on doit suivre. Par définition, dans la plupart de cas, une théorie symétrique est un cas particulier de cette théorie "chaotique", où les différents paramètres (constantes de couplage) satisfont des relations entre eux, impliquées par la symétrie en question. D'autre part, il est évident que la théorie symétrique est toujours un point fixe de la théorie de départ. En conséquence, pour examiner la stabilité infrarouge de cette symétrie, il est nécessaire et suffisant d'étudier les trajectoires des constantes de couplage autour du point fixe de la théorie symétrique. On a alors besoin des différentes fonctions $-\beta$ calculées au premier ordre de la série des perturbations des constantes de couplage supplémentaires qui brisent la symétrie considérée. Ensuite, il faut vérifier si le rapport des constantes de couplages supplémentaires à celles de la théorie symétrique s'annule bien dans le domaine infrarouge.
Présentons maintenant brièvement les résultats obtenus jusqu'à maintenant :

(i) l'invariance de Lorentz est une symétrie attractive si la théorie non symétrique du départ est une théorie de jauge[19] ;

(ii) certaines symétries internes globales sont attractives seulement pour les petits groupes (O(2)) et répulsives pour les grands groupes (O(2N), $N > 2$)[17,18] ;

(iii) la supersymétrie globale est aussi une symétrie attractive[18,77] ;

(iv) en gardant l'invariance de Lorentz, l'électrodynamique spinorielle est une théorie attractive, tandis que l'électrodynamique scalaire est répulsive[18] ;

(v) en commençant avec des théories définies sur un réseau et en utilisant des variables définies sur un groupe compact (variables des "links") un argument non-perturbatif semble démontrer que toutes les théories de jauge non abéliennes sont attractives dans l'infrarouge[78]. Néanmoins, dans cette approche, la limite continue de toutes ces théories considérées sur le réseau, est toujours la même théorie invariante de jauge.

Dans le chapitre suivant, nous étudions la stabilité infrarouge des théories de jauge par une méthode perturbative, comme celle expliquée auparavant. Nous étudions aussi la stabilité de la supersymétrie N = 4.

ON THE INFRARED STABILITY OF GAUGE THEORIES

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Abstract : We consider a class of field theories which contains a Lorentz and gauge invariant theory as a fixed point, but whose generic member possesses none of these symmetries. We show that this fixed point is an infrared repulsor for all non-abelian groups. We also discuss the case of the N = 4 supersymmetric gauge theory whose stability properties are shown to depend on the gauge group.

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1. INTRODUCTION

The occurrence of symmetries (space-time as well as internal) in Nature has fascinated and puzzled physicists for a long time. The more symmetric a physical theory is, the more attractive it is considered to be. We can hardly imagine a theory which obeys no symmetry principle at all. The quest for symmetry has been proven to be one of the most powerful tools in discovering the laws of Nature. Usually one tries to guess the right symmetry and then incorporates it in the fundamental dynamical equations. Almost every step in our theoretical understanding of the interactions among elementary particles can be associated with the introduction of such a new symmetry. In particular, the great progress which was accomplished in this direction during the last years, was due to the realization that all such interactions are described by theories invariant under a group of gauge transformations. Leaving aside gravitational interactions, which, although they seem to obey a gauge principle, are at present beyond our understanding, all experimental results indicate that the invariance of the world, at least at low energies, is based on the group $\left[\text{Poincaré}\right]_{\text{global}} \times \left[\text{U(1)} \times \text{SU(2)} \times \text{SU(3)}\right]_{\text{local}}$. Notwithstanding present experimental evidence, theorists often like, on essentially aesthetic grounds, to add a supersymmetry with one, or more, spinorial generators.

A question which is rarely asked is the origin of these symmetries. Poincaré invariance is usually taken for granted because it reflects the four-dimensional continuous structure of space and time. There is no obvious justification for the U(1) x SU(2) x SU(3) gauge symmetry of the standard model and it is in fact puzzling that only small unitary groups seem to be relevant. The prevailing philosophy is that of "grand-unification" which assumes that this is the remnant of a larger, probably semi-simple, group G which is spontaneously broken at very high energies. SU(5), SO(10) or E_7, with or without supersymmetric extensions, are often considered as candidates for G. This does not answer the question of the origin of the observed symmetries, it only transfers it to higher

energies, since the choice of G is usually made by guess. N = 8 supergravity is the only attempt to at least a partial answer to this problem. Assuming one excludes fundamental particles with spin higher than two, N = 8 is the largest supergravity one can write and SU(8) seems to be the largest gauge symmetry it admits with only the basic multiplet. The remaining problem is the origin of supergravity itself at the Planck mass.

The unconventional way to answer these questions is to say that, since no group is more special than the others, none will be chosen. At extremely high energy there is no symmetry. The idea is that our present equations with their symmetries and the specific values of their parameters, are infrared attractive fixed points of some generalized renormalization group transformations. Since the philosophy of this approach has already been explained [1] , we briefly review the results : global internal symmetries present the interesting feature of being infrared attarctors for small groups (O(2)) and repulsors for large ones (O(2N) $N > 2$) [2,3] ; global supersymmetry is also an infrared attractor [3,4] .

For the important case of gauge theories, one may consider a model which is invariant under gauge transformations but not under Lorentz ones. The model contains several independent coupling constants because the different "components" of the gauge potential couple independently. For the particular case in which all these coupling constants are equal one recovers the ordinary Lorentz invariant theory. The interesting result is that this symmetric theory is an infrared attractive fixed point. If we start somewhere in its vicinity we shall come across it. This result is true for pure Yang-Mills theories [5] as well as for the abelian spinor electrodynamics [6] .

One may consider the opposite case, namely one may try to keep Lorentz invariance and attempt to derive gauge invariance by use of the same principle. The only case which has been studied is that of abelian theories [3] . Spinor electrodynamics turns out to be an infrared attractor but scalar electrodynamics is not.

Finally, starting from theories defined on a space-time lattice in terms of compact group-valued variables, a non-perturbative argument indicates that all gauge theories are infrared attractors [7].

In this paper we want to report on some further results along these lines. Our approach is perturbative but we shall try to recover both Lorentz and gauge invariance. Unfortunately, the results are not very encouraging. In sect. 2 we study the stability of N = 4 supersymmetry for arbitrary gauge group G. We keep the invariance under one spinorial generator and we investigate the conditions under which the full N = 4 invariance is recovered in the infrared. We find that the N = 4 theory is an infrared attractor only for G different from SU(N) ($N \geqslant 3$) and E_6. Sect. 3 contains the results for Yang-Mills theories. We find that gauge symmetry is an infrared repulsor for all groups. This unwelcome result is due to the property of asymptotic freedom. We can change it by introducing a sufficient number of fermions in order to reverse the sign of the β-function of the symmetric theory. In sect. 4 we study the possibility of spontaneous breaking through radiative corrections of some global symmetries which we had kept exact. In our case these were a global SU(N) symmetry and a symmetry under three dimensional rotations. Finally, in sect. 5 we consider the special case of the unitary groups U(1), SU(2) and SU(3).

2. SUPERSYMMETRY

The infrared stability of N = 1 global supersymmetry has already been studied in several models [3,4]. Among the approaches, though, which have recently been proposed for the eventual unification of the gravitational with the rest of the interactions, the most attractive ones seem to involve more than one local supersymmetry. In particular it is suspected, although far from being demonstrated, that the N = 8 supergravity theory [8] could be something more than an interesting mathematical construction. We would like to offer an independent argument pro or against symmetries with more than one spinorial generators by studying the stability of the $N > 1$ fixed points. The largest N for

which a globally supersymmetric theory can be constructed is N = 4 and since our method, which requires renormalizability, cannot be applied to locally supersymmetric theories, we will restrict ourselves to the discussion of the stability of the N = 4 supersymmetric fixed point. As we will see in what follows its properties depend on the gauge group chosen and this is the first example of such a distinction that we encounter.

In terms of N = 1 superfields, the field content of the N = 4 supersymmetric Yang-Mills theory (SYM) with gauge algebra G is : a gauge superfield V and three chiral superfields Φ^i , i = 1 , 2 , 3 , each being in the adjoint representation of G. Their components are labelled Φ^i_a with a = 1,2,..., dim-G (the dimension of the algebra G). For simplicity we will consider the set of field theories with the above field content which apart from being invariant under the gauge group G explicitly realize one of the four supersymmetries of the N = 4 theory as well as the SO(3) under which the three Φ^i 's form a triplet.

The generic theory belonging to this space is described by :

$$\mathcal{L} = \int d^4\theta \sum_{i=1}^{3} Tr\left(e^{-gV} \bar{\Phi}_i \, e^{gV} \Phi^i \right) + \frac{1}{64 g^2} \int d^2\theta \, Tr \, W^\alpha W_\alpha$$

$$+ \left\{ \frac{i\lambda}{3!} \int d^2\theta \, \epsilon_{ijk} \, Tr\left(\Phi^i \left[\Phi^j , \Phi^k \right] \right) + h.c. \right\} \qquad (2.1)$$

$$+ \text{Gauge fixing} + \text{Ghost terms}$$

where $\Phi^i = \sum_a \Phi^i_a \, t^a$, $t^{a^+} = t^a$ are the generators of G in the defining representation (N x N matrices for O(N)) and

$$W_\alpha = \bar{D}^2 \left(e^{-gV} D_\alpha \, e^{gV} \right).$$

Notice that (a) The N = 4 SYM corresponds to $\lambda = g$ [9] (b) For G $\neq$ SU(N $\geq$ 3) or E_6, i.e. for G one of the so-called safe algebras [10] the SO(3) invariance mentioned above does not have to be imposed for $\mathcal{L}$ to take the form (2.1). It is automatic if we assume the gauge symmetry and N = 1 supersymmetry. The reason is simply that, identically, $Tr \, \Phi^i \left\{ \Phi^j , \Phi^k \right\}$ is equal to zero in these cases.

To study the stability of the $\lambda = g$ fixed point we need the β-functions for the couplings in the theory. The graphs involved in the calculation of β_g at the one-loop level do not contain any Φ^3 coupling and, consequently, their values are the same as in the N = 4 symmetric theory. Thus, at this order [11]

$$\beta_g = 0 \quad . \tag{2.2}$$

As for β_λ, because of the finiteness of the $\Phi_i - \Phi_j - \Phi_k$ three-point 1PI vertex to any loop order, one only has to calculate the $\Phi_i - \bar{\Phi}_i$ propagator. It receives the 1-loop contributions given in (Fig. 1) where we used the fact that for $\lambda = g$ the sum of the two graphs on the left is equal to zero [12]. It is now evident that up to a factor const $\times \pi^2$, which is eventually absorbed into the definition of the scale parameter t,

$$\beta_\lambda = \lambda\left(\lambda^2 - g^2\right) \quad . \tag{2.3}$$

(The sign of the λ^3 term in β_λ is exactly what is dictated by the absence of asymptotic freedom in the theory with g = 0). The linearized equation

$$\frac{d}{dt}(\lambda - g) = \beta_\lambda \approx 2 g^2 (\lambda - g) \tag{2.4}$$

leads to the conclusion that the N = 4 SYM is an infrared attractor.

In the spirit of our approach, however, we should not impose any restriction on the "theory space" around the fixed point. Thus, for G = SU($N \geqslant 3$) or E_6 we must include couplings of the form $f_{ijk}\, \mathrm{Tr}\, \Phi^i \left\{\Phi^j, \Phi^k\right\}$ in the Lagrangian. Using arguments similar to those presented above, one can easily conclude that in this case

$$\begin{aligned} \beta_g &= 0 \quad , \\ \beta_\lambda &= \lambda\left(\lambda^2 - g^2 + \cdots\right) \quad , \\ \beta_{f_{ijk}} &= A\, f_{ijk}\left(\lambda^2 - g^2 + \cdots\right) \quad , \end{aligned} \tag{2.5}$$

where the dots denote terms of order f^2 and A is a numerical constant.

The stability properties of the N = 4 SYM fixed point ($\lambda = g$, $f_{ijk} = 0$ $\forall\, i, j, k$) are determined by the linearized RG equations :

$$\frac{dg}{dt} = 0 \quad ,$$

$$\frac{d}{dt}(\lambda - g) \approx 2 g^2 (\lambda - g) \tag{2.6}$$

and $$\frac{d}{dt} f_{ijk} \approx 2 A g f_{ijk} (\lambda - g) \quad .$$

The evolution of f_{ijk} depends upon the sign of the product $Ag(\lambda - g)$ which is arbitrary and, consequently, the above-fixed point is not an attractor (neither infrared nor ultraviolet).

3. YANG-MILLS THEORIES

In this section we study the infrared limit of a class of theories which contain, as a fixed point, a theory which is invariant under Lorentz transformations and SU(N) gauge transformations. The generic member of this class, however, possesses none of these symmetries, although, for simplicity, we shall restrict ourselves to theories which have a global SU(N) invariance. We thus consider the Lagrangian :

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} Tr \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 + \frac{1}{\alpha \eta^2} Tr \left(\eta_\mu A_\mu\right)^2 - \xi Tr \left(\partial_\mu A_\mu\right)^2 - \xi \frac{1}{\eta^2} Tr \left(\eta_\mu \partial_\mu A_\nu\right)^2 \\ & + 2 i g Tr \, \partial_\mu A_\nu [A_\mu, A_\nu] + \frac{1}{2} G \, Tr [A_\mu, A_\nu]^2 \\ & - \frac{1}{2} \lambda_1 Tr \left([A_\mu, A_\nu]_+\right)^2 - \lambda_2 \left(Tr A_\mu A_\nu\right)^2 - \frac{1}{2} \lambda_3 \left(Tr A_\mu A_\mu\right)^2 \end{aligned} \tag{3.1}$$

where A_μ belongs to the adjoint representation of SU(N). The definitions and conventions we are using are :

$$A_\mu = \frac{1}{2} A_\mu^a t^a \qquad a = 1, \ldots, N^2 - 1 \quad . \tag{3.2a}$$

The N x N traceless Hermitian matrices t^a are normalized according to

$\mathrm{Tr}\, t^a t^b = 2\delta^{ab}$ and satisfy

$$[t^a, t^b] = 2i f^{abc} t^c \,,$$
$$\{t^a, t^b\} = \frac{4}{N}\delta^{ab} + 2 d^{abc} t^c \,. \qquad (3.2b)$$

The symbol $[\ ,\ ]_+$ is defined by $[t^a, t^b]_+ \equiv 2 d^{abc} t^c$.

η_μ is a fixed vector which we may choose to be of the form $(\eta, 0, 0, 0)$ and (3.1) is supposed to be taken at the limit $\alpha = 0$. The reason is that it is essential in our approach to have renormalizability. With ξ treated as a coupling, the A^a_μ propagator has the appropriate $1/k^2$ -large momentum behavior (necessary for the theory (3.1) to be power counting renormalizable) only for $\alpha = 0$. Therefore the second term implies that $A_0 = 0$. As it is written (3.1) is invariant under three-dimensional rotations as well as SU(N) global transformations. The extra symmetry under $A_\mu \to -A^T_\mu$ forbids the appearance of a term proportional to $\mathrm{Tr}\, \partial_\mu A_\nu [A_\mu, A_\nu]_+$, while the use of dimensional regularization allows us to consider a purely massless theory. The symmetric limit is obtained for

$$\xi = \zeta = 0 \quad ; \quad \lambda_1 = \lambda_2 = \lambda_3 = 0 \quad ; \quad G = g^2 \,. \qquad (3.3)$$

At this point we find the SU(N) Yang-Mills theory in the axial gauge and, consequently, for physical quantities, we recover both Lorentz and gauge invariance. We wish to study the nature of the fixed point (3.3) in the infrared limit. We must first compute the corresponding β -functions at one loop. We start from those of ξ and ζ . They are given by the diagram of Fig. 2 where the cross represents a $(\partial_\mu A_\mu)^2$ or a $(\eta_\mu \partial_\mu A_\nu)^2$ insertion. A straightforward calculation gives :

$$8\pi^2 \beta_\xi = -\frac{11}{3} N \xi g^2 \,, \qquad (3.4)$$

$$8\pi^2 \beta_\zeta = \frac{32}{3} N \xi g^2 \,. \qquad (3.5)$$

Already (3.4) shows that $\xi = 0$ is a fixed point which is repulsive in the

infrared and, therefore, the symmetric theory (3.3) is not an infrared attractor. It is instructive to trace the origin of this failure : The diagram of Fig. 2, with either one of the insertions, gives no counterterm of the form $(\partial_\mu A^a_\mu)^2$. Therefore, to this order, the entire contribution to β_ξ comes from Z_3, the wave function renormalization of the field which, in the axial gauge, gives the β -function of the symmetric theory. The asymptotic freedom of the latter is responsible for the rise of ξ in the infrared. Notice that this result is true for every non-abelian global group G, not only SU(N).

Although this result is conclusive, we give for future reference the β - functions of the remaining coupling constants. We are interested in the relative evolution of λ_i and $G - g^2$ with respect to g^2. We thus define the ratios $\delta_i = \lambda_i / g^2$ $i = 1,2,3$ and $\delta = (G - g^2)/g^2$. To lowest order in the small parameters we find :

$$8\pi^2 \beta_{\delta_i} = - \frac{11}{3} N \delta_i g^2 \quad , \tag{3.6a}$$

$$8\pi^2 \beta_\delta = - \frac{11}{3} N \delta g^2 \quad . \tag{3.6b}$$

We conclude that these ratios also grow in the infrared and this behaviour is again due to the asymptotic freedom of the symmetric theory. Indeed, we find that the counterterms for all the non-symmetric terms vanish at this order and the entire contribution to the β -functions comes again from the wave function renormalization of the field. This conclusion suggests a way to change the nature of the symmetric fixed point. We add to the theory a number of fermion multiplets sufficient to reverse the sign of the β -function of the symmetric theory. For definiteness, let us take n multiplets, each belonging to an r -dimensional representation of the global group. This introduces a new coupling constant, e, which multiplies the $\bar{\psi} - A - \psi$ term (For simplicity, we choose the fermion representation such that $\bar{r} \times r$ contains the adjoint only once. In addition we impose discrete symmetries which forbid terms of the form $\bar{\psi}_i - A - \psi_j$ for $i \neq j$). In the symmetric limit $e = g$, therefore we choose $| e^2 - g^2 | \ll g^2$. The counterterms computed previously

remain the same, i.e. they still vanish at first order in the small parameters. Therefore the β-functions (3.4) and (3.6) change signs following the change in sign of the symmetric β - function. On the other hand (3.5) becomes :

$$8\pi^2 \beta_\zeta = \frac{32}{3} N \zeta g^2 + \frac{4}{3} T_F e^2 \zeta \ , \tag{3.7}$$

where T_F is the trace of the sum of the squares of the generators in the fermion representation. It is now obvious that ζ also approaches zero in the infrared. We still have to check whether $e^2 - g^2$ goes to zero. We define $\rho = (e - g)/g$ and compute the β - function. We get

$$16\pi^2 \beta_\rho = \left(-3N + \frac{8}{3} T_F\right) g^2 \rho + \cdots \tag{3.8}$$

which shows that $\rho \longrightarrow 0$. By loosing asymptotic freedom, we have gained infrared attractiveness.

4. THE FATE OF THE GLOBAL SYMMETRIES

In the previous section we studied the infrared nature of the symmetric fixed point. For this purpose it was sufficient to start somewhere in its vicinity, namely at a point where all symmetry breaking coupling constants are smaller than g^2. In the spirit of our approach, however, this is a very special choice ; the generic theory is the one in which all coupling constants take arbitrary (although small) values. In this section we examine this general case. The introduction of the gauge non-invariant four boson couplings in the Lagrangian (3.1) gives rise to a non-trivial effective potential V(A). We want to study the properties of V(A) and, in particular, to answer the following two questions : i) Is V(A) bounded from below and ii) does V(A) develop a minimum away from the origin ? If this happens, we obtain a spontaneous breaking of the remaining symmetries, namely the global SU(N) and the three dimensional rotational symmetry. The mechanism we have in mind is the one suggested by S. Coleman and E. Weinberg [13] for theories with scalar mesons. Let us briefly recall their argument for the simple case of massless scalar electro-

dynamics. We consider the Lagrangian :

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 + \frac{1}{2}(\partial_\mu \varphi)^2 + e A_\mu (\partial_\mu \varphi)\tilde{\varphi} + \frac{1}{2} e^2 A_\mu^2 \varphi^2 - \frac{\lambda}{4!}(\varphi^2)^2 \tag{4.1}$$

where φ is a two-component real scalar field (φ_1, φ_2), $\varphi^2 = \varphi_1^2 + \varphi_2^2$ and $\tilde{\varphi} = (\varphi_2, -\varphi_1)$. It is important to notice that φ is massless. (4.1) is invariant under U(1) gauge transformations. In order to study the possibility of spontaneous symmetry breaking we construct the effective potential of the scalar fields. The result at the one loop order, is :

$$V = \frac{\lambda}{4!}\varphi^4 + \frac{3e^4}{64\pi^2}\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right) + O(\lambda^2) \tag{4.2}$$

where μ is a subtraction point used to define the coupling constant $\lambda(\mu)$. In the tree approximation the minimum of V is at $\varphi = 0$ and no spontaneous symmetry breaking occurs. At the one-loop order, however, the minimum is at $\varphi = \mu$, provided

$$\lambda(\mu) = \frac{33}{8\pi^2} e^4 \quad . \tag{4.3}$$

This is not a restriction on the values of the physical parameters because λ depends on the subtraction point μ and varies through the renormalization group equation. It is easy to show that, when μ varies, $\lambda(\mu)$ runs over the entire positive real axis (naturally, when λ grows large, the perturbation expansion is no more reliable) and, in particular, there is always a value $\mu = \mu_c$ such that $\lambda(\mu_c)$ satisfies the equation (4.3). μ_c is the value of the field φ at the minimum of the potential and this shows that this model always exhibits spontaneous symmetry breaking.

We repeat the analysis for the Lagrangian (3.1). The role of φ is played by the field A_μ. If it takes a non-zero vacuum expectation value, both rotation and global SU(N) symmetries will be spontaneously broken. The effective potential

depends on three constant, hermitian, traceless matrices and, in the tree approximation, it is, up to a minus sign, the sum of the last four terms in (3.1). The lesson we learnt from the previous example is that, for the Coleman-Weinberg phenomenon to occur in a natural way, the lowest order expression of the effective potential must become of the same order of magnitude as the one-loop corrections for some value of the subtraction point. Furthermore, this should happen while all coupling constants remain small and the potential is bounded from below. In our case, the necessary condition for spontaneous symmetry breaking, the analogue of eq.(4.3), is of the form :

$$\text{A linear combination of } (G, \lambda_1, \lambda_2, \lambda_3) \sim O(g^4, G^2, \lambda_i^2, \dots) \tag{4.4}$$

where the precise form of the linear combination of the left-hand side depends on the matrix configuration in the vacuum expectation value of A_μ . In order to check whether (4.4) can be satisfied, we need the β -functions of the coupling constants. They read :

$$\begin{aligned} 8\pi^2 \beta_{\lambda_0} = &\left(-\frac{22}{3} N g^2 + \frac{8}{3} T_F e^2\right)\lambda_0 - \frac{4}{3} T_F g^2 \rho^2 e^2 - 2N\lambda_0^2 \\ &+\left(2N - \frac{76}{3N}\right)\lambda_1^2 + \lambda_1\left(\frac{2}{3}\lambda_2 + \frac{4}{3}\lambda_3\right) \\ &+ \lambda_0\left[\frac{4}{3}\left(2N - \frac{9}{N}\right)\lambda_1 + 6\lambda_2 - \frac{8}{3}\lambda_3\right] \end{aligned} \tag{4.5}$$

$$\begin{aligned} 8\pi^2 \beta_{g^2} = &\left(-\frac{11}{3} N + \frac{4}{3} T_F\right) g^4 - 4 T_F g^2 f^2 - \frac{8}{3} T_F g f^3 + 3N\lambda_0 g^2 \\ &+\left(-2N + \frac{8}{N}\right)\lambda_1 g^2 - 4\lambda_2 g^2 + 4\lambda_3 g^2 \end{aligned} \tag{4.6}$$

$$\begin{aligned} 8\pi^2 \beta_{\lambda_1} = &\left(-\frac{22}{3} N g^2 + \frac{8}{3} T_F e^2\right)\lambda_1 + \frac{1}{6} N \lambda_0^2 + \left(\frac{3N}{2} - \frac{104}{3N}\right)\lambda_1^2 \\ &+ \frac{5}{3} N \lambda_0 \lambda_1 + \lambda_1\left(8\lambda_2 + \frac{4}{3}\lambda_3\right) \end{aligned} \tag{4.7}$$

$$\begin{aligned} 8\pi^2 \beta_{\lambda_2} = &\left(-\frac{22}{3} N g^2 + \frac{8}{3} T_F e^2\right)\lambda_2 + \lambda_0^2 + \left(\frac{23}{3} - \frac{196}{3N^2}\right)\lambda_1^2 \\ &+\left[\left(-\frac{2N}{3} + \frac{4}{N}\right)\lambda_2 + \frac{8}{3N}\lambda_3\right]\lambda_1 + \left(-\frac{N^2}{3} + \frac{11}{3}\right)\lambda_2^2 + \frac{2}{3}\lambda_3^2 \\ &+\left(-\frac{2}{3}\lambda_1 + \frac{2N}{3}\lambda_2\right)\lambda_0 \end{aligned} \tag{4.8}$$

$$8\pi^2 \beta_{\lambda_3} = \left(-\frac{22}{3}N g^2 + \frac{8}{3}T_F e^2\right)\lambda_3 + \frac{2}{3}\lambda_0^2 + \left(-2 + \frac{232}{3N^2}\right)\lambda_1^2$$
$$+ \left[\left(4N - \frac{56}{3N}\right)\lambda_2 + \left(\frac{16}{3}N - \frac{80}{3N}\right)\lambda_3\right]\lambda_1 + \left(\frac{2N^2}{3} + 2\right)\lambda_2^2$$
$$+ \left(\frac{4}{3}\lambda_1 + \frac{4}{5}N\lambda_2 + \frac{8}{3}N\lambda_3\right)\lambda_0 + \left(\frac{4}{3}N^2 + \frac{20}{3}\right)\lambda_2\lambda_3 + \left(N^2 - 1\right)\lambda_3^2 \qquad (4.9)$$

where we have put $\lambda_0 \equiv G - g^2$. In these formulae N must be larger than three, otherwise the different couplings are not all independent. We shall come back to the case of SU(2) and SU(3) in the next section. The β -functions (4.5)-(4.9) are too complicated and a complete analysis of the general case can only be done numerically. Furthermore, we expect the results to depend on the initial values assigned to the coupling constants. Indeed, a detailed numerical analysis confirms this fact. If one starts from what seems to be the most "natural" region of initial values, namely all couplings small (in the numerical computations values between 10^{-2} and 10^{-3} were used) and of the same order of magnitude one finds that those that vary most significantly are λ_2 and λ_3 . Fig. 3 shows these variations for the case of SU(10). This can be easily understood if one looks at the β -functions in the large N limit. The dominant terms are those proportional to N^2 and β_{λ_2} is negative while β_{λ_3} is positive. Consequently λ_2 increases fast in the infrared and λ_3 decreases and becomes negative. All other coupling constants present a slower rate of variation and remain positive. It follows from this analysis that the prefered direction for symmetry breaking (if breaking occurs) is the one that will make the coefficient of λ_3 in the left-hand side of (4.4) maximum, while, at the same time, it will minimize the coefficients of the other coupling constants and specially λ_2 . There is no non-zero matrix configuration that makes the coefficient of λ_2 vanish since $(\mathrm{Tr}\, A_\mu A_\nu)^2$ is positive-definite but one can, for $N \geq 6$, put the coefficients of G and λ_1 to zero. We can then easily convince ourselves that the most "favorable" direction for symmetry breaking for SU(N), $N \geq 6$ is when

$$A_i = \rho\, t_i \qquad i = 1, 2, 3 \qquad (4.10)$$

where ρ is a constant and t_i are three elements of the Cartan sub-algebra of SU(N) such that $\{t_i, t_j\} = 0$ for $i \neq j$. In this case we have

$$\left(\mathrm{Tr}\, A_\mu A_\nu\right)^2 = \frac{1}{3}\left(\mathrm{Tr}\, A_\mu A_\mu\right)^2 = 12\,\rho^4 \tag{4.11}$$

and the effective potential in the tree approximation is :

$$V_{tree} = 12\left(\lambda_2 + \frac{3}{2}\lambda_3\right)\rho^4 \quad . \tag{4.12}$$

It follows that spontaneous symmetry breaking will occur if $\sigma = \lambda_2 + \frac{3}{2}\lambda_3$ can become as small as $O(\lambda_i^2)$ while all coupling constants remain small. Fig. 3 shows the variation of σ for SU(10). We see that it does not become arbitrarily small and, in fact, it is driven upwards by the rise of λ_2. Soon λ_2 grows too much, it becomes of order one, and the perturbation expansion can no longer be trusted. Spontaneous symmetry breaking does not occur.

A similar numerical analysis shows that this result is also true for SU(4) and SU(5). When we start with all coupling constants positive and of the same order of magnitude, there is no symmetry breaking. Notice though, that, by changing the initial conditions, we can obtain a different picture. For example, for the case of SU(10), if we start with λ_2 considerably smaller than the other coupling constants (a factor of five is sufficient) we find that the decreasing values of λ_3 force σ to fall in the infrared, thus triggering a spontaneous symmetry breaking.

The conclusion of this section can be stated as follows : If one considers the example of massless scalar electrodynamics, one finds that the φ^4 coupling constant λ of eq. (4.1) becomes smaller as one moves towards the infrared. From this one might naively conclude that this theory makes no sense since, for a subtraction point μ smaller than a certain μ_0, λ becomes negative. This however is wrong. As soon as the relation (4.3) is satisfied, the Coleman-Weinberg phenomenon occurs and the theory spontaneously develops a mass. In other words, the infrared behaviour of the theory cannot be studied by looking at its massless phase. This may explain the fact that gauge symmetry was found to be an infrared repulsor for this

theory. We tried here to see if something similar was happening for the theories in the class (3.1). We showed that, like scalar Q.E.D., the gauge symmetry is never recovered, but here stop the similarities. For a large domain of initial values of the coupling constants, which includes the most "natural" values of all couplings of the same order of magnitude, we found no evidence for spontaneous symmetry breaking for all groups SU(N) with $N \geqslant 4$. These theories do not seem to save their infrared behaviour by developing a mass spontaneously.

5. U(1) , SU(2) , SU(3)

As we already said, the results of the previous section apply only to gauge theories based on the groups SU(N) $N \geqslant 4$. In this section we shall study the small groups separately.

The abelian theories have been studied in ref. [3] using a Lorentz covariant formalism. Spinor electrodynamics was found to be an infrared attractor. We can add that this property remains true even if we break Lorentz invariance and add the ξ and ζ terms of eq. (3.1). These terms do not get renormalized at the one-loop level and, therefore, their β - functions are determined by the vector-boson wave function renormalization. Since this theory is not asymptotically free, all these couplings go to zero in the infrared. Concerning the possibility of spontaneous symmetry breaking through radiative corrections, we remark that in this theory, there is only one symmetry breaking parameter : the coefficient of the A_μ^4 term. It follows that every term in the effective potential, to any order in perturbation theory, will be proportional to this coupling constant. Therefore, as long as all coupling constants remain small, the Coleman-Weinberg phenomenon does not occur.

Scalar electrodynamics is not an infrared attractor [3] . Furthermore, it suffers from spontaneous symmetry breaking through radiative corrections. However this model is very interesting because it describes the nematic-smectic A phase transition in liquid crystals [14] . For this reason we have performed a detailed analysis of its infrared properties which we shall present elsewhere.

Coming to SU(2) Yang-Mills theory, we notice that we can write only two

independent couplings. We choose them to be the terms proportional to G and λ_3. It is easy to check that, starting from a point where all coupling constants are small and equal, we arrive, as we move towards the infrared at a region where $\lambda_3 \sim \lambda_0^2$ (Fig. 4). At this point the Coleman-Weinberg dimensional transmutation occurs, A_r^a acquires a non-zero vacuum expectation value and the global SU(2) symmetry as well as the three dimensional rotation symmetry become spontaneously broken.

Finally, the SU(3) Yang-Mills theory behaves in a similar way. One has three independent coupling constants, say G, λ_2 and λ_3. Again, λ_3 decreases in the infrared. Consequently we consider a matrix configuration which gives a large λ_3 coefficient in the effective potential. We choose $\langle A_1 \rangle = \rho t_3$, $\langle A_2 \rangle = \rho t_8$ and $\langle A_3 \rangle = 0$. We obtain :

$$V_{tree} = 8 (\lambda_2 + \lambda_3) \rho^4 . \qquad (5.1)$$

Fig. 5 shows the variation of the coupling constants as well as the combination $\sigma = \lambda_2 + \lambda_3$. We see that σ decreases in the infrared region and becomes of order λ_i^2 . Radiative corrections induce again a spontaneous symmetry breaking and the theory becomes massive.

6. CONCLUSION

We studied the possibility of obtaining the symmetries of the standard model as infrared attractive fixed points. Our results do not confirm the arguments based on the compact version of gauge theories formulated on a space-time lattice [7] . All gauge symmetries are found to be infrared repulsors. However, the intriguing special feature of SU(2) and SU(3) is that they develop a spontaneous mass generation and, consequently, like scalar electrodynamics, their infrared properties cannot be consistently investigated using the class of massless theories we considered here.

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FIGURE CAPTIONS

1 : The diagrams contributing to β_λ in (2.1).

2 : The diagram contributing to the β -functions of ξ and ζ .

3 : The evolution of the coupling constants for SU(10).

4 : The evolution of the coupling constants for SU(2).

5 : The evolution of the coupling constants for SU(3).

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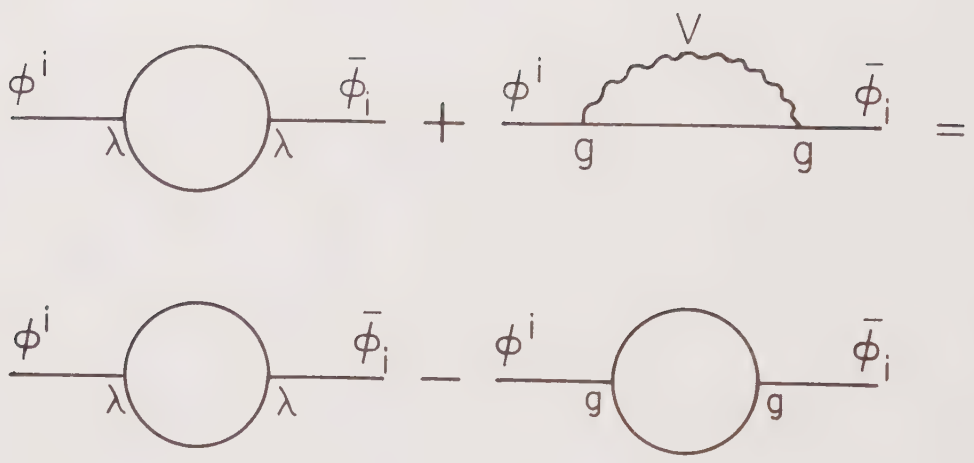

Fig. 1

$(\partial_\mu A^{a\mu})^2$ or $(\eta \cdot \partial A^a_\mu)^2$

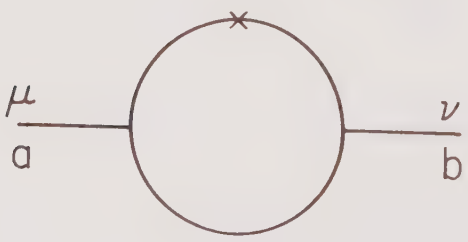

Fig. 2

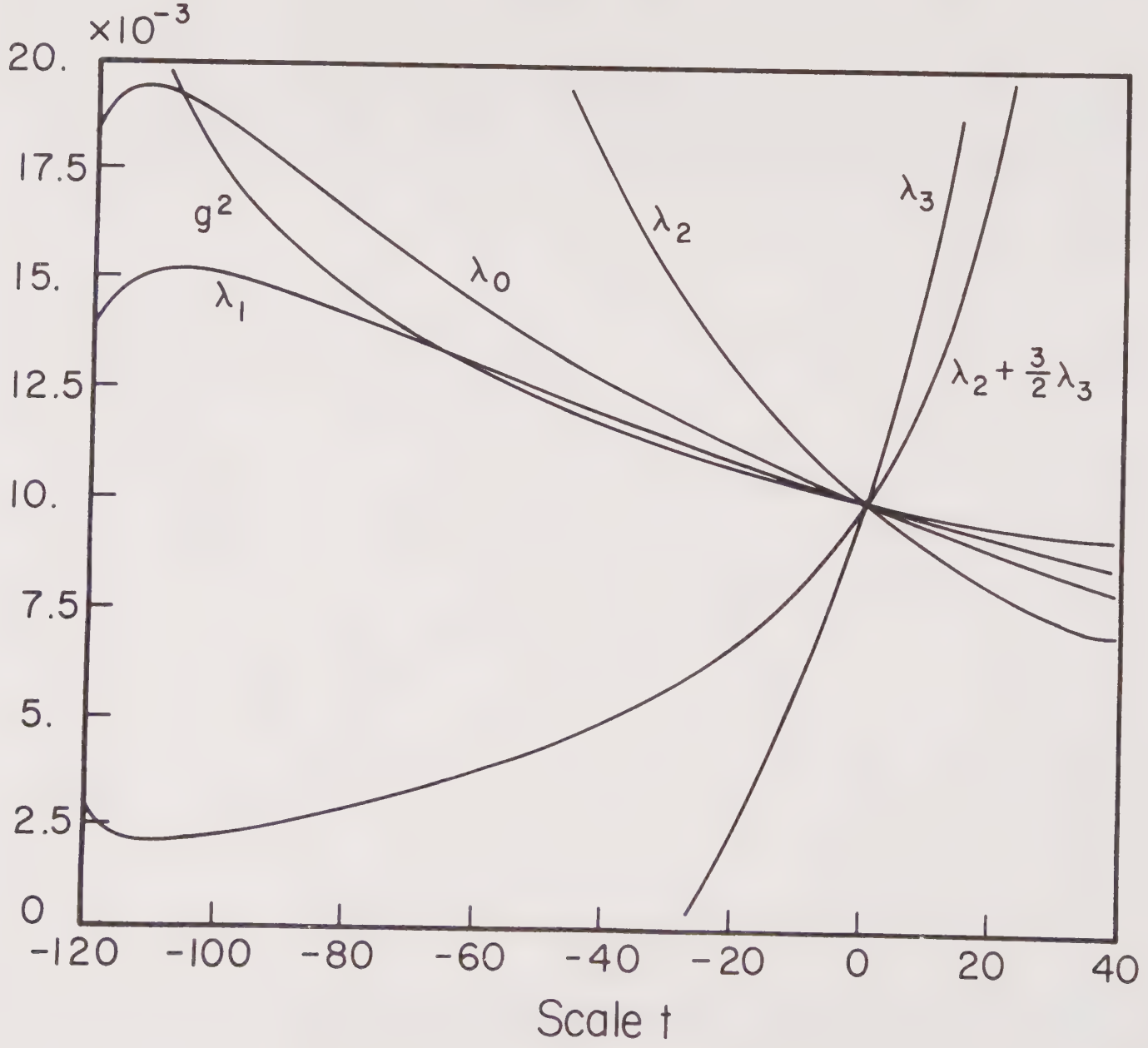
×10⁻³
20.
17.5
15.
12.5
10.
7.5
5.
2.5
0
g²
λ₀
λ₁
λ₂
λ₃
λ₂ + 3/2 λ₃
-120
-100
-80
-60
-40
-20
0
20
40
Scale t

Fig. 3

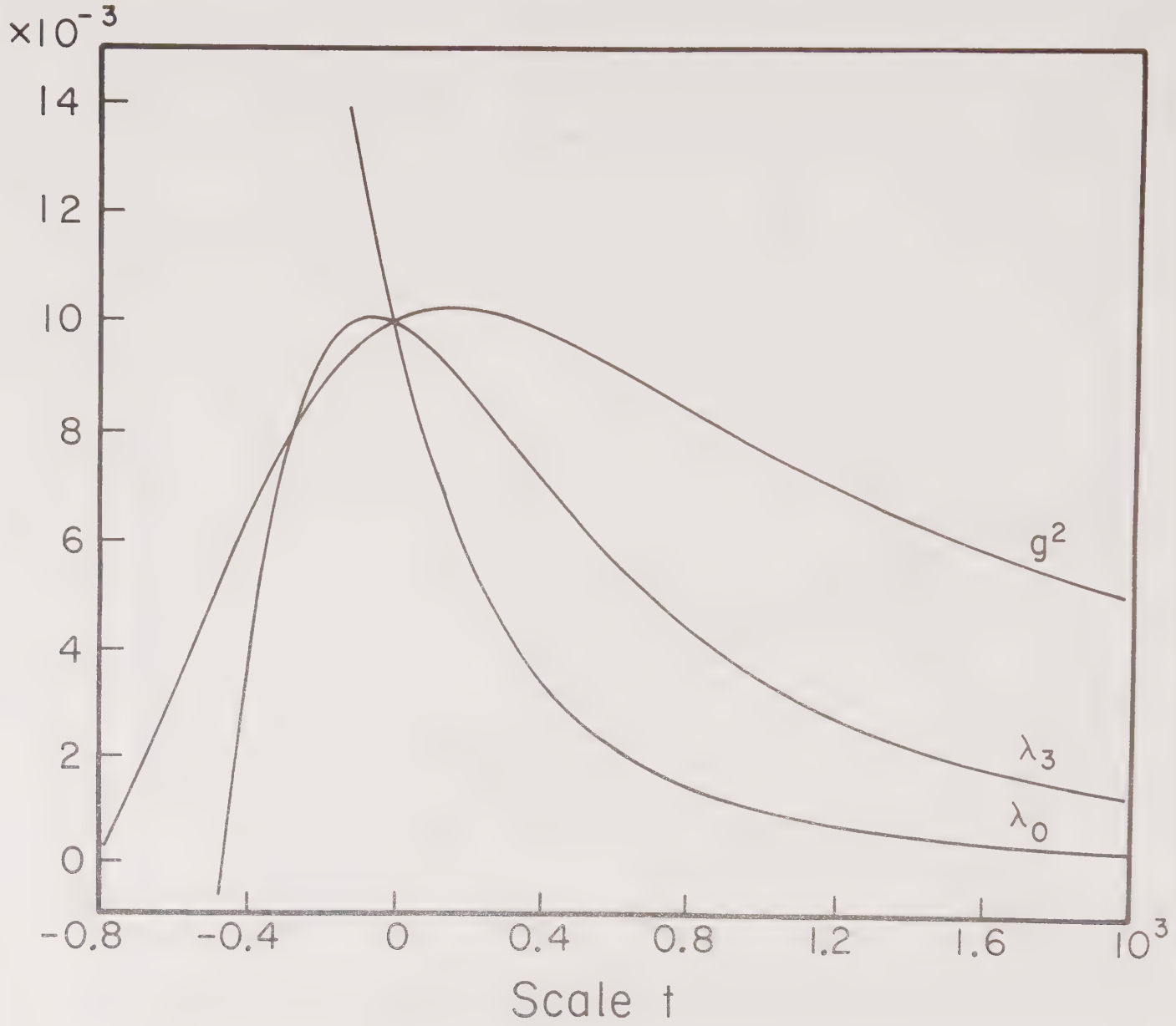

Fig. 4

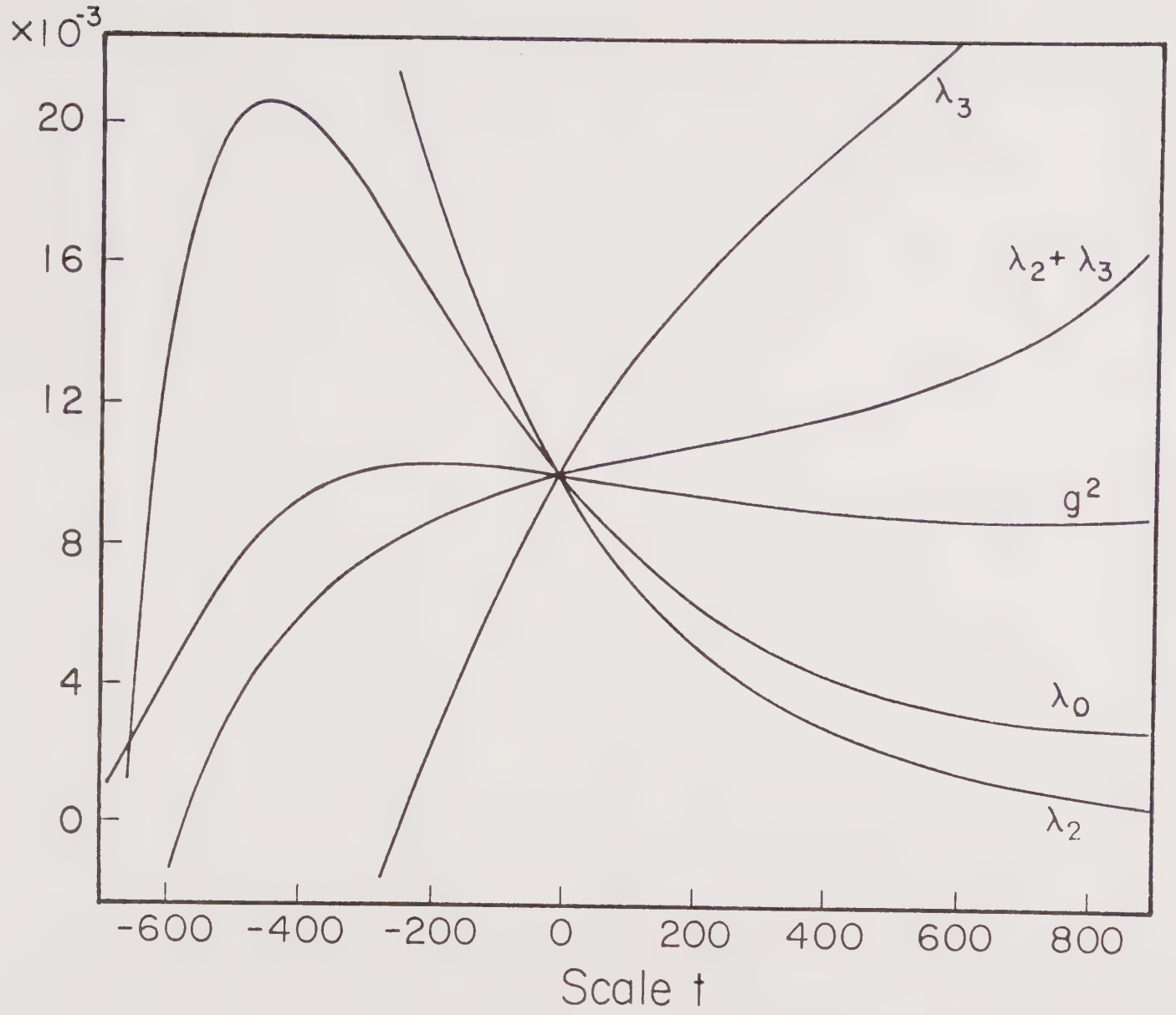
×10⁻³
20
16
12
8
4
0
λ3
λ2+ λ3
g²
λ0
λ2
-600
-400
-200
0
200
400
600
800
Scale t

Fig. 5

VIII.3 - Indépendance de jauge des résultats

Dans ce chapitre, nous voulons donner des arguments sur l'indépendance par rapport à la procédure suivie des résultats obtenus au chapitre précédent. En effet, dans la théorie chaotique de départ, l'invariance de jauge ainsi que l'invariance de Lorentz ont été brisées en gardant seulement une invariance globale interne et une invariance O(3), au lieu de celle du groupe de Lorentz. Ainsi le Lagrangien de la théorie symétrique se trouvait dans la jauge axiale. On pourrait se demander si nos résultats sont indépendants de la procédure suivie.

Essayons alors une autre procédure où le Lagrangien de la théorie symétrique se trouve dans une jauge covariante. Dans ce cas, la théorie chaotique du départ pouvait conserver l'invariance de Lorentz ainsi que l'invariance globale interne, mais elle doit contenir des scalaires anticommutants qui jouent le rôle des fantômes dans la théorie symétrique. Pour cela, introduisons un nombre n de scalaires complexes Φ_i, i = 1,..,n appartenant à la représentation adjointe du groupe ; pour simplifier, demandons aussi une invariance supplémentaire O(n). La densité de Lagrangien la plus générale qui a ces invariances et qui est renormalisable s'écrit :

$$\mathcal{L} = -\frac{1}{4} Tr\, F_{\mu\nu}^2 + \frac{1}{2\alpha} Tr(\partial_\mu A_\mu)^2 + Tr(\partial_\mu \Phi_i)^2 + ie\, Tr\left(\partial_\mu \Phi_i^* [A_\mu, \Phi_i] - h.c.\right) + \left\{\text{interactions}\right.$$

$$\left.\text{en quatre } A_\mu \text{ ou deux } A_\mu \text{ et deux } \Phi_i \text{ ou quatre } \Phi_i\right\}. \qquad \text{(VIII.1)}$$

La théorie symétrique s'obtient comme point fixe de (VIII.1) en mettant tous les couplages des interactions dans la dernière parenthèse égaux à zéro, n =-1, e = g et α = 0. Le prolongement à n =-1 du nombre de scalaires est nécessaire pour obtenir les fantômes anticommutants, et la limite α = 0 donne la théorie symétrique dans la jauge de Landau. (C'est seulement dans cette jauge que le Lagrangien effectif de jauge de Faddeev-Popov est hermitique). En posant alors α = 0 et n = -1, nous allons examiner d'abord si le point fixe e = g est attractif infrarouge on non.

Nous voulons calculer les fonctions $-\beta$ des constantes de couplage e et g près du point fixe symétrique : $g^2 \gg (e-g)^2 \sim \lambda_i$, où λ_i représentent

les différentes constantes de couplages à quatre champs. Dans cette limite, les diagrammes qui contribuent à la fonction $-\beta$ de g sont :

et ceux qui contribuent à la fonction $-\beta$ de e sont :

Dans la jauge de Landau ($\alpha = 0$), les deux derniers diagrammes sont finis, c'est pourquoi la fonction $-\beta$ de e est donnée seulement par le propagateur de Φ_i. Le résultat du calcul est :

$$16\pi^2 \beta_g = -\frac{29}{4} C g^3 - \frac{1}{4} C e^2 g + \frac{1}{12} C e^3 \qquad \text{(VIII.2a)}$$

$$16\pi^2 \beta_e = -\frac{19}{12} C e^3 - \frac{25}{12} C g^2 e \qquad \text{(VIII.2b)}$$

où C est le casimir quadratique du groupe (par ex. N pour SU(N)). En définissant la constante $\eta = e/g - 1$, il est facile de trouver que la fonction $-\beta$ de η est donnée par

$$16\pi^2 \beta_\eta = -\frac{35}{12} C \eta g^2 + O(\eta^2) \qquad \text{(VIII.3)}$$

résultat qui montre que la constante de couplage e ne devient pas égale à g dans la limite infrarouge. Par conséquent, la théorie de jauge symétrique n'est pas un point fixe attractif pour tous les groupes. La conclusion est la même quelle que soit la procédure utilisée.

CONCLUSION

Dans cette thèse, nous avons examiné un ensemble de problèmes concernant des applications des théories de jauge à la physique des particules élémentaires. Nous avons vu qu'aux énergies actuelles, toutes les interactions fondamentales entre les particules élémentaires sont décrites d'une façon satisfaisante par le modèle standard, qui est une théorie de jauge basée sur le groupe de symétrie $U(1) \times SU(2) \times SU(3)$. Nous nous sommes intéressés à des extensions de ce modèle pour essayer de comprendre de façon plus profonde les lois de la nature.

Nous avons étudié deux possibilités dans la direction d'unification des interactions fondamentales entre les particules élémentaires : les théories grandement unifiées, et la supersymétrie. Nous avons présenté leurs motivations et leurs propriétés principales, ainsi qu'un ensemble de travaux que nous avons fait dans leur cadre. Nous avons conclu qu'elles ont des prédictions théoriques précises intéressantes, qui peuvent déjà être confrontées à l'expérience, comme par exemple la valeur de $\sin^2\theta_w$. Il faudra cependant la nouvelle génération d'expériences avec les grands accélérateurs pour pouvoir tester la plupart des phénomènes nouveaux, prédits par ces théories. Ainsi des perspectives intéressantes sont offertes aux théoriciens.

Malgré les avantages de principe de tous ces modèles généralisant le modèle minimal $U(1) \times SU(2) \times SU(3)$, ils introduisent un certain nombre de difficultés et de problèmes nouveaux que nous avons discutés. A la fin de cette thèse nous avons examiné une autre possibilité, assez spéculative pour l'instant, selon laquelle les symétries de la nature pourraient s'obtenir comme points fixes attracteurs infrarouges, provenant d'une théorie chaotique à très haute énergie.

En conclusion, au-delà du modèle standard beaucoup de directions nouvelles sont ouvertes, le formalisme de base restant celui des théories de jauge. Aucun modèle satisfaisant à tout point de vue n'a encore

été construit. En outre, on n'a pas trouvé de façon convaincante d'unifier les interactions courantes avec la gravitation, dont la quantification est un problème encore non résolu. L'espoir, partagé actuellement par la communauté des physiciens, est que la supergravité permettra de réaliser le but ultime d'unification de toutes les interactions fondamentales entre les particules élémentaires.

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RESUME

Dans cette thèse nous étudions des extensions du modèle standard $U(1) \times SU(2) \times SU(3)$, le formalisme de base étant celui des théories de jauge. Nous considérons deux possibilités d'unification des interactions fondamentales entre les particules élementaires, la "grande unification" et la supersymétrie, et nous présentons un ensemble de travaux dans leur cadre.

Nous donnons une description précise du phénomène de la brisure spontanée de symétrie et nous calculons les prédictions de la théorie unifiée minimale SU(5) (ordinaire ou supersymétrique) pour le temps de vie du proton et la valeur de l'angle faible θ_w. Nous examinons le problème de la violation forte de la symétrie CP et nous donnons des arguments qui excluent l'existence de l'axion à la Peccei - Quinn. Nous étudions une symétrie d'origine supersymétrique entre les quarks et les gluons ainsi que la renormalisation de la supersymétrie N=1. Nous calculons les prédictions principales des théories supersymétriques pour la diffusion profondément inélastique, la production des "jets" à grand moment transverse dans les réactions proton-antiproton, et les effets de polarisation dans le processus de production d'un photon direct.

A la fin de cette thèse nous présentons l'idée de l'"anti-grande unification" et nous étudions la stabilité infrarouge des théories de jauge.

MOTS CLES

Théorie de jauge , Unification, Symétrie, Renormalisation.